20/22

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ANSWERS:

<u>Question 1:</u>

When there were no ties, 3 medals were awarded.
6x5x4 = 120

When there are ties,

MEDAL AWARDED	GOLD	SILVER	BRONZE	CALCULATION	
6 gold	C (6,6)	-	-	C (6,6) = 1	Since there are at least 3 gold medals awarded, there are no more medals left for silver and bronze.
5 gold	C (6,5)	-	-	C (6,5) = 6	
4 gold	C (6,4)	-	-	C (6,4) = 15	
3 gold	C (6,3)	-	-	C (6,3) = 20	
2 gold 4 bronze	C (6,2)	-	C (4,4)	C (6,2) x C (4,4) = 15	Since there are only 2 gold medals awarded, no silver is available but there is still a bronze medal. There is possible that at least one people and less than four people get ties and awarded bronze medal since two people get gold medal already.
2 gold 3 bronze	C (6,2)	-	C (4,3)	C (6,2) x C (4,3) = 60	
2 gold 2 bronze	C (6,2)	-	C (4,2)	C (6,2) x C (4,2) = 90	

2 gold 1 bronze	C (6,2)	-	C (4,1)	C (6,2) x C (4,1) = 90	
1 gold 5 silver	C (6,1)	C (5,5)	-	C (6,1) x C (5,5) = 6	Since there is only 1 gold medal awarded, a silver medal is awarded. It is possible for 2 to 5 people to get ties and be awarded silver medals. There is no bronze medal given because there are at least 1 person awarded gold and 2 people awarded silver and there are only 3 medals that can be awarded.
1 gold 4 silver	C (6,1)	C (5,4)	-	C (6,1) x C (5,4) = 30	
1 gold 3 silver	C (6,1)	C(5,3)	-	C (6,1) x C (5,3) = 30	
1 gold 2 silver	C (6,1)	C(5,2)	-	C (6,1) x C (5,2) = 60	
1 gold 1 silver 4 bronze	C (6,1)	C(5,1)	C (4,4)	C (6,1) x C (5,1) x C (4,4) = 30	When there is only 1 gold and 1 silver medal awarded, a bronze medal can be given because we can have 3 medals given.

1 gold 1 silver 3 bronze	C (6,1)	C(5,1)	C (4,3)	C (6,1) x C (5,1) x C (4,3) = 120	Since there are 2 people awarded gold and silver medals respectively, the others (1 to 4 people) have possible ties and can be awarded bronze.
1 gold 1 silver 2 bronze	C (6,1)	C(5,1)	C (4,2)	C (6,1) x C (5,1) x C (4,2) = 180	
1 gold 1 silver 1 bronze	C (6,1)	C(5,1)	C (4,1)	C (6,1) x C (5,1) x C (4,1) = 120	4

In conclusion, the total ways are 1 + 6 + 15 + 20 + 15 + 60 + 90 + 90 + 6 + 30 + 30 + 60 + 30 + 120 + 180 + 120 = 873 total of ways obtained.

Question 2:

a) This problem is a combination without repetition as there are no two donuts from one same variety.

Combination without repetition, C (n,r) = $\frac{n!}{r!\,(n-r)!}$

Number of ways = $^{20}C_{12}$

$= \frac{20!}{12!\,(20-12)!}$

= 125970 ways

b) No restriction by means the chosen donuts can be any flavour or any same flavour for a dozen of donuts one could possibly obtain.

Combination with repetition, C (n + r -1,r) = $\frac{(n+r-1)!}{r!\,(n-1)!}$

Number of ways = $^{20+12-1}C_{20-1}$

$= \frac{(20+12-1)!}{12!\,(20-1)!}$

$= \frac{31!}{12!(\,19!)}$

= 141120525 ways

c) If there must be at least six kaya-filled donuts, then to calculate it use a combination with repetition.

Combination with repetition, $C(n + r - 1, r) = \frac{(n+r-1)!}{r!\,(n-1)!}$

We assumed that six kaya-filled donuts had been chosen, so left 6 more donuts to choose.

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Number of ways = ${}^{20+6-1}C_{20-1}$

$= {}^{25}C_{19}$

$= \frac{(20+6-1)!}{6!\,(20-1)!}$

$= \frac{25!}{6!(\,19!)}$

= 177100 ways

Question 3:

a) Since five people sit around the table, we use circular permutations, $P_n = (n-1)!$

Number of ways = (5-1)!

= 4!

= 24

b) A (President), B (Vice President), C, D, E

Since President and Vice president must sit together, then (A,B), C, D, E

Number of ways = (4-1)! × 2

= 12

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c) If five people were labelled as A, B, C, D and E, and A is President, B is Vice President with five spaces to fill in, the situations are as follows:

- A/B _ A/B _ _ as A or B can sit and switch either on these two spaces. So the calculations will be 2! = 2 since only one person can sit in each space.
- For the remaining people, C,D and E, same as the first situation where only one person can sit in each space. Therefore, 3! = 6.
- Next situation is to calculate the circular permutations as these five people are sitting around a table by using $P_n = (n-1)!$. The arrangement is (A/B _ A/B)(_)(_) which represents 3 groups. So, (3-1)! = 2.
- Combining all situations, we can obtain the number of ways is 2× 6× 2 = 24 ways.

(3-1)! 3. 2! =12

Question 4:

a) Since is choose, so is selection, not arrangement
Number of ways = $^{13}C_{10}$
= 286 ways

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b) Since positions are assigned, so is arrangement.
Number of ways = 13!
= $13 \times 12 \times 11 \times 10 \times 9 \times 8 \times 7 \times 6 \times 5 \times 4$
= 1037836800 ways

c) Since is choose, so is selection
Number of ways = (1 woman & 9 man) + (2 woman & 8 man) + (3 woman & 7 man)
= $(^{3}C_{1} \times {}^{10}C_{9}) + (^{3}C_{2} \times {}^{10}C_{8}) + (^{3}C_{3} \times {}^{10}C_{7})$
= 285 ways

Question 5:

Let f be a function from finite set X to finite set Y. |X| = n and |Y| = m, and k = $\lceil \frac{n}{m} \rceil$

|X| = |number of students| = 30, |Y| = |number of alphabetical letters| = 26

Pigeons - number of students

Pigeon holes - number of letters

k= $\lceil \frac{n}{m} \rceil$ = $\lceil \frac{30}{26} \rceil$ = 2 (Shown)

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Question 6:

To calculate the minimum student, we must obtain a minimum number of students that each comes from a different state, which is 13 states combined. Then, as the question says “at least”, therefore 99 students come from different states. Make it 100 students to obtain the number of students from the same state. The 1st form of the pigeonhole principle applies in this question as k < n. In detail, k represents the number of states and n represents the number of students of different states, 13 states < 99 of students in different states.

1287

13 states $\times$ 99 students on different state = 1288 students

1

To obtain the minimum number of students in the same state, which is 100, we will add 1.

1287 1288

1288 students + 1 = 1289 students on same state pigeon hole principle

$\left\lceil \frac{9}{2} \right\rceil = 5$

Question 7:

a) Since there are 9 students in a small college, and the class has at least five male students or five female students, then k, or the pigeonhole is 9 students and n, the pigeon at least 5 male or 5 female.

By using contradiction, the class must have at least 5 male and 5 female students, which is 4 + 4 = 8. This contradiction proved false as the class consists of 9 students.

Therefore, the statement is true for either one student is male or female to make the class consist of 9 students.

b) Assume that there are at least three male students or five female students in the class, then there are less than nine students in the discrete mathematics class.
Let the class have 4 male students and 5 students in the class, 4+5 = 9, or the class has 3 male students and 6 female students in the class, 3+6 = 9.
Contradiction formed, hence there will be at least three male students or five female students in the class to let the class have nine students.

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