ASSIGNMENT 1

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Question 1

point: (1, −1)

$x = 1, \quad y = -1$

subs into $2x - y = 3$

$2(1) - (-1) = 3 \ (=3)$

$\therefore (1,-1)$ is a solution to $2x - y = 3$

subs into $x + 3y = 5$

$1 + 3(-1) = -2 \ (\neq 5)$

$\therefore (1,-1)$ is not a solution to $x + 3y = 5$

point (2, 1)

$x = 2, \quad y = 1$

subs into $2x - y = 3$

$2(2) - 1 = 3 \ (=3)$

$\therefore (2,1)$ is a solution to $2x - y = 3$

subs into $x + 3y = 5$

$2 + 3(1) = 5 \ (=5)$

$\therefore (2,1)$ is a solution to $x + 3y = 5$

Question 2

a) $x - y = 2$, $2x - 2y = 4$

$$\begin{bmatrix} 1 & -1 \\ 2 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 2 \\ 2 & -2 & 4 \end{array}\right] \xrightarrow{R_2 \to 2R_1 - R_2} \left[\begin{array}{cc|c} 1 & -1 & 2 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore$ y is a free variable. The final row of row reduced matrix is equal to 0. hence, the system linear equation has many solution.

b) $x - y = 1$

$x - y = 3$

$$\begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 1 & -1 & 1 \\ 1 & -1 & 3 \end{array}\right] \xrightarrow{R_2 \to R_1 - R_2} \left[\begin{array}{cc|c} 1 & -1 & 1 \\ 0 & 0 & -2 \end{array}\right]$$

$\therefore$ final row states than $0x + 0y = -2$. However 0 cannot equal to 2.
Hence the system linear equation has no solution.

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Qustion 3

a) $\begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix}$

$$\det \begin{bmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{bmatrix} = (-1)^{1+1}(1)\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} + (-1)^{1+2}(0)\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + (-1)^{1+3}(3)\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= (1)(1)(2-1) + (-1)(0)(10-0) + (1)(3)(5-0)$$

$$= 1 + 0 + 15$$

$$= 16$$

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b) $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix}$

$$\det \begin{bmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{bmatrix} = (-1)^{1+1}(1)\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} + (-1)^{1+2}(-1)\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + (-1)^{1+3}(0)\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= (1)(1)(0-1) + (-1)(-1)(1-0) + (1)(0)(-1-0)$$

$$= -1 + 1 + 0$$

$$= 0$$

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Question 4

a) let $A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{3(1)-(0)(0)} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$$

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b) let $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$$\det A = 1 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(1-0) - 0 + 0$$

$$= 1$$

$\det A_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$ $\quad$ $\det A_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$ $\quad$ $\det A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} = 0$

$\det A_{12} = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = 0$ $\quad$ $\det A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ $\quad$ $\det A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = -2$

$\det A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$ $\quad$ $\det A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$ $\quad$ $\det A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

cofactor matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ $\quad$ Adjoint $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

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Question 5

a) $x_1 + 2x_2 - 3x_3 = 9$

$2x_1 - x_2 + x_3 = 0$

$4x_1 - x_2 + x_3 = 4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \to 2R_1 - R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 5 & -7 & 18 \\ 4 & -1 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to 4R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 5 & -7 & 18 \\ 0 & 9 & -13 & 32 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 1 & -7/5 & 18/5 \\ 0 & 9 & -13 & 32 \end{array}\right]$$

$$\xrightarrow{R_3 \to 9R_2 - R_3} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 1 & -7/5 & 18/5 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} \end{array}\right]$$

$\frac{2}{5}x_3 = \frac{2}{5}$

$x_3 = 1$

$x_2 - \frac{7}{5}x_3 = \frac{18}{5}$

when $x_3 = 1$

$x_2 - \frac{7}{5}(1) = \frac{18}{5}$

$x_2 = 5$

$x_1 + 2x_2 - 3x_3 = 9$

when $x_3 = 1$, $x_2 = 5$

$x_1 + 2(5) - 3(1) = 9$

$x_1 = 2$

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$x_1 = 2$

$x_2 = 5$

$x_3 = 1$

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b) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow{R_2 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$\xrightarrow{R_3 \to 3R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 10 & 2 & 14 \end{array}\right]$$

$$\xrightarrow{R_3 \to 10R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & 52 & 104 \end{array}\right]$$

$52x_3 = 104$

$x_3 = 2$

$-x_2 + 5x_3 = 9$

when $x_3 = 2$

$-x_2 + 5(2) = 9$

$x_2 = 1$

$x_1 + x_2 + 2x_3 = 8$

when $x_3 = 2$, $x_2 = 1$

$x_1 + 1 + 2(2) = 8$

$x_1 = 3$

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$\therefore$ $x_1 = 3$

$x_2 = 1$

$x_3 = 2$

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Question 6

a) $x_1 - 3x_2 - 2x_3 = 0$

$-x_1 + 2x_2 + x_3 = 0$

$2x_1 + 4x_2 + 6x_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow{R_2 \to R_1 + R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to 2R_1 - R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -10 & -10 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to 10R_2 + R_3} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to 3R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_1 + x_3 = 0$, x_3 is a free variable.

$x_2 + x_3 = 0$

let $x_3 = a$

$x_1 + a = 0$ $\qquad x_2 + a = 0$

$x_1 = -a$ $\qquad x_2 = -a$

$\therefore x_1 = -a, \; x_2 = -a, \; x_3 = a$, where $a \in \mathbb{R}$

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b) $2r + s = 3$

$4r + s = 7$

$2r + 5s = -1$

$$\left[\begin{array}{ccc|c} 2 & 1 & 0 & 3 \\ 4 & 1 & 0 & 7 \\ 2 & 5 & 0 & -1 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & \frac{3}{2} \\ 4 & 1 & 0 & 7 \\ 2 & 5 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{R_2 \to 4R_1 - R_2} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & \frac{3}{2} \\ 0 & 1 & 0 & -1 \\ 2 & 5 & 0 & -1 \end{array}\right]$$

$$\xrightarrow{R_3 \to 2R_1 - R_3} \left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & \frac{3}{2} \\ 0 & 1 & 0 & -1 \\ 0 & -4 & 0 & 4 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - \frac{1}{2}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & -4 & 0 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 4R_2} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \quad r = 2$

$s = -1$

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Question 7

$2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

a) $$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$U_{11} = 2, \quad U_{12} = 1, \quad U_{13} = 4$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$2l_{21} = 8 \qquad 2l_{31} = 4$

$l_{21} = 4 \qquad l_{31} = 2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & l_{32} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$4 + U_{22} = -3 \qquad 16 + U_{23} = 2$

$U_{22} = -7 \qquad U_{23} = -14$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & l_{32} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$2 - 7l_{32} = 11$

$-7l_{32} = 9$

$l_{32} = -\frac{9}{7}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$8 + 18 + U_{33} = -1$

$U_{33} = -27$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$x_2 = 12, \quad 4x_2 + y_2 = 20, \quad 2x_2 - \frac{9}{7}y_2 + z_2 = 33$

$4(12) + y_2 = 20 \qquad 2(12) - \frac{9}{7}(-28) + z_2 = 33$

$y_2 = -28 \qquad z_2 = -27$

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$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$-27z_1 = -27$, $-7y_1 - 14z_1 = -28$, $2x_1 + y_1 + 4z_1 = 12$

$z_1 = 1$ $\quad$ $-7y_1 - 14(1) = -28$ $\quad$ $2x_1 + 2 + 4(1) = 12$

$y_1 = 2$ $\quad$ $x_1 = 3$

$\therefore \; x = 3$

$y = 2$

$z = 1$

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b) $\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$

$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\quad l_{11} = 2, \; l_{21} = 8, \; l_{31} = 4$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & l_{22} & 0 \\ 4 & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\quad 2u_{12} = 1, \; u_{12} = \frac{1}{2}$ $\quad , \; 2u_{13} = 4, \; u_{13} = 2$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & l_{22} & 0 \\ 4 & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\quad 8(\frac{1}{2}) + l_{22} = -3, \; l_{22} = -7$ $\quad , \; 4(\frac{1}{2}) + l_{32} = 11, \; l_{32} = 9$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & l_{33} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\quad 8(2) + (-7)u_{23} = 2, \; u_{23} = 2$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & l_{33} \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$ $\quad 4(2) + 9(2) + l_{33} = -1, \; l_{33} = -27$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$

$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$

$2x_2 = 12$, $8x_2 - 7y_2 = 20$, $4x_2 + 9y_2 - 27z_2 = 33$

$x_2 = 6$ $\quad 8(6) - 7y_2 = 20$ $\quad 4(6) + 9(4) - 27z_2 = 33$

$y_2 = 4$ $\quad z_2 = 1$

$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$

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$z_1 = 1$, $y_1 + 2z_1 = 4$, $x_1 + \frac{1}{2}y_1 + 2z_1 = 6$

$y_1 + 2(1) = 4$ $\quad x_1 + \frac{1}{2}(2) + 2(1) = 6$

$y_1 = 2$ $\quad x_1 = 3$

$\therefore x = 3$

$y = 2$

$z = 1$

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Question 8

$d(u,v) = \sqrt{(\sqrt{2}-0)^2 + (1-2)^2 + (-1-2)^2}$

$= 2\sqrt{3}$

Question 9

a) $u \cdot v = (2)(3) + (1)(3) + (2)(1)$

$= 6 + 3 + 2$

$= 11$

b) $u \times v = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} k$

$= (1-6)i - (2-6)j + (6-3)k$

$= -5i - (-4)j + 3k$

$= -5(1\ 0\ 0) - (-4)(0\ 1\ 0) + 3(0\ 0\ 1)$

$= (-5\quad 4\quad 3)$

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Question 10

a)
$$c_1\begin{bmatrix}1\\1\\1\end{bmatrix}+c_2\begin{bmatrix}1\\2\\3\end{bmatrix}+c_3\begin{bmatrix}2\\-1\\1\end{bmatrix}=\begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$$\begin{bmatrix}1&1&2\\1&2&-1\\1&3&1\end{bmatrix}\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&1&2&1\\1&2&-1&-2\\1&3&1&5\end{array}\right]\xrightarrow{R_2\to R_2-R_1}\left[\begin{array}{ccc|c}1&1&2&1\\0&1&-3&-3\\1&3&1&5\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-R_1}\left[\begin{array}{ccc|c}1&1&2&1\\0&1&-3&-3\\0&2&-1&4\end{array}\right]$$

$$\xrightarrow{R_3\to R_3-2R_2}\left[\begin{array}{ccc|c}1&1&2&1\\0&1&-3&-3\\0&0&5&10\end{array}\right]$$

$$\xrightarrow{R_3\to \frac{1}{5}R_3}\left[\begin{array}{ccc|c}1&1&2&1\\0&1&-3&-3\\0&0&1&2\end{array}\right]$$

$$\xrightarrow{R_1\to R_1-R_2}\left[\begin{array}{ccc|c}1&0&5&4\\0&1&-3&-3\\0&0&1&2\end{array}\right]$$

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$$\xrightarrow{R_1\to R_1-5R_3}\left[\begin{array}{ccc|c}1&0&0&-6\\0&1&-3&-3\\0&0&1&2\end{array}\right]$$

$$\xrightarrow{R_2\to R_2+3R_3}\left[\begin{array}{ccc|c}1&0&0&-6\\0&1&0&3\\0&0&1&2\end{array}\right]$$

$c_1=-6$, $c_2=3$, $c_3=2$

$\therefore$ v is linear combination to u_1, u_2, u_3.

b)
$$c_1\begin{bmatrix}1\\2\\5\end{bmatrix}+c_2\begin{bmatrix}2\\5\\1\end{bmatrix}+c_3\begin{bmatrix}1\\5\\2\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

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$c_1+2c_2+c_3=0$, $2c_1+5c_2+5c_3=0$, $5c_1+c_2+2c_3=0$

$c_1=-2c_2-c_3$

when $c_3=0$, $c_2=0$

$c_1=0$

when $c_1=-2c_2-c_3$

$2(-2c_2-c_3)+5c_2+5c_3=0$

$-4c_2-2c_3+5c_2+5c_3=0$

$c_2=-3c_3$

when $c_3=0$

$c_2=0$

$5(-2c_2-c_3)+(-3c_3)+2c_3=0$

$-10c_2-5c_3-3c_3+2c_3=0$

$-10(-3c_3)-6c_3=0$

$30c_3-6c_3=0$

$24c_3=0$

$c_3=0$

$\therefore$ since $c_1=c_2=c_3=0$, hence $u=(1,2,5)$, $v=(2,5,1)$, $w(1,5,2)$ are linearly independent