

## ASSIGNMENT DISCRETE STRUCTURE 1 (PART 2)

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ANSWERS:

1. Since Set Z is integer number, let Set Z = {1,2,3,4,5,6}

$$R = \{(1,3), (3,1), (2,4), (4,2), (3,5), (5,3), (4,6), (6,4)\}$$

- Symmetric

$$\forall x, y \in Z, (x, y) \in R \rightarrow (y, x) \in R$$

Eg: There exist (1,2) and (2,1) in relation R.

- Not transitive

$$(x, y), (y, z) \in R, (x, z) \notin R$$

In relation R on set Z, (1,3), (3,1) is shown but no (1,1) available in set relation.

- Irreflexive

$$\forall x \in Z, (x, x) \notin R$$

x is not related to x

As shown in relation to set Z, there is no (1,1) or (2,2).

- Not antisymmetric

$$\exists x, y \in Z, (x, y) \in R \rightarrow (y, x) \notin R$$

Eg: (1,3) and (3,1)  $\in R$

So, it is not an antisymmetric relation.

- Not asymmetric

To be asymmetric, the set of relations must be antisymmetric and irreflexive.

For relation R on set Z, it is irreflexive but not antisymmetric. Therefore, it is not an asymmetric relation.

not reflexive

2.

$$M_R = \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

-R is reflexive because the matrix of relation has 1's on the main diagonal.

-R is not symmetric because the matrix of relation  $M_R$  is not equal to  $M_R^T$ .

-R is not transitive because  $M_R \otimes M_R \neq M_R$ .

-Hence, R is not an equivalence relation.

show the matrix

3.a) Show that  $f$  is one to one.

Function is one to one when  $\forall x_1 \forall x_2 ((f(x_1) = f(x_2)), x_1 = x_2)$ .

$$f(x,y) = (2x-y, x-2y)$$

$$(2x_1-y_1, x_1-2y_1) = (2x_2-y_2, x_2-2y_2)$$

$$(2x_1-y_1 = 2x_2-y_2) \times -2$$

$$-4x_1+2y_1 = -4x_2+2y_2$$

$$(+)\quad x_1-2y_1 = x_2-2y_2$$

$$-3x_1 = -3x_2$$

$$x_1 = x_2$$

$$2x_1-y_1 = 2x_2-y_2$$

$$-y_1 = -y_2$$

$$y_1 = y_2$$

Thus,  $f(x,y) = (2x-y, x-2y)$  is a one-to-one function.

b) Find  $f^{-1}$

$$f(x) = 2x - y$$

$$\text{let } f(x) = n$$

$$n = 2x - y$$

$$2x = n + y$$

$$x = (n + y)/2$$

$$f^{-1}(n) = (n + y)/2$$

✓

$$f(y) = x - 2y$$

$$\text{let } f(y) = m$$

$$m = x - 2y$$

$$m - x = -2y$$

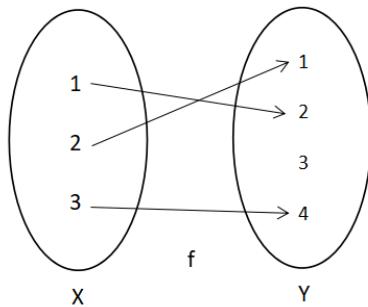
$$y = (x - m)/2$$

$$f^{-1}(m) = (x - n)/2$$

✓

✓

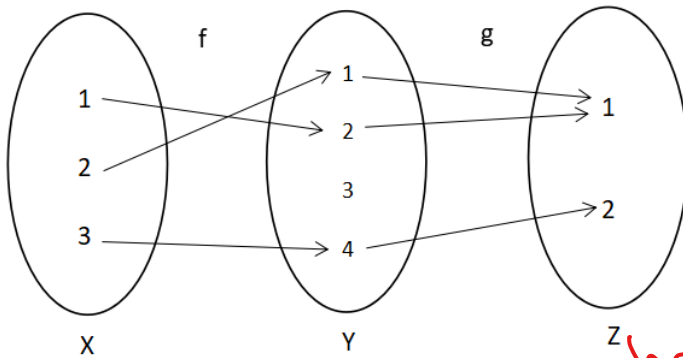
4. a)



✓

✓

b)  $g(x) = \{(1,1), (2,1), (3,2)\}$



5.  $f(x) = x^3$  and  $g(x) = x-1$  for all real number  $x$

i) find  $(g \circ f)$  and  $(f \circ g)$

$(g \circ f) = g(f(x)) = x^3 - 1$

$(f \circ g) = f(g(x)) = (x-1)^3 = x^3 - 3x^2 + 3x - 1$

ii) determine whether  $g \circ f$  equals  $f \circ g$ :

$g \circ f = x^3 - 1$

$f \circ g = x^3 - 3x^2 + 3x - 1$

Thus,  $(g \circ f) \neq (f \circ g)$ .

*show counter example*

*g(x<sup>3</sup>)*  
*f(x-1)*

*show the steps!*  
*4*

*2*

6.

Let  $a_n$  = the number of strings of length  $n$  in  $A^*$  that do not contain 01

$a_n = a_{n-1} + 1$ , and assume that  $a_0 = 1$

When  $n=1$ ,  $a_1 = a_{1-1} + 1 = a_0 + 1 = 1 + 1 = 2$        $A^* = \{1, 0\}$

When  $n=2$ ,  $a_2 = a_{2-1} + 1 = a_1 + 1 = 2 + 1 = 3$        $A^* = \{00, 01, 10, 11\}$

When  $n=3$ ,  $a_3 = a_{3-1} + 1 = a_2 + 1 = 3 + 1 = 4$        $A^* = \{000, 001, 010, 011, 100, 101, 110, 111\}$

When  $n=4$ ,  $a_4 = a_{4-1} + 1 = a_3 + 1 = 4 + 1 = 5$

$A^* = \{0000, 0001, 0010, 0011, 0100, 0101, 0110, 0111, 1000, 1001, 1010, 1011, 1100, 1101, 1110, 1111\}$

$A_n = 2, 3, 4, 5, 6, \dots$

So,  $a_n = a_{n-1} + 1$  when  $n \geq 1$ , with  $a_0 = 1$ .

$a_{n-1}$

2

So, when  $n=8$ ,  $c(8) = 4$