ASSIGNMENT 2

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Question 1

1) Axiom 3 : $(u+v)+w = u+(v+w)$

let $u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$, $v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$, $w = \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$

$$(u+v)+w = \left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} \right) + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2+t_3) \\ 2-(t_1+t_2+t_3) \\ 3+(2t_1-2t_2-2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+(t_2+t_3) \\ 2-(t_2+t_3) \\ 3+(2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \left(\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix} \right)$$

$$= u+(v+w)$$

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$\therefore$ Addition is associative.

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Axiom 7: $k(u+v) = ku + kv$

let $u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$, $v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$

$$k(u+v) = k\left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}\right)$$

$$= k\begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+k(t_1+t_2) \\ 2-k(t_1+t_2) \\ 3+k(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(kt_1+kt_2) \\ 2-(kt_1+kt_2) \\ 3+(2kt_1+2kt_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+kt_1 \\ 2-kt_1 \\ 3+2kt_1 \end{bmatrix} + \begin{bmatrix} 1+kt_2 \\ 2-kt_2 \\ 3+2kt_2 \end{bmatrix}$$

$$= k\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + k\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$$

$$= ku + kv$$

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Axiom 8 : $(k+m)u = ku + mu$

let $u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$

$$(k+m)u = (k+m)\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(k+m)t_1 \\ 2-(k+m)t_1 \\ 3+(k+m)2t_1 \end{bmatrix}$$

$$ku+mu = k\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + m\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+kt_1 \\ 2-kt_1 \\ 3+2kt_1 \end{bmatrix} + \begin{bmatrix} 1+mt_1 \\ 2-mt_1 \\ 3+2mt_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(kt_1+mt_1) \\ 2-(kt_1+mt_1) \\ 3+(2kt_1+2mt_1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(k+m)t_1 \\ 2-(k+m)t_1 \\ 3+(k+m)2t_1 \end{bmatrix}$$

$$= (k+m)u$$

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$\therefore$ V is a vector space.

Question 2

a) let $u = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix}$

$V = \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 - c_2 & a_2 - c_2 \end{bmatrix}$

$u + v = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix} + \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 - c_2 & a_2 - c_2 \end{bmatrix}$

$= \begin{bmatrix} (a_1 - b_1) + (a_2 - b_2) & a_1 + a_2 \\ (b_1 + c_1) + (b_2 + c_2) & (a_1 - c_1) + (a_2 - c_2) \end{bmatrix}$

$\therefore$ S is closed under addition.

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Let k be any real number

$ku = k \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix}$

$= \begin{bmatrix} k(a_1 - b_1) & ka_1 \\ k(b_1 + c_1) & k(a_1 - c_1) \end{bmatrix}$

$= \begin{bmatrix} ka_1 - kb_1 & ka_1 \\ kb_1 + kc_1 & ka_1 - kc_1 \end{bmatrix}$

$\therefore$ S is closed under multiplication.

b) let $\begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$

$a - b = 5$, $a = 3$, $b + c = -2$, $a - c = 3$

when $a = 3$, when $b = -2$, when $a = 3$, $c = 0$

$3 - b = 5$ $\quad$ $-2 + c = -2$ $\quad$ $3 - 0 = 3$ $(=3)$

$b = -2$ $\quad$ $c = 0$

$\therefore \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S.

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Question 3

a) Let

$$c_1\begin{bmatrix}1\\2\\3\end{bmatrix}+c_2\begin{bmatrix}-1\\3\\-1\end{bmatrix}+c_3\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\6\\1\end{bmatrix}+c_5\begin{bmatrix}-3\\4\\5\end{bmatrix}=\begin{bmatrix}a\\b\\c\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\2&3&2&6&4&b\\3&-1&-1&1&5&c\end{array}\right]\xrightarrow{R_2\to R_2-2R_1}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\0&5&0&6&10&b-2a\\3&-1&-1&1&5&c\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-3R_1}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\0&5&0&6&10&b-2a\\0&2&-4&1&14&c-3a\end{array}\right]$$

$$\xrightarrow{R_2\to R_2-2R_3}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\0&1&8&4&-18&b+4a-2c\\0&2&-4&1&14&c-3a\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-2R_2}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\0&1&8&4&-18&b+4a-2c\\0&0&-20&-7&50&5c-2b-11a\end{array}\right]$$

$$\xrightarrow{R_3\to -\frac{1}{20}R_3}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&a\\0&1&8&4&-18&b+4a-2c\\0&0&1&\frac{7}{20}&\frac{5}{2}&\frac{2b+11a-5c}{20}\end{array}\right]$$

c_4 and c_5 are leading variable, hence let $c_4 = x_1$, $c_5 = x_2$.

$$c_3+\frac{7}{20}c_4+\frac{5}{2}c_5=\frac{2b+11a-5c}{20}$$

$$c_3+\frac{7}{20}x_1+\frac{5}{2}x_2=\frac{2b+11a-5c}{20}$$

$$c_3=\frac{2b+11a-5c}{20}-\frac{7}{20}x_1-\frac{5}{2}x_2$$

$$c_3=\frac{2b+11a-5c-7x_1-50x_2}{20} \quad \text{— (1)}$$

$$c_2+8c_3+4c_4-18c_5=b+4a-2c \quad \text{— (2)}$$

subs (1), x_1, x_2 into (2)

$$c_2+8\left(\frac{2b+11a-5c-7x_1-50x_2}{20}\right)+4x_1-18x_2=b+4a-2c$$

$$c_2+\left(\frac{4b+22a-10c-14x_1-100x_2}{5}\right)=b+4a-2c-4x_1+18x_2$$

$$c_2=b+4a-2c-4x_1+18x_2-\left(\frac{4b+22a-10c-14x_1-100x_2}{5}\right)$$

$$c_2=\frac{5b+20a-10c-20x_1+90x_2-4b-22a+10c+14x_1+100x_2}{5}$$

$$c_2=\frac{b-2a-6x_1+190x_2}{5} \quad \text{— (3)}$$

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$c_1 - c_2 + c_3 - 3c_5 = a$ — (4)

subs (1), (3), x_1 and x_2 into (4)

$$c_1 - \left(\frac{b - 2a - 6x_1 + 190x_2}{5}\right) + \left(\frac{2b + 11a - 5c - 7x_1 - 50x_2}{20}\right) - 3x_2 = a$$

$20c_1 - (4b - 8a - 24x_1 + 760x_2) + (2b + 11a - 5c - 7x_1 - 50x_2 - 60x_2 = 20a$

$20c_1 - 4b + 8a + 24x_1 - 760x_2 + 2b + 11a - 5c - 7x_1 - 50x_2 - 60x_2 = 20a$

$20c_1 - 2b + 19a - 5c + 17x_1 - 870x_2 = 20a$

$20c_1 = a + 2b + 5c - 17x_1 + 870x_2$

$$c_1 = \frac{a + 2b + 5c - 17x_1 + 870x_2}{20}$$

$\therefore$ show that a solution exists for all choice of a, b, c

Thus, span $(S) = \mathbb{R}^3$.

b) let $\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 2 & 3 & 2 & 6 & 4 \\ 3 & -1 & -1 & 1 & 5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 2 & 3 & 2 & 6 & 4 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 0 & 2 & -4 & 1 & 14 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 2R_3} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 1 & 8 & 4 & -18 & 0 \\ 0 & 2 & -4 & 1 & 14 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 1 & 8 & 4 & -18 & 0 \\ 0 & 0 & -20 & -7 & 50 & 0 \end{array}\right]$$

c_4 and c_5 are leading variable, let $c_4 = a$, $c_5 = b$

$-20c_3 - 7c_4 + 50c_5 = 0$

$-20c_3 - 7a + 50b = 0$

$-20c_3 = 7a - 50b$

$c_3 = \frac{50b - 7a}{20}$

$c_2 + 8c_3 + 4c_4 - 18c_5 = 0$

$c_2 + 8\left(\frac{50b - 7a}{20}\right) + 4a - 18b = 0$

$20c_2 + 8(50b - 7a) + 80a - 360b = 0$

$20c_2 + 400b - 56a + 80a - 360b = 0$

$20c_2 = -40b - 24a$

$c_2 = \frac{-40b - 24a}{20}$

$c_1 - c_2 + c_3 - 3c_5 = 0$

$c_1 - \left(\frac{-40b - 24a}{20}\right) + \left(\frac{50b - 7a}{20}\right) - 3b = 0$

$20c_1 + 40b + 24a + 50b - 7a - 60b = 0$

$20c_1 + 30b + 17a = 0$

$c_1 = \frac{-30b - 17a}{20}$

$\therefore$ Span $(S) = \mathbb{R}^3$,

S is not linearly independent since does not exists a trivial solution.

$\therefore$ S is not basis of $\mathbb{R}^3$

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Question 4

a) let $c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

$$\left[\begin{array}{cc|c}1 & 3 & 1\\1 & -1 & 0\end{array}\right]\xrightarrow{R_2\to R_1-R_2}\left[\begin{array}{cc|c}1 & 3 & 1\\0 & 4 & 1\end{array}\right]$$

$4c_2=1$ $\quad$ $c_1+3c_2=1$

$c_2=\frac{1}{4}$ $\quad$ when $c_2=\frac{1}{4}$

$c_1+3(\frac{1}{4})=1$

$c_1=\frac{1}{4}$

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$\therefore \frac{1}{4}\begin{bmatrix}1\\1\end{bmatrix}+\frac{1}{4}\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

coordinates of $\begin{bmatrix}1\\0\end{bmatrix}$ relative to B_2 is $\begin{bmatrix}c_1\\c_2\end{bmatrix}=\begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\end{bmatrix}$

b) let $c_1\begin{bmatrix}1\\-1\end{bmatrix}+c_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$, $d_1\begin{bmatrix}1\\-1\end{bmatrix}+d_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$

$c_1=1$, $-c_1+2c_2=1$ $\quad$ $d_1=3$, $-d_1+2d_2=-1$

when $c_1=1$ $\quad$ when $d_1=3$

$-1+2c_2=1$ $\quad$ $-3+2d_2=-1$

$2c_2=2$ $\quad$ $2d_2=2$

$c_2=1$ $\quad$ $d_2=1$

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$\therefore c_1=1,\ c_2=1,\ d_1=3,\ d_2=1$

$[v_1]_{B_1}=\begin{bmatrix}1\\1\end{bmatrix}$, $[v_2]_{B_2}=\begin{bmatrix}3\\1\end{bmatrix}$

$\therefore$ transition matrix $[I]^{B_1}_{B_2}=\begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}$

c) $[v_1]_{B_1}=[I]^{B_1}_{B_2}[v]_{B_2}$

$[v_1]_{B_1}=\begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$

$[v_1]_{B_1}=\begin{bmatrix}11\\5\end{bmatrix}$

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