

GROUP (2) MEMBERS

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ASSIGNMENT 2

Question 1

Suppose we have an experiment that consists of flipping a coin 5 times.

- i. Find the total number for the outcomes for this experiment?
- ii. List all the elements in the sample space?
- iii. Find the probability that tails will be occurring at least 2 times on five coin flips?
- iv. Find the probability of getting all Head and all Tail on five coin flips?

i) i) $2 \times 2 \times 2 \times 2 \times 2$
 $= 2^5$
 $= 32 \text{ outcomes}$

ii) Let S be the sample space

$$S = \{ \text{HHHHH, HHHHT, HHHTH, HHTTT, HTTTT, TTTT}, \\ \text{TTTTH, TTTHH, TTHHH, TNNHH, HTHTH, THTHT,} \\ \text{HHTHH, TTHTT, HTTHT, THTHT, THHHT, HTTTH,} \\ \text{THHTT, HTTHH, HHTTH, TTHTT, TTHTH, HHTHT,} \\ \text{HTHTT, THTHH, THTTH, HTHHT, HTHTT, THTHH,} \\ \text{HHHTH, TTTHT} \}$$

iii) Let A be the event that tails will be occurring at least 2 times

$$A = \{ \text{HHHTH, HHTTT, HTTTT, TTTT}, \text{TTTTH, TTTHH, TTHHH,} \\ \text{HTHTH, THTHT, TTHTT, HTTHT, THTHT, THHHT, HTTTH,} \\ \text{THHTT, HTTHH, HHTTH, TTHTT, TTHTH, HHTHT, HTHTT} \\ \text{THTHH, THTTH, HTHHT, HTHTT, THTHH, TTTHT} \}$$

$$P(A) = \frac{27}{32}$$

iv) Let B be the event of getting all heads and all tails

$$B = \{ \text{HHHHH, TTTTT} \}$$

$$P(B) = \frac{2}{32} = \frac{1}{16}$$

Question 2

A bag has 10 blue balls and 5 red balls. 2 balls are to be drawn successively and without replacement. What is the probability that the first ball is red and the second is blue?

$$\begin{aligned}
 \text{blue} &= 10 \\
 \text{red} &= 5 \\
 \text{total} &= 15
 \end{aligned}
 \quad
 \begin{aligned}
 &\text{Let A be the event that the first ball is red and the second ball is blue} \\
 P(A) &= \frac{5}{15} \times \frac{10}{14} \\
 &= \frac{5}{21} = 0.2381
 \end{aligned}$$

Question 3

Two coins are loaded so that each comes up heads 80% of the time when tossed. If the coins are both tossed simultaneously,

- What probability will the result be both are Tail?
- What probability will the result give Head and Tail?

i) Let two coins be coin 1 and coin 2
 Let A be the event "coin 1 obtain tail"
 Let B be the event "coin 2 obtain tail"

$$\begin{aligned}
 P(A) &= P(B) = 1 - 0.8 \\
 &= 0.2 \quad \therefore A \text{ and } B \text{ are independent events}
 \end{aligned}$$

$$\begin{aligned}
 &\text{Let C be the event "the result be both tail"} \\
 P(C) &= P(A) \cdot P(B) \\
 &= 0.2 \times 0.2 \\
 &= 0.04
 \end{aligned}$$

Question 4

A UTM received 1000 applications for the undergraduate program every semester. Of all applicants, 150 had a CGPA score above 3.70. 520 applicants had a CGPA score between 3.50 and 3.70. 100 applicants had a score between 3.00 and 3.50. All other applicants had a score lower than 3.00.

- What probability will a randomly selected applicant have a score lower than 3.00?
- If this semester, the total of students who get the CGPA score lower than 3.00 is 20% of the total student received, and the number of the other students who get CGPA above 3.50 is the same. Find the number of students who get CGPA between 3.00 and 3.50?

$$\begin{aligned}
 4) \text{ i) Let D be the event that applicants have a score lower than 3.00} \\
 D &= 1000 - 150 - 520 - 100 \\
 &= 230
 \end{aligned}$$

$$P(D) = \frac{230}{1000} = 0.23$$

$$\begin{aligned}
 \text{ii) Lower than 3.00} &= \frac{20}{100} \times 1000 \\
 &= 200 \text{ students}
 \end{aligned}$$

Let A be the event that students get CGPA between 3.00 and 3.5

$$\begin{aligned}
 A &= 1000 - 200 - 520 - 150 \\
 &= 130
 \end{aligned}$$

Question 5

A boy has 10 red and 10 blue marbles. He only needed to take out 15.

- What is the probability of choosing 10 marbles that is blue and 5 marbles are red?
- The boy bought new marble, and now he has 100 marbles in total. He has 30 red, 50 green, and 20 blue from the marbles. What is the probability that a randomly chosen marble is new OR green?
- If the boy wants to choose 10 marbles. The probability he will get the marble are new is 0.8, and the probability for the boy to get red and new is 0.24. Find the probability for the marble to be red given the marble is new?

5) i) $\text{red} = 10$
 $\text{blue} = 10$
 $\text{total} = 20$

$$\frac{{}^{10}C_{10} \times {}^{10}C_5}{{}^{20}C_{15}} = 0.0163$$

ii) $\text{red} = 30$
 $\text{green} = 50$
 $\text{blue} = 20$

New marbles = $100 - 20 = 80$

Let N be the event of new marbles

$$P(N) = \frac{80}{100} = 0.8$$

Let G be the event of green marbles

$$P(G) = \frac{50}{100} = 0.5$$

	red	blue	green	total
old	10	10	0	20
new	30	50	50	80
total	40	60	50	100

$$P(N \cap G) = 0.5$$

$$\begin{aligned} \therefore P(N \cup G) &= P(N) + P(G) - P(N \cap G) \\ &= 0.8 + 0.5 - 0.5 \\ &= 0.8 \end{aligned}$$

iii) Let R be the event of red marbles

$$P(N) = 0.8$$

$$\therefore P(R|N) = \frac{P(R \cap N)}{P(N)}$$

$$P(R \cap N) = 0.24$$

$$P(R|N) = ?$$

$$= \frac{0.24}{0.8}$$

$$= 0.3$$

Question 6

Ali is a Software Engineering student taking Discrete Structure this semester. The tables below show the percentage of his result from Quiz, Assignment, Test 1 and Final Exam and his time on each activity.

	Quiz	Assignment	Test 1	Final Exam
% Mark for the activity	10	15	35	40
% Time spend on study	20	40	60	70

Let A denote the event "the mark for the Quiz".

Let B denote the event "the mark for the Assignment".

Let C denote the event "the mark for the Test 1".

Let D denote the event "the mark for the Final Exam".

Let H denote the event "time Ali spent"

- Find the probability of events A, B, C and D
- Find the probability, $P(H|A)$, $P(H|B)$, $P(H|C)$ and $P(H|D)$
- Find the probability, $P(A|H)$, $P(B|H)$, $P(C|H)$ and $P(D|H)$
- Find the probability, $P(H)$

6) i) $P(A) = 0.1$

$$P(B) = 0.15$$

$$P(C) = 0.35$$

$$P(D) = 0.40$$

ii) $P(H|A) = 0.2$

$$P(H|B) = 0.4$$

$$P(H|C) = 0.6$$

$$P(H|D) = 0.7$$

$$\begin{aligned}
 \text{iii) } P(A|H) &= \frac{P(H|A) \cdot P(A)}{P(H|A) \cdot P(A) + P(H|B) \cdot P(B) + P(H|C) \cdot P(C) + P(H|D) \cdot P(D)} \\
 &= \frac{(0.2)(0.1)}{(0.2)(0.1) + (0.4)(0.15) + (0.6)(0.35) + (0.7)(0.4)} \\
 &= \frac{0.02}{0.57} = \frac{2}{57} \\
 &= 0.0351
 \end{aligned}$$

$$\begin{aligned}
 P(B|H) &= \frac{P(H|B) \cdot P(B)}{P(H|A) \cdot P(A) + P(H|B) \cdot P(B) + P(H|C) \cdot P(C) + P(H|D) \cdot P(D)} \\
 &= \frac{(0.4)(0.15)}{(0.2)(0.1) + (0.4)(0.15) + (0.6)(0.35) + (0.7)(0.4)} \\
 &= \frac{0.06}{0.57} = \frac{2}{19} \\
 &= 0.1053
 \end{aligned}$$

$$\begin{aligned}
 P(C|H) &= \frac{P(H|C) \cdot P(C)}{P(H|A) \cdot P(A) + P(H|B) \cdot P(B) + P(H|C) \cdot P(C) + P(H|D) \cdot P(D)} \\
 &= \frac{(0.6)(0.35)}{(0.2)(0.1) + (0.4)(0.15) + (0.6)(0.35) + (0.7)(0.4)} \\
 &= \frac{0.21}{0.57} = \frac{7}{19} \\
 &= 0.3684
 \end{aligned}$$

$$\begin{aligned}
 P(D|H) &= \frac{P(H|D) \cdot P(D)}{P(H|A) \cdot P(A) + P(H|B) \cdot P(B) + P(H|C) \cdot P(C) + P(H|D) \cdot P(D)} \\
 &= \frac{(0.7)(0.4)}{(0.2)(0.1) + (0.4)(0.15) + (0.6)(0.35) + (0.7)(0.4)} \\
 &= \frac{0.28}{0.57} \\
 &= 0.4912
 \end{aligned}$$

$$\begin{aligned}
 \text{iv) } P(H) &= P(H|A) \cdot P(A) + P(H|B) \cdot P(B) + P(H|C) \cdot P(C) + P(H|D) \cdot P(D) \\
 &= (0.2)(0.1) + (0.4)(0.15) + (0.6)(0.35) + (0.7)(0.47) \\
 &= 0.57
 \end{aligned}$$