

Subject : SEK11013 - 01

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1. $U = \{10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30\}$

$B = \{12, 14, 16, 18, 20, 22, 24, 26, 28\}$

$C = \{11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

a) Proper subset of C :

$\{3\}, \{11\}, \{12\}, \{13\}, \dots, \{12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29\}$

$|C| = 19$

b) Proper subset of B :

$\{3\}, \{12\}, \{14\}, \{16\}, \dots, \{14, 16, 18, 20, 22, 24, 26, 28\}$

$|B| = 9$

c)

U

10

27

C

B

11

12

14

13

30

16

18

20

22

24

26

15

29

28

17

25

23

19

21

$$d) B \oplus C = (B - C) \cup (C - B)$$

$$= \emptyset \cup \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$$

$$= \{11, 13, 15, 17, 19, 21, 23, 25, 27, 29\}$$

$$|B \oplus C| = 10$$

$$2. A = \{s, u, b\}, B = \{s, e, t\}, C = \{n, e, t\}$$

$$a) |A| = 3 \quad P(A) = \{\emptyset, \{s\}, \{u\}, \{b\}, \{s, u\}, \{s, b\}, \{u, b\}, \{s, u, b\}\}$$

$$|P(A)| = 2^3$$

$$= 8$$

$$b) A \cup B \cap C = (A \cup B) \cap C$$

$$= (\{s, u, b\} \cup \{s, e, t\}) \cap \{n, e, t\}$$

$$= \{s, u, b, e, t\} \cap \{n, e, t\}$$

$$= \{e, t\}$$

$$c) A \cap B \cup C = (A \cap B) \cup C$$

$$= (\{s, u, b\} \cap \{s, e, t\}) \cup \{n, e, t\}$$

$$= \{s\} \cup \{n, e, t\}$$

$$= \{s, n, e, t\}$$

$$d) A - B = \{u, b\}$$

$$e) B \times C = \{s, e, t\} \times \{n, e, t\}$$

$$= \{(s, n), (s, e), (s, t), (e, n), (e, e), (e, t), (t, n), (t, e), (t, t)\}$$

$$3. a) \text{ True}$$

$$b) \text{ True}$$

$$c) \text{ False}$$

$$d) \text{ False}$$

$$e) \text{ False}$$

4. a) $(p \rightarrow q) \wedge (\neg p \leftrightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$(p \rightarrow q)$	$(\neg p \leftrightarrow \neg q)$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	F	F
F	T	T	F	T	F	F
F	F	T	T	T	T	T

b) $(p \leftrightarrow q) \vee (\neg p \rightarrow \neg q)$

p	q	$\neg p$	$\neg q$	$(p \leftrightarrow q)$	$(\neg p \rightarrow \neg q)$	$(p \leftrightarrow q) \vee (\neg p \rightarrow \neg q)$
T	T	F	F	T	T	T
T	F	F	T	F	T	T
F	T	T	F	F	F	F
F	F	T	T	T	T	T

5. $A = \neg p \wedge (\neg q \vee \neg r)$, $B = p \vee (q \wedge r)$, is $A \equiv B$?

p	q	r	$\neg p$	$\neg q$	$\neg r$	$(\neg q \vee \neg r)$	$(q \wedge r)$	$\neg p \wedge (\neg q \vee \neg r)$	$p \vee (q \wedge r)$
T	T	T	F	F	F	F	T	F	T
T	T	F	F	F	T	T	F	F	T
T	F	T	F	T	F	T	F	F	T
T	F	F	F	T	T	T	F	F	T
F	T	T	T	F	F	F	T	F	T
F	T	F	T	F	T	T	F	T	F
F	F	T	T	T	F	T	F	T	F
F	F	F	T	T	T	T	F	T	F

Since $\neg p \wedge (\neg q \vee \neg r) \neq p \vee (q \wedge r)$, hence $A \not\equiv B$

6. $A = p \wedge (p \vee q)$, $B = p \vee (p \wedge q)$

p	q	$p \vee q$	$p \wedge q$	$p \wedge (p \vee q)$	$p \vee (p \wedge q)$
T	T	T	T	T	T
T	F	T	F	T	T
F	T	T	F	F	F
F	F	F	F	F	F

Since $p \wedge (p \vee q) = p \vee (p \wedge q)$, hence $A \equiv B$.

7. $P(x) = x$ is a student $Q(x) = x$ is smart $R(x) = x$ is shy

a) Some students are shy.

$$\exists x (P(x) \rightarrow R(x))$$

b) All smart people are not shy.

$$\forall x [(P(x) \rightarrow Q(x)) \rightarrow \neg R(x)]$$

$$(P \wedge Q) \vee Q = B, (P \vee Q) \wedge Q = A$$

$$(P \wedge Q) \vee Q = B, (P \vee Q) \wedge Q = A$$

$$(P \wedge Q) \vee Q = B, (P \vee Q) \wedge Q = A$$

$$(P \wedge Q) \vee Q = B, (P \vee Q) \wedge Q = A$$

$$(P \wedge Q) \vee Q = B, (P \vee Q) \wedge Q = A$$

$$P \equiv B \text{ mod } 2, (P \wedge Q) \vee Q = (P \vee Q) \wedge Q \text{ mod } 2$$

8. For all real numbers, if x is a negative number, then $(x)^2$ is a positive number.

Before writing the proof, we can verify the theorem by Substituting x with certain values.

$$\text{If } x = -2, \text{ then } (x)^2 = 4$$

$$\text{If } x = -3, \text{ then } (x)^2 = 9$$

Let $P(x) = x$ is a negative number.

$Q(x) = (x)^2$ is a positive number.

$$\forall x (P(x) \rightarrow Q(x))$$

Let a be a negative real number, then

$$a = -n$$

$$(a)^2 = (-n) \times (-n)$$

$$(a)^2 = (n)^2$$

$(a)^2$ is a positive number.

$\therefore$ All real numbers x , if x is a negative number, then $(x)^2$ is a positive number.

9. Contradiction of $A \cap (B \cap A') = \{\}$

- Assume that $A \cap (B \cap A') \neq \{\}$
- $\exists x \in U$ such that $x \in A \cap (B \cap A')$
- This implies that there exists $x \in A$ and $x \in B \cap A'$
- From the statement $x \in B \cap A'$, we know that $x \in B$ but $x \notin A$.
- Element of $x \in A$ cannot be found in $x \in B \cap A'$.
- This is the contradiction to the hypothesis.
- Hence, $A \cap (B \cap A') = \{\}$