ASSIGNMENT 3

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SECTION/COURSE CODE	**: [SECI 2143] [01] – [PROBABILITY & STATISTICAL DATA ANALYSIS]**
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Case Study Report submitted to School of Computing, Univerisiti Teknologi Malaysia in partial fulfilment to complete the SECI 2143 Assignment for Semester 2 session 2020/2021

Question 1

(a) $n=25$ $\bar{x}=19.5$ $s=9.88$

degree of freedom $=25-1$
$=24$

$\alpha = \frac{(1-0.90)}{2}$
$= 0.05$

90% confidence interval for mean $= \bar{x} \pm t_{24,0.1}\left(\frac{s}{\sqrt{n}}\right)$

$= 19.5 + 1.711\left(\frac{9.88}{\sqrt{25}}\right)$ 1

$= 19.5 \pm (3.380936)$ 2

$= 19.5 + (3.380936),\ 19.5 - (3.380936)$

$= 22.8809$ $= 16.1191$

$= (16.1191, 22.8809)$ 1

we are 90% confident that mean number is between 16.1191 and 22.8809

(b) $\hat{p} = \frac{160}{360}$, $\alpha = 0.01$ $n = 360$

$\hat{p} = 0.4444$ (4.dp) 1

$E = Z_{\frac{\alpha}{2}}\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$

$= Z_{\frac{0.01}{2}}\sqrt{\frac{0.4444(1-0.4444)}{360}}$ 2

$= 2.575\sqrt{\frac{0.444(0.5556)}{360}}$

$= 2.575(0.0262)$ 2

$= 0.06744$

$\hat{p} \pm E = 0.4444 \pm 0.06744$

$= 0.4444 + 0.06744$, $0.4444 - 0.06744$

$= 0.51184$ $= 0.3770.$

$(0.3770, 0.51184)$ 1

10

we are 99% confident that the population of food stores that offer promotions is between 0.3770 and 0.5118.

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2.

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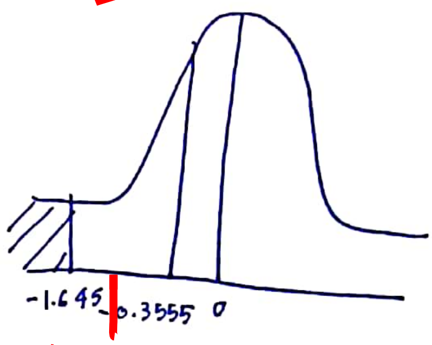
(a) $H_0: p = 0.10$ $\quad H_1: p < 0.10$ 2 $\quad \alpha = 0.50$ $\quad n = 350$ $\quad x = 33$

$\hat{p} = \frac{x}{n} = \frac{33}{350} = 0.0943$ 1 $\quad z_\alpha = -1.645$ 1

test statistic

$$z = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.0943 - 0.10}{\sqrt{\frac{(0.10)(1-0.10)}{350}}} = \frac{-0.0057}{\sqrt{\frac{9}{35000}}} = -0.3555 \ (4 \text{ d.p})$$ 2



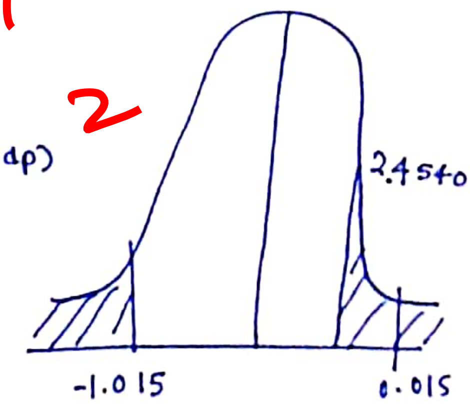
Since $z = -0.3555 > z_\alpha = -1.645$. We fail to reject the null hypothesis, H_0. we can conclude that at 0.05 significant level, there is insufficient evidence that the incorrect test results will be less than 10%. 1

(b) $H_0: \mu = 58000$ 2 $\quad \alpha = 0.01$ $\quad \sigma = 652$

$H_1: \mu \neq 58000$ $\quad \bar{x} = 58400$ $\quad n = 16$

critical value, $z_{\frac{0.01}{2}} < -2.575$ $\quad z_{\frac{0.01}{2}} > 2.575$ 1

$$z = \frac{\bar{x} - \mu}{\frac{\sigma}{\sqrt{n}}} = \frac{58400 - 58000}{\frac{652}{\sqrt{16}}} = \frac{400}{163} = 2.4540 \ (4 \text{ d.p})$$ 2



Since $z = 2.4540$ is between -2.575 and 2.575. hence, we fail to reject null hypothesis, H_0. We can conclude that at 0.01 significant level, there is sufficient evidence that the average compressive strength of steel is 58000 psi 2

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Question 3

$H_0 : \mu = 10$ ✓ 2

$H_1 : \mu \neq 10$ ✓

7.5

$$\bar{x} = \frac{\sum x}{n} = \frac{(10.3+9.9+10.2+10.1+9.7+9.7+9.8+10.3+10.0+10.4)}{10}$$

$$= \frac{100.6}{10} = 10.06$$ ✓ 1/2

$$s = \sqrt{\frac{[(10.3-10.06)^2 + (9.9-10.06)^2 + (10.2-10.06)^2 + (10.1-10.06)^2 + (9.7-10.06)^2 + (9.9-10.06)^2 + (9.8-10.06)^2 + (10.3-10.06)^2 + (10.0-10.06)^2 + (10.4-10.06)^2}{10-1}}$$

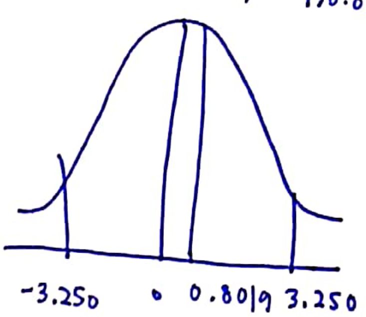
$$= \sqrt{\frac{0.0576 + 0.0256 + 0.0196 + 0.0016 + 0.1296 + 0.0256 + 0.0676 + 0.0576 + 0.0036 + 0.1156}{9}}$$

$$= \sqrt{\frac{0.504}{9}} = 0.2366$$ ✓ 1/2

degree of freedom $= 10-1 = 9$ $\alpha = 0.01$

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{10.06 - 10}{\frac{0.2366}{\sqrt{10}}} = \frac{0.06}{\frac{0.2366}{\sqrt{10}}} = 0.80193 = 0.8019 \ (4 \text{ d.p.})$$ ✓ 1/2

$t_{9,0.01} > 3.250$, $t_{9,0.01} < -3.250$



✓ 1

Since $t = 0.8019$ is between -3.250 and 3.250, hence we fail to reject null hypothesis, H_0. We can conclude that at 0.01 significance level, there is sufficient evidence that average content volume of the brand X car lubricant is 10 liters. 2

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(c)

$H_0 : \sigma = 0.44$ 　 $n = 14$

$H_1 : \sigma \neq 0.44$ 　 $\alpha = 0.05$ 　 2

7.5

$$\bar{x} = \frac{3.70 + 4.31 + 3.73 + 4.33 + 3.33 + 2.58 + 4.47 + 3.55 + 4.66 + 3.68 + 3.02 + 4.09 + 2.36 + 3.35}{14}$$

$$\bar{x} = \frac{51.1602}{14}$$

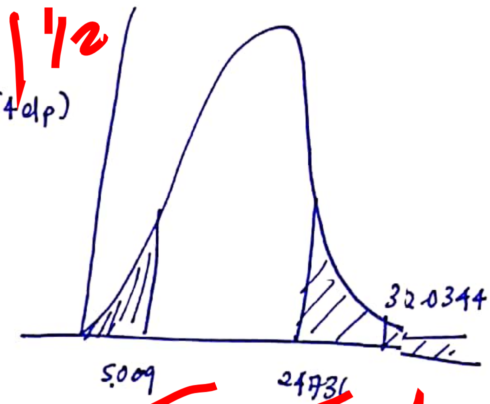
$= 3.6543$ (4.d.p) 1/2

$$s = \sqrt{\frac{(3.70-3.6543)^2 + (4.31-3.6543)^2 + (3.73-3.6543)^2 + (2.58-3.6543)^2 + (4.47-3.6543)^2 + (3.55-3.6543)^2 + (4.33-3.6543)^2 + (3.33-3.6543)^2 + (3.02-3.6543)^2 + (4.09-3.6543)^2 + (2.36-3.6543)^2 + (4.66-3.6543)^2 + (3.68-3.6543)^2 + (3.35-3.6543)^2}{14-1}}$$

$$s = \sqrt{\frac{6.20186437}{13}} = 0.6907 \text{ (4.dp)}$$ 1/2

degree of freedom $= 14-1$
$= 13$

$$x^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{(13)(0.6907)^2}{(0.44)^2} = \frac{6.20186437}{0.1936} = 32.0344 \text{ (4 dp)}$$ 1 1/2



1

since $x^2 = 32.0344 > \chi^2_{13, 0.025} = 24.736$

we will reject the null hypothesis, H_0. We can conclude that at 0.05 significance level, there is sufficient evidence that the vitamin supplement has many effect in standard deviation of the birth weight. 2

7.5

Question 4

$H_0 : \sigma^2 = 0.18$ 2

$H_1 : \sigma^2 \neq 0.18$

$\alpha = (1-0.95)$, $n = 101$, $s^2 = 0.165$, $\sigma^2 = 0.18$
$= 0.05$

$df = 101 - 1$
$= 100$

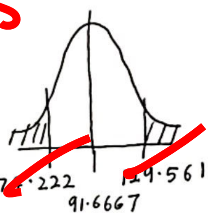
chi square

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(101-1)(0.165)}{0.18}$$

$$= \frac{16.5}{0.18} = 91.6667$$

2.5



1

Since 91.6667 does not fall between $\chi^2_{100, 1-0.025} < 74.222$ and $\chi^2_{100, 0.025} > 129.561$, hence we fail to reject Ho. Therefore we can conclude that at 0.05 significance level, there is sufficient evidence that the new joint inspector is making satisfactory measurements.

2

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Question 5

$H_0: \mu_1 = \mu_2$

$H_1: \mu_1 \neq \mu_2$ 2

$\alpha = 0.05$, $s_1^2 = 25.30$, $s_2^2 = 22.20$

$n_1 = 15$, $n_2 = 18$

$\bar{x}_1 = 76.40$, $\bar{x}_2 = 71.20$

degree of freedom , $v = \dfrac{\left(\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}\right)^2}{\left[\dfrac{(s_1^2/n_1)^2}{n_1 - 1} + \dfrac{(s_2^2/n_2)^2}{n_2 - 1}\right]}$

$$v = \frac{\left(\dfrac{25.30}{15} + \dfrac{22.20}{18}\right)^2}{\left[\dfrac{(25.30/15)^2}{15-1} + \dfrac{(22.20/18)^2}{18-1}\right]}$$ 5

$$v = \frac{8.5264}{0.2927}$$

$v = 29.1321$ (4 d.p)

$v \approx 29$

$t_{0.05/2,\,29} = 2.045$, reject H_0 if $t_0^* < -2.045$, $t_0^* > 2.045$ 1

$$t_0^* = \frac{\bar{x}_1 - \bar{x}_2 - 0}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}}$$ 5

$$t_0^* = \frac{76.40 - 71.20 - 0}{\sqrt{\dfrac{25.30}{15} + \dfrac{22.20}{18}}}$$

$$t_0^* = \frac{5.2}{1.7088}$$

$t_0^* = 3.0431$ (4 d.p)

since $t_0^* = 3.0431 > t_{0.05/2,\,29} = 2.045$, hence reject H_0. We can conclude that at 0.05 significance level there is sufficient evidence to conclude that there is any difference in the mean spending between two populations. 2

Question 6

7

$H_0: \sigma_1 = \sigma_2$

$H_1: \sigma_1 > \sigma_2$ 2

$\alpha = 1 - 0.95 = 0.05$, $v_1 = 16 - 1 = 15$, $v_2 = 21 - 1 = 20$ 1

$F_{0.05,15,20} = 2.33$

$$F = \frac{S_1^2}{S_2^2} = \frac{3.6^2}{2.5^2} = \frac{12.96}{6.25} = 2.0736$$ 2

2.203

$\therefore$ Since $F = 2.0736 < F_{0.05,15,20} = 2.33$, hence we fail to reject H_0. We can conclude that at 0.05 significance level there is insufficient evidence to conclude the standard deviation of processors from batch 1 is greater than the standard deviation in batch 2. 2

Question 7

$n = 10$, $\alpha = 0.05$, 2

$H_0 : \mu_0 = 0$ $df = 10 - 1$
$H_1 : \mu_0 < 0$ $= 9$

15

Before	After	Difference	d^2
203	225	-22	484
390	410	-20	400
389	402	-13	169
279	285	-6	36
333	355	-22	484
213	240	-27	729
410	444	-34	1156
364	370	-6	36
470	501	-31	961
464	490	-26	676
		$\Sigma_{difference} = -207$	$\Sigma d^2 = 5131$

3

$\bar{x} = \frac{\Sigma d}{n}$

$\bar{x} = \frac{-207}{10}$

$= -20.7$ 1

standard deviation

$$S_D = \sqrt{\frac{\Sigma d^2 - \frac{(\Sigma d)^2}{n}}{n-1}}$$

$$= \sqrt{\frac{5131 - \frac{(-207)^2}{10}}{10-1}} = 9.6959$$ 3

t-test value

$$t = \frac{\bar{x} - \mu_0}{S_D / \sqrt{n}}$$

$$= \frac{-20.7 - 0}{\frac{9.6959}{\sqrt{10}}}$$

$= -6.7512$ 3

$\therefore$ Since $t = -6.7512 < -1.833$, we reject the H_0 at 0.05 significance level. We can conclude that there is convincing evidence to suggest that training can increase the number of words spelled correctly. 3

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Question 8

i)

x	y	xy	x^2	y^2
0.27	2	0.54	0.0729	4
1.41	3	4.23	1.9881	9
2.19	3	6.57	4.7961	9
2.83	6	16.98	8.0089	36
2.19	4	8.76	4.7961	16
1.81	2	3.62	3.2761	4
0.85	1	0.85	0.7225	1
3.05	5	15.25	9.3025	25
$\Sigma x = 14.60$	$\Sigma y = 26$	$\Sigma xy = 56.80$	$\Sigma x^2 = 32.9632$	$\Sigma y^2 = 104$

2.5

$$r = \frac{\Sigma xy - (\Sigma x \Sigma y)/n}{\sqrt{[\Sigma x^2 - (\Sigma x)^2/n][\Sigma y^2 - (\Sigma y)^2/n]}}$$

$$r = \frac{56.80 - (14.60)(26)/8}{\sqrt{(32.9632 - 14.60^2/8)(104 - 26^2/8)}}$$

$$r = \frac{9.35}{11.0998}$$ 1/2

$r = 0.8424$ (4 d.p) 1

ii) $H_0 : \rho = 0$

$H_1 : \rho \neq 0$ 1/2

$\alpha = 0.05$, $df = 8 - 2$

$= 6$

$t_{0.05/2, 6} = 2.447$ 1/2 $\therefore$ reject H_0 if $t < -2.447$, $t > 2.447$.

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

$$t = \frac{0.8424}{\sqrt{\frac{1-0.8424^2}{8-2}}}$$

$$t = \frac{0.8424}{0.21999}$$ 1

$t = 3.8293$ (4 d.p)

since $t = 3.8293 > t_{0.05/2, 6} = 2.447$, hence reject H_0. We can conclude that at 0.05 significance level, there is sufficient evidence to conclude that variable x and Y are really correlate. 1

iii) $t_{0.01/2, 6} = 3.707$. 1 reject H_0 if $t < -3.707$ and $t > 3.707$. 1

since $t = 3.8293 > t_{0.01/2, 6} = 3.707$, hence reject H_0. The decision is same as (ii).

Question 9

6

i)

x	y	xy	x^2
97	24	2328	9409
85	29	2465	7225
98	26	2548	9604
105	24	2520	11 025
120	24	2880	14 400
151	22	3322	22 801
140	23	3220	19 600
134	23	3082	17 956
146	21	3066	21 316
$\Sigma x = 1076$	$\Sigma y = 216$	$\Sigma xy = 25431$	$\Sigma x^2 = 133\,336$

2

$\bar{y} = \frac{216}{9}$, $\bar{x} = \frac{1076}{9}$

$\bar{y} = 24$ $\quad \bar{x} = 119.5556$ (4d.p)

$$b_1 = \frac{\Sigma xy - \frac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \frac{(\Sigma x)^2}{n}}$$

$$b_1 = \frac{25431 - \frac{(1076)(216)}{9}}{133\,336 - \frac{1076^2}{9}}$$

$$b_1 = -\frac{393}{4694.2222}$$

$b_1 = -0.08372$ (5d.p) 1

$b_0 = \bar{y} - b_1\bar{x}$

$b_0 = 24 - (-0.08372)(119.5556)$

$b_0 = 24 + 10.0092$

$b_0 = 34.0092$ (4d.p) 1

equation ?

ii) when $x = 125$,

$\hat{y} = 34.0092 - 0.08372(125)$

$\hat{y} = 23.5442$ 2

$\therefore$ the mileage for a car which has 125 cubic km engine volume is 23.5442 km.

5

Question 10

$H_0 : \mu_1 = \mu_2 = \mu_3$

H_1 : at least one is different

For memory booster :

$n = 5$

$$\bar{x} = \frac{70 + 77 + 83 + 90 + 97}{5} = \frac{417}{5}$$

$$\bar{x} = 83.4$$

$$s = \sqrt{\frac{(70-83.4)^2 + (77-83.4)^2 + (83-83.4)^2 + (90-83.4)^2 + (97-83.4)^2}{5-1}}$$

$$s = \sqrt{\frac{449.2}{4}} = 10.5972 \text{ (4d.p)}$$

For category Placebo :

$n = 5$

$$\bar{x} = \frac{37 + 43 + 50 + 57 + 63}{5} = \frac{250}{5}$$

$$\bar{x} = 50$$

$$s = \sqrt{\frac{(37-50)^2 + (43-50)^2 + (50-50)^2 + (57-50)^2 + (63-50)^2}{5-1}}$$

$$s = \sqrt{\frac{436}{4}} = 10.4403 \text{ (4d.p)}$$

For category Without Treatment :

$n = 5$

$$\bar{x} = \frac{3 + 10 + 17 + 23 + 30}{5} = \frac{83}{5}$$

$$\bar{x} = 16.6$$

$$s = \sqrt{\frac{(3-16.6)^2 + (10-16.6)^2 + (17-16.6)^2 + (23-16.6)^2 + (30-16.6)^2}{5-1}}$$

$$s = \sqrt{\frac{449.2}{4}} = 10.5972 \text{ (4d.p)}$$

$$\bar{\bar{x}} = \frac{83.4 + 50 + 16.6}{3} = \frac{150}{3}$$

$$\bar{\bar{x}} = 50$$

1

$$s_{\bar{x}} = \sqrt{\frac{(83.4-50)^2 + (50-50)^2 + (16.6-50)^2}{3-1}}$$

$$s_{\bar{x}} = \sqrt{\frac{2231.12}{2}} = 33.4$$

$$s_p^2 = \frac{10.5972^2 + 10.4403^2 + 10.5972^2}{3}$$

$$s_p^2 = \frac{333.6012}{3} = 111.2004 \text{ (4d.p)}$$

1

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$$F = \frac{n s^2_{\bar{x}}}{s^2_p}$$

$$F = \frac{(5)(33.4)^2}{111.2004}$$

$$F = \frac{5577.8}{111.2004} = 50.1599 \text{ (4d.p)}$$

numerator degree of freedom $= 3-1$

$= 2$

denominator degree of freedom $= 3(5-1)$

$= 12$

$F_{2,12,0.01} = 6.93$

since $F = 50.1599 > F_{2,12,0.01} = 6.93$, hence reject H_0. We can conclude that at 0.01 significance level, there is sufficient evidence to conclude that the treatments will have different effects.