**ASSIGNMENT 3**

GROUP MEMBERS:

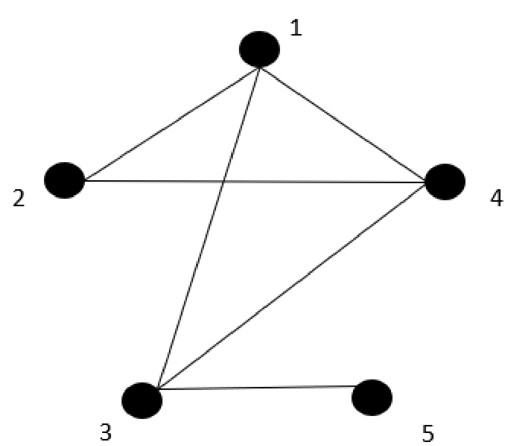
- CHUA XIN LIN (A21EC0020)
- FARAH AUNI MARDHATI BINTI ZAKARIA (A21EC0173)
- MAYSARA MOHAMED SHOKRY SAYED MOHAMED (A21EC4002)

ANSWERS:

**<u>Question 1:</u>**

- Transpose the adjacency matrix to prove the graph that will be drawn is an undirected graph.
- $A_G = A_G{}^T$
- Graph is symmetric therefore it is an undirected graph.

$$A_G = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array} \qquad A_G{}^T = \begin{array}{c c} & \begin{matrix} 1 & 2 & 3 & 4 & 5 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{matrix} & \begin{bmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix} \end{array}$$



## Question 2:

i) Incidence Matrix

V= {1, 2, 3, 4, 5, 6}

E= {a, b, c, d, e, f, g, h, i, k}

Incidence matrix =

| | *a* | *b* | *c* | *d* | *e* | *f* | *g* | *h* | *i* | *k* |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 1 | 2 | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 2 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | 1 | 0 | 1 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 4 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 5 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 |
| 6 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 1 | 1 |

ii) Adjacency Matrix

V= {1, 2, 3, 4, 5, 6}

Adjacency matrix =

| | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1 | 0 | 2 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 | 1 | 0 | 0 |
| 3 | 2 | 0 | 0 | 1 | 1 | 1 |
| 4 | 1 | 1 | 1 | 0 | 0 | 1 |
| 5 | 0 | 0 | 1 | 0 | 0 | 1 |
| 6 | 0 | 0 | 1 | 1 | 1 | 0 |

## Question 3:

- Both graph Y and graph Z have the same number of vertices (6) and the same number of edges (9).
- Graph Y: deg(A) = 2, deg(B) = 4, deg(C) = 3, deg(D) = 4, deg(E) = 2, deg(F) = 3
- Graph Z: deg(1) = 3, deg(2) = 2, deg(3) = 4, deg(4) = 3, deg(5) = 4, deg(6) = 2
- Both graphs Y and Z have the same degree for corresponding vertices.
- Both graphs Y and Z have the same number of connected components.
- Both graphs Y and Z are connected graphs.
- Both graphs Y and Z do not have loops or parallel edges.
- f(A) = 6, f(B) = 5, f(C) = 4, f(D) = 3, f(E) = 2, f(F) = 1
- Both graphs are isomorphic.

$$\text{Adjacency matrix} = \text{(Graph Y)} \quad \begin{array}{c} \\ A \\ B \\ C \\ D \\ E \\ F \end{array} \begin{array}{c} \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

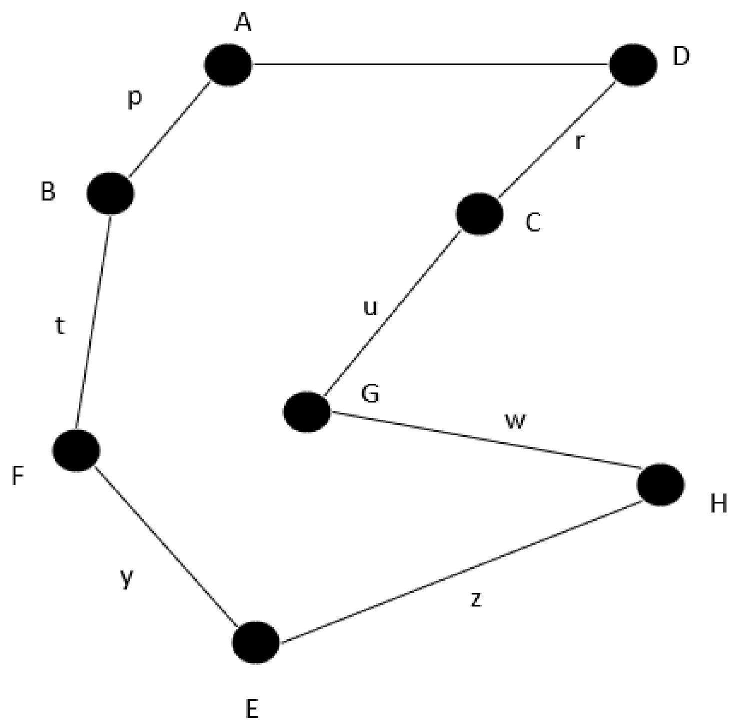
$$\text{Adjacency matrix} = \text{(Graph Z)} \quad \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \end{array} \begin{array}{c} \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & 6 \end{array} \\ \begin{bmatrix} 0 & 0 & 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \end{array}$$

**<u>Question 4:</u>**

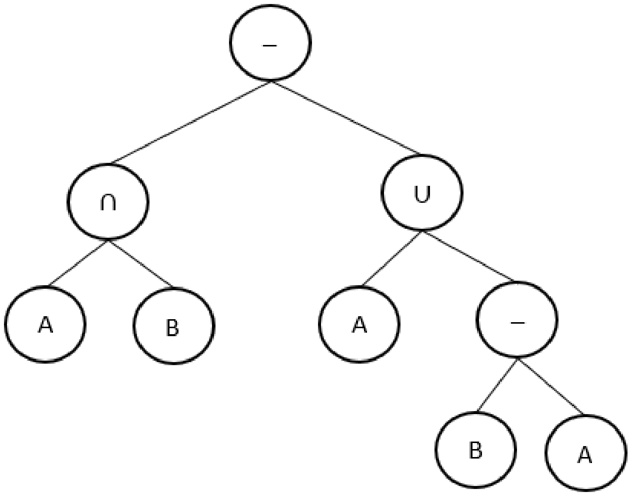
- 1- r - 2 - q - 3 - s - 1 - t - 4 - u - 3 - z - 5 - w - 4 - v - 4 - x - 6 - y - 5
- Graph above is a Euler Trail.
- The graph above starts at 1 and ends at 5, passes through every vertex of the graph at least once, and traverses every edge of the graph exactly once.

**Question 5:**

- A – p – B – t – F – y - E – z – H – w – G – u – C – r – D – s – A
- The graph above is a Hamiltonian circuit because it is a simple circuit that has every vertex appears exactly once except for the first and last vertex.



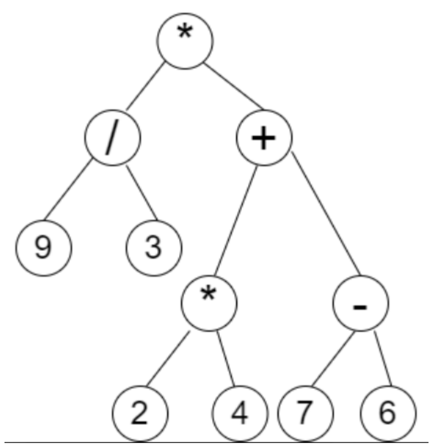
**Question 6:**




a) Prefix notation = - ∩ A B ∪ A – B A
b) Postfix notation = A B ∩ A B A - ∪ -
c) Infix notation = A ∩ B – A ∪ B – A

**Question 7:**

a) Ordered rooted tree




b) Expression in infix notation
((9/3) * (2*4) + (7-6))

c) Value of this arithmetic expression
* / 9 3 + * 2 4 - 7 6
* / 9 3 + * 2 4 1
* / 9 3 + 8 1
* / 9 3 9
* 3 9
27

**Question 8:**

a) Children of vertex j = n, o, p

b) Ancestor of vertex s = a, b, d, i, m, s

c) Siblings of vertex q = r, s, t

d) Number of leaves = 11 (e, h, k, l, n, o, p, q, r, s, t)

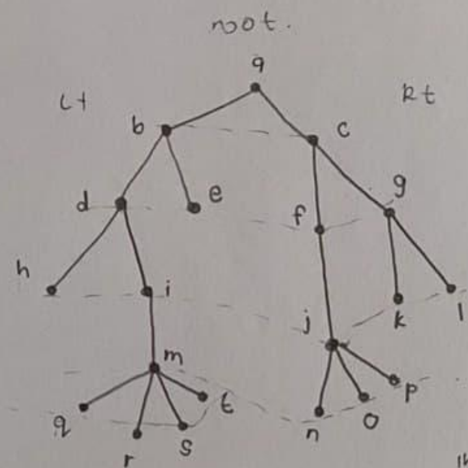
e) Level 3 vertices = h, i, j, k, l

f) Least m for rooted 4-ary tree = 2

g) Height of this rooted tree = 5

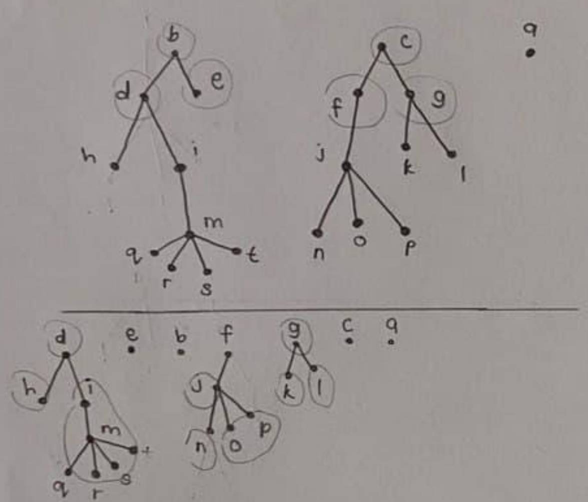
h) Postorder = h, q, r, s, t, m, i, d, e, b, n, o, p, j, f, k, l, g, c, a

Inorder = h, d, q, m, r, s, t, i, b, e, a, n, j, o, p, f, c, k, g, l

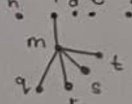



**post order**

Lt, Rt, root.



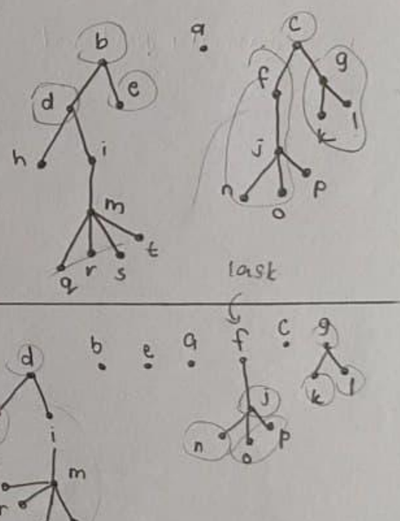
h i d e b n o p j f k l g c a



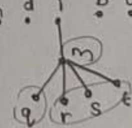
h q r s t m i d e b n o p j f k l g c a

**in order**

Lt, root, Rt

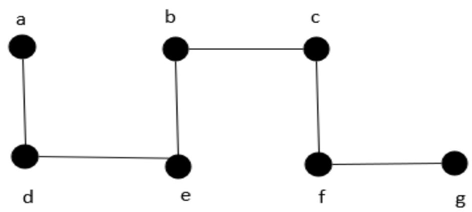


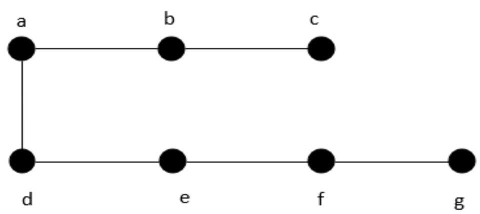

h d i b e a n j o p f c k g l

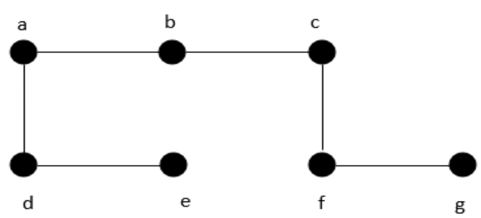


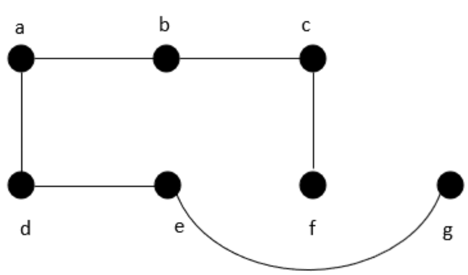
h d q m r s t i b e a n j o p f c k g l

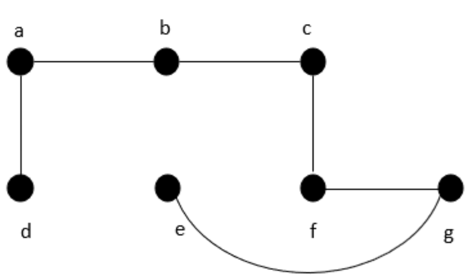
## Question 9:



**Question 10:**

BF = 2

EG = 2

CG = 3

CE = 4

AD = 5

AF = 7

DF = 13

AB = 14

EF = 15

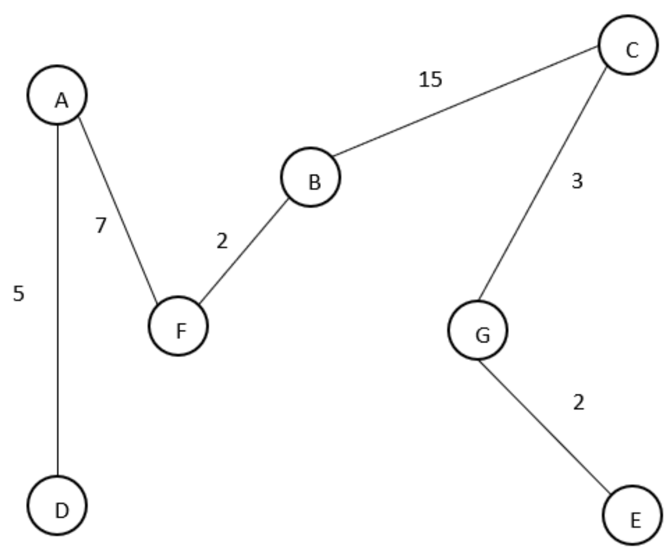
BC = 15

GF = 18

BG = 19

AC = 21

DE = 22




Length = 2 + 2 + 3 + 5 + 7 + 15 = 34

Length = BF + EG + CG + AD + AF + BC = 34

**Question 11:**

a) Edges of a full 5-ary tree with 121 leaves

$n = \frac{ml - 1}{m - 1}$

$n = \frac{(5)(121) - 1}{(5) - 1}$

n = 151 nodes

To find edges, use nodes, n = n - 1 edges

edges = n - 1

= 151 - 1

= 150

b) Vertices of a full 7-ary tree with 121 internal vertices

n = mi + 1

= (7 x 121) + 1

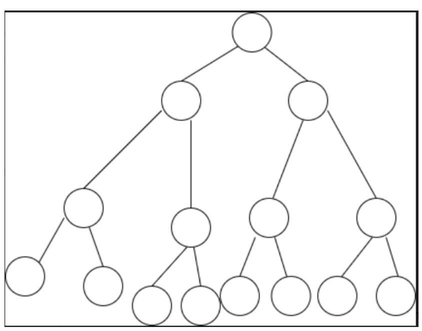
= 848 vertices

c) Largest and smallest number of nodes T can have

T = binary tree of height 3

Largest number of nodes = 15 nodes

2^(3 + 1) - 1 = 15 nodes



Smallest number of nodes = h + 1

= 3 + 1 = 4 nodes

d) Since the result of the tournament is represented in binary tree, hence m = 2.

Tournament limit to 5 rounds shows that this tree is level 5.

l (leaves) = teams can registered for the tournament

$= 2^5$

= 32

Let i (internal vertices) = matches need to schedule by organizer

$I = \frac{(m-1)n + 1}{m}$

$$32 = \frac{(2-1)(n+1)}{2}$$

n = 63

Total nodes (total of team on every round) = 63

To get internal nodes = Total nodes - leaves

= 63 - 32

= 31 matches need to schedule by organizer

**Question 12:**

| Iteration | S | N | L(A) | L(B) | L(C) | L(D) | L(E) | L(F) |
|---|---|---|---|---|---|---|---|---|
| 0 | {} | {A,B,C,D,E,F} | 0 | ∞ | ∞ | ∞ | ∞ | ∞ |
| 1 | {A} | {B,C,D,E,F} | 0 | 18 | 5 | ∞ | ∞ | ∞ |
| 2 | {A,C} | {B,D,E,F} | 0 | 15 | 5 | 8 | 11 | ∞ |
| 3 | {A,C,D} | {B,E,F} | 0 | 10 | 5 | 8 | 10 | 16 |
| 4 | {A,C,D,E} | {B,F} | 0 | 10 | 5 | 8 | 10 | 16 |
| 5 | {A,C,D,E,F} | {B} | 0 | 10 | 5 | 8 | 10 | 16 |
| | | | | | | | | |

a) Shortest path = 15 + 3 + 8

= 16

b) Path = A - C - D - F