

DISCRETE STRUCTURE ASSIGNMENT PART 2

1. MAATHUREE A/P VEERABALAN (A21E(0051)
2. HARCHANA A/P ARULAPPAN (A21E(0028)
3. PUTERI NUR ELEEYA SYAFIKA BINI MUHD ZABIDI (A21E(0124)

QUESTION 1

i) Both heads (H) and tails (T) have probability, $p = 0.5$

Total outcome = 2^3

= 32 outcomes

[illegible]

iii) $S = \{HHH, HHH, HTH, HTH, THH, THH, TTH, TTH\}$
 $= 8$

At least 2 tails (T) : { HTT, THT, TTH, TTT }

$$P(E) = \frac{4}{8}$$

≈ 0.5

iv) All head and tail = { HHHH, TTTT }

$$P(E) = \frac{2}{32}$$

$$= 0.0625$$

Question 2

First case has 10 blue balls and 5 red balls

blue = 10, red = 5

The first ball is red, then the probability is $P(R) = \frac{5}{15}$

Second case has 10 blue balls and 4 red balls

blue = 10, red = 4

The second ball is blue, then the probability is $P(B) = \frac{10}{14}$

The final probability is (Occur together AND)

$$P(A) = \left(\frac{5}{15} \right) \times \left(\frac{10}{14} \right)$$

$$= \frac{5}{21}$$

Question 3

number of coins = 2, probability of getting head = 0.8, probability of getting tail = 0.2

i) probability that both coins are tail

$$= (0.2)^2$$

$$= 0.04$$

ii) probability that give result head and tail

$$= (0.2 \times 0.8) + (0.8 \times 0.2)$$

$$= 0.16 + 0.16$$

$$= 0.32$$

Question 4

$$\text{score above } 3.70 = 150$$

$$\text{score between } 3.50 \text{ and } 3.70 = 520$$

$$\text{score between } 3.00 \text{ and } 3.50 = 100$$

$$\text{score lower than } 3.00 = 230$$

$$i) P(\text{score lower than } 3.00)$$

$$= \frac{230}{1000}$$

$$= 0.23$$

$$ii) \text{ score lower than } 3.00$$

$$= 20\%$$

$$= \frac{20}{100} \times 1000$$

$$= 200 \text{ students}$$

$$\text{score more than } 3.50$$

$$= 150 + 520$$

$$= 670 \text{ students}$$

$$\text{number of students who gets GPA between } 3.00 \text{ and } 3.50$$

$$= 1000 - 200 - 670$$

$$= 130 \text{ students}$$

Question 5

i) Size of sample = $(20, 15)$ ways

10 blue marbles out = $(10, 10)$ ways

5 red marbles out = $(10, 5)$ ways

$$\text{Probability} = \frac{C(10, 10) \times C(10, 5)}{C(20, 15)}$$

$$= \frac{1 \times 252}{15504}$$

$$= \frac{21}{1242}$$

$$= 0.0163$$

$$\begin{aligned} \text{ii) } P(N) &= \frac{20}{100} + \frac{50}{100} + \frac{10}{100} \\ &= \frac{80}{100} \end{aligned}$$

$$= \frac{4}{5}$$

$$= 0.8$$

$$\begin{aligned} P(H) &= \frac{50}{100} \\ &= 0.5 \end{aligned}$$

$$\begin{aligned} P(N \cup H) &= P(N) + P(H) - P(N \cap H) \\ &= 0.8 + 0.5 - 0.5 \\ &= 0.8 \quad \text{X} \end{aligned}$$

$$\text{iii) } P(N) = 0.8$$

$$P(R \cap N) = 0.24$$

$$P(R|N) = \frac{P(R \cap N)}{P(N)}$$

$$= \frac{0.24}{0.8}$$

$$= 0.3$$

Question 6

i) $P(A) = 0.08$

$P(B) = 0.15$

$P(C) = 0.32$

$P(D) = 0.40$

ii) $P(H|A) = 0.20$

$P(H|B) = 0.40$

$P(H|C) = 0.60$

$P(H|D) = 0.70$

iii) $P(A|H) = \frac{P(H|A)P(A)}{P(H|A)P(A) + P(H|B)P(B) + P(H|C)P(C) + P(H|D)P(D)}$

$$= \frac{(0.2)(0.08)}{(0.2)(0.08) + (0.4)(0.15) + (0.6)(0.32) + (0.7)(0.4)}$$
$$= \frac{0.016}{0.548}$$
$$= 0.0292$$

$P(B|H) = \frac{P(H|B)P(B)}{P(H|A)P(A) + P(H|B)P(B) + P(H|C)P(C) + P(H|D)P(D)}$

$$= \frac{(0.4)(0.15)}{(0.2)(0.08) + (0.4)(0.15) + (0.6)(0.32) + (0.7)(0.4)}$$
$$= \frac{0.06}{0.548}$$
$$= 0.1095$$

$$\begin{aligned}
 P(C|H) &= \frac{P(H|C) P(C)}{P(H|A) P(A) + P(H|B) P(B) + P(H|C) P(C) + P(H|D) P(D)} \\
 &= \frac{(0.6)(0.32)}{(0.2)(0.08) + (0.4)(0.15) + (0.6)(0.32) + (0.7)(0.4)} \\
 &= \frac{0.192}{0.548} \\
 &= 0.3504
 \end{aligned}$$

$$\begin{aligned}
 P(D|H) &= \frac{P(H|D) P(D)}{P(H|A) P(A) + P(H|B) P(B) + P(H|C) P(C) + P(H|D) P(D)} \\
 &= \frac{(0.7)(0.4)}{(0.2)(0.08) + (0.4)(0.15) + (0.6)(0.32) + (0.7)(0.4)} \\
 &= \frac{0.28}{0.548} \\
 &= 0.5109
 \end{aligned}$$