

## Assignment 1 (Chapter 1 and 2)

### Question 1

Determine whether either of the points  $(1, -1)$  and  $(2, 1)$  is a solution to the given system of equations.

$$2x - y = 3$$

$$x + 3y = 5$$

### Question 2

Solve the following systems of linear equations:

$$\begin{aligned} \text{a) } x - y &= 2 \\ 2x - 2y &= 4 \end{aligned}$$

$$\begin{aligned} \text{b) } x - y &= 1 \\ x - y &= 3 \end{aligned}$$

### Question 3

Compute the determinants using cofactor expansion along the first row

a)

$$\begin{vmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}$$

b)

$$\begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}$$

### Question 4

Find the inverse of the matrix below.

a)

$$\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$$

b)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

### Question 5

Solve the given system of equations using Gaussian Elimination

a)

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 9 \\ 2x_1 - x_2 + x_3 &= 0 \\ 4x_1 - x_2 + x_3 &= 4 \end{aligned}$$

b)

$$\begin{aligned} x_1 + x_2 + 2x_3 &= 8 \\ -x_1 - 2x_2 + 3x_3 &= 1 \\ 3x_1 - 7x_2 + 4x_3 &= 10 \end{aligned}$$

**Question 6**

Solve the given system of equations using Gauss-Jordan elimination

a)

$$\begin{aligned}x_1 - 3x_2 - 2x_3 &= 0 \\ -x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + 4x_2 + 6x_3 &= 0\end{aligned}$$

b)

$$\begin{aligned}2r + s &= 3 \\ 4r + s &= 7 \\ 2r + 5s &= -1\end{aligned}$$

**Question 7**

Solve the following system using:

$$\begin{aligned}\text{a) Doolittle} \quad & 2x + y + 4z = 12 \\ \text{b) Crout} \quad & 8x - 3y + 2z = 20 \\ & 4x + 11y - z = 33\end{aligned}$$

**Question 8**

Find the distance between **u** and **v** below

$$u = \begin{bmatrix} \sqrt{2} \\ 1 \\ -1 \end{bmatrix}, \quad v = \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix}$$

**Question 9**

Let  $u = (2, 1, 2)$  and  $v = (3, 3, 1)$ . Find

- a)  $u \cdot v$
- b)  $u \times v$

**Question 10**

a) Determine whether  $v = (1, -2, 5)$  in  $\mathbb{R}^3$  is a linear combination of the following vectors

$$u_1 = (1, 1, 1), \quad u_2 = (1, 2, 3), \quad u_3 = (2, -1, 1)$$

b) Determine whether the vectors in  $\mathbb{R}^3$ ,  $u = (1, 2, 5)$ ,  $v = (2, 5, 1)$ ,  $w = (1, 5, 2)$  are linearly independent?