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Assignment 1 (Part 2)

Group members :

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1. $a \, R \, b$ if and only if $|a-b| = 2$

reflexive :
Z is not a reflexive relation because $\forall a \in Z$, $(a,a) \in R$.

irreflexive :
Z is an irreflexive relation because $\forall a \in Z, (a,a) \notin R$.

symmetric :
Z is a symmetric relation because $\forall (a,b) \in Z$, $(a,b) \in R \wedge (b,a) \in R$.

asymmetric :
Z is not an asymmetric relation because $\forall (a,b) \in Z$, $(a,b) \in R$ but $(b,a) \in R$.

antisymmetric :
Z is not an antisymmetric relation because $\forall (a,b) \in Z$, if $(a,b) \in R \wedge (b,a) \in R \rightarrow a=b$ or if $(a,b) \in R \wedge a \neq b \rightarrow R$

transitive :
Z is not a transitive relation because $\forall a,b,c \in A$, if $(a,b) \in R \wedge (b,c) \in R \rightarrow (a,c) \neq R$.

2. $R = \{(a,a), (a,b), (a,d), (b,b), (b,c), (c,c), (c,d), (d,a), (d,d)\}$

$$M_R = \begin{array}{c c} & \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{array}{c} a \\ b \\ c \\ d \end{array} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

$\therefore$ R is reflexive because it has value 1 at it's diagonal.

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$$M_R^T = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 0 & 0 & 1 \\ b & 1 & 1 & 0 & 0 \\ c & 0 & 1 & 1 & 0 \\ d & 1 & 0 & 1 & 1 \end{array}$$

∴ R is not symmetric because $M_R \neq M_R^T$

$$\begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 0 & 1 \end{array} \otimes \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 0 & 1 \\ b & 0 & 1 & 1 & 0 \\ c & 0 & 0 & 1 & 1 \\ d & 1 & 0 & 0 & 1 \end{array} = \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 1 & 1 & 1 & 1 \\ b & 0 & 1 & 1 & 1 \\ c & 1 & 0 & 1 & 1 \\ d & 1 & 1 & 0 & 1 \end{array}$$

∴ R is not a transitive because $M_R \otimes M_R \neq M_R$

∴ R is reflexive.

3 a) $f(x) = 2x - y$
$f(y) = x - 2y$

$(2x_1 - y_1, x_1 = 2y_1) = (2x_2 - y_2, x_2 = 2y_2)$

For 1^{st} eqn :

$2x_1 - y_1 = 2x_2 - y_2$
$2x_1 - 2x_2 = y_1 - y_2$
$2(x_1 - x_2) = y_1 - y_2$
$y_1 - y_2 = 2(x_1 - x_2)$ — ①

For 2^{nd} eqn :

$x_1 - 2y_1 = x_2 - 2y_2$
$x_1 - x_2 = 2y_1 - 2y_2$
$x_1 - x_2 = 2(y_1 - y_2)$ — ②

Substitude ② into ①,

$y_1 - y_2 = 2[2(y_1 - y_2)]$
$y_1 - y_2 = 4(y_1 - y_2)$
$3(y_1 - y_2) = 0$
$y_1 - y_2 = 0$
$y_1 = y_2$

Substitude ① into ②,

$x_1 - x_2 = 2[2(x_1 - x_2)]$
$x_1 - x_2 = 4(x_1 - x_2)$
$3(x_1 - x_2) = 0$
$x_1 - x_2 = 0$
$x_1 = x_2$

∴ f is one-to-one because $f(x_1, y_1) = f(x_2, y_2)$, $x_1 = x_2$ and $y_1 = y_2$.

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b) $f(x,y) = (2x-y, x-2y)$

Let $f(x,y) = f(a,b)$, $f^{-1}(a,b) = f^{-1}(x,y)$

Let $a = (2x-y)$ — ①

$y = 2x - a$ — ③

Let $b = x - 2y$ — ②

$x = b + 2y$ — ④

Substitude ④ into ① ;

$a = 2(b+2y) - y$

$a = 2b + 4y - y$

$a = 2b + 3y$

$3y = a - 2b$

$y = \frac{a-2b}{3}$

Substitude ③ into ② ;

$b = x - 2(2x-a)$

$b = x - 4x + 2a$

$b = -3x + 2a$

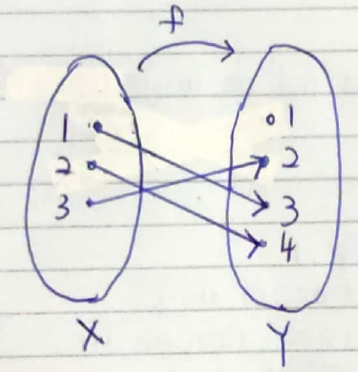
$3x = 2a - b$

$x = \frac{2a-b}{3}$

$$f^{-1}(a,b) = \left(\frac{2a-b}{3}, \frac{a-2b}{3}\right)$$

$$\therefore f^{-1}(x,y) = \left(\frac{2x-y}{3}, \frac{x-2y}{3}\right)$$

4 a)



b) $g = \{(1,1), (2,2), (3,2)\}$

5 $f(x) = x^3$, $g(x) = x-1$

i) $g \circ f = g[f(x)]$

$= g[x^3]$

$= x^3 - 1$

$f \circ g = f[g(x)]$

$= f(x-1)$

$= (x-1)^3$

ii) $g \circ f \neq f \circ g$ because two function from one set to another are equal if and only if they taken the same value.

6 $a_n = a_{n-1} + 1$, $a_0 = 1$
$n \geq 1$

$a_1 = (0, 1)$
$a_2 = (00, 10, 11)$
$a_3 = (000, 100, 110, 111)$
$a_4 = (0000, 1000, 1100, 1111)$

7 Input : n
Output : c(n)

```
   c(n)
{if (n = 1) or (n = 2) or (n = 3)
      return 0
  else if (n = 3 or n = 4)
     return 1
 else
 return   c(n-2) + C(n-3)
}
```

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