ASSIGNMENT 3

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Faculty of Engineering

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Question 1

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x-y \\ x+3y \end{bmatrix}$$

let $T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2(0)-0 \\ 0+3(0) \end{bmatrix}$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

since $T(0) = 0$, hence T is linear transformation.

need to check $T(u+v) = T(u)+T(v)$

$T(ku) = kT(u)$

Question 2

let $u_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$, $v = \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$.

$$T(u) + T(v) = T\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + T\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+y_1-z_1 \\ 2x_1y_1 \\ x_1+z_1+1 \end{bmatrix} + \begin{bmatrix} x_2+y_2-z_2 \\ 2x_2y_2 \\ x_2+z_2+1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+x_2+y_1+y_2-z_1-z_2 \\ 2x_1y_1+2x_2y_2 \\ x_1+x_2+z_1+z_2+2 \end{bmatrix}$$

$$T(u+v) = T\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right)$$

$$= T\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+x_2+y_1+y_2-z_1-z_2 \\ 2(x_1+x_2)(y_1+y_2) \\ x_1+x_2+z_1+z_2+1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+x_2+y_1+y_2-z_1-z_2 \\ 2x_1y_1+2x_1y_2+2x_2y_1+2x_2y_2 \\ x_1+x_2+z_1+z_2+1 \end{bmatrix}$$

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let c be any real number,

$$cT(u) = c\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = c\begin{bmatrix} x_1+y_1-z_1 \\ 2x_1y_1 \\ x_1+z_1+1 \end{bmatrix} = \begin{bmatrix} cx_1+cy_1-cz_1 \\ 2cx_1y_1 \\ cx_1+cz_1+c \end{bmatrix}$$

$$T(cu) = T\begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \end{bmatrix} = \begin{bmatrix} cx_1+cy_1-cz_1 \\ 2c^2x_1y_1 \\ cx_1+cz_1+1 \end{bmatrix}$$

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$\therefore$ since $T(u)+T(v) \neq T(u+v)$ and $cT(u) \neq T(cu)$, hence T is not linear transformation.

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Question 3

a) let $u_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $u_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $u_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$T(e_1) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \checkmark$$

$$T(e_2) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$
$$= \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \checkmark$$

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$$T(e_3) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$
$$= \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \checkmark$$

$\therefore$ $T(e_1) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$, $T(e_2) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$, $T(e_3) = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$

b) $T(3e_1 - 4e_2 + 6e_3) = 3T(e_1) - 4T(e_2) + 6T(e_3)$

$$= 3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} + \begin{bmatrix} 18 \\ 18 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 20 \\ 3 \end{bmatrix}$$

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Question 4

$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ -2x+4y \end{bmatrix}$, $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

let $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1-2(3) \\ -2(1)+4(3) \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 10 \end{bmatrix}$

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since $T(v) \neq 0$, hence $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is not in $N(T)$.

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Question 5

a) let
$$c_1\begin{bmatrix}-1\\2\\1\end{bmatrix}+c_2\begin{bmatrix}0\\5\\0\end{bmatrix}+c_3\begin{bmatrix}-1\\-1\\2\end{bmatrix}=\begin{bmatrix}-2\\1\\2\end{bmatrix}$$

$-c_1-c_3=-2$, $2c_1+5c_2-c_3=1$, $c_1+2c_3=2$

$c_1=2-2c_3$ — ①

subs ①

$-(2-2c_3)-c_3=-2$

$-2+4c_3-c_3=-2$

$3c_3=0$

$c_3=0$

when $c_3=0$,

$c_1=2-2(0)$

$c_1=2$

when $c_1=2$, $c_3=0$,

$2(2)+5c_2-0=1$

$4+5c_2=1$

$5c_2=-3$

$c_2=-\frac{3}{5}$

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since $c_1=2$, $c_2=-\frac{3}{5}$, $c_3=0$, exist a solution , Thus $w\in R(T)$

b)
$$\begin{bmatrix}-1&0&-1\\2&5&-1\\1&0&2\end{bmatrix}\xrightarrow{R_1\to -R_1}\begin{bmatrix}1&0&1\\2&5&-1\\1&0&2\end{bmatrix}$$

$$\xrightarrow{R_2\to R_2-2R_1}\begin{bmatrix}1&0&1\\0&5&-3\\1&0&2\end{bmatrix}$$

$$\xrightarrow{R_2\to \frac{1}{5}R_2}\begin{bmatrix}1&0&1\\0&1&-\frac{3}{5}\\1&0&2\end{bmatrix}$$

$$\xrightarrow{R_3\to R_3-R_1}\begin{bmatrix}1&0&1\\0&1&-\frac{3}{5}\\0&0&1\end{bmatrix}$$

since leading 1s are in columns 1, 2 and 3 , basic for $R(T)$ is

$$R(T)=\text{span}\left\{\begin{bmatrix}-1\\2\\1\end{bmatrix},\begin{bmatrix}0\\5\\0\end{bmatrix},\begin{bmatrix}-1\\-1\\2\end{bmatrix}\right\}$$

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e) let $c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{R_1 \to -R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 5 & -3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & -\frac{3}{5} & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \to R_2 + \frac{3}{5}R_3]{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ unique solution for all choices of c_1, c_2 and c_3.

$\therefore$ Hence, no vector span in $N(T)$.

$\therefore$ no vector as basis for $N(T)$.

$\therefore$ $\dim(N(T)) = 0$

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