SECI1013 – DISCRETE STRUCTURE

ASIGNMENT 2 (PART 1)

DEADLINE 12 DECEMBER 2021

1. There are six runners in the 100-yard dash. How many ways are there for three medals gold, silver and bronze to be awarded if ties are possible.

2. How many ways are there to choose a dozen of donuts from 20 verities
   a) If there are no two donuts of the same variety
   b) If there is no restriction
   c) If there must be at least six kaya-filled donuts

3. At a corporate dinner, five people including the president and vice president are sitting around a circular table
   a) How many ways for these people to be seated around the table
   b) If the president and vice president should be seated next to each other
   c) If exactly one person seated between president and vice president

4. Thirteen people on a softball team show up for a game
   a) how many ways are there to choose 10 players to take the filed
   b) How many ways are there to assign 10 position by selecting players from the 13 people who show up
   c) of the 13 people show up, three are women. How many ways are there to choose 10 players to take the field if at least one player must be a woman

5. Show that if there are 30 students in a class, then at least two have last name begin with the same letter.

6. What is the minimum number of students, each of whom comes from one of the 13 states, who must be enrolled in a university to guarantee that are at least100 come from the same state.

7. Suppose that there are nine students in a discrete mathematics class at a small college
   a) show that the class must have at least five male students or five female students
   b) show that the class must have at least three male students or five female students

ASSIGNMENT 2

1. There are six runners in the 100-yard dash. How many ways are there for three medals gold, silver and bronze to be awarded if ties are possible.

6 gold = ${}^6C_6 = 1$

5 gold = ${}^6C_5 = 6$

4 gold = ${}^6C_4 = 15$

3 gold = ${}^6C_3 = 20$

2 gold, 4 bronze = ${}^6C_2 \times {}^4C_4 = 15$

2 gold, 3 bronze = ${}^6C_2 \times {}^4C_3 = 60$

2 gold, 2 bronze = ${}^6C_2 \times {}^4C_2 = 90$

2 gold, 1 bronze = ${}^6C_2 \times {}^4C_1 = 60$

1 gold, 5 silver = ${}^6C_1 \times {}^5C_5 = 6$

1 gold, 4 silver = ${}^6C_1 \times {}^5C_4 = 30$

1 gold, 3 silver = ${}^6C_1 \times {}^5C_3 = 60$

1 gold, 2 silver = ${}^6C_1 \times {}^5C_2 = 60$

1 gold, 1 silver, 4 bronze = ${}^6C_1 \times {}^5C_1 \times {}^4C_4 = 30$

1 gold, 1 silver, 3 bronze = ${}^6C_1 \times {}^5C_1 \times {}^4C_3 = 120$

1 gold, 1 silver, 2 bronze = ${}^6C_1 \times {}^5C_1 \times {}^4C_2 = 180$

1 gold, 1 silver, 1 bronze = ${}^6C_1 \times {}^5C_1 \times {}^4C_1 = 120$

$\therefore$ Total $= 1 + 6 + 15 + 20 + 15 + 60 + 90 + 60 + 6 + 30 + 60 + 60$
$+ 30 + 120 + 180 + 120$
$= 873$

2. How many ways are there to choose a dozen of donuts from 20 verities
   a) If there are no two donuts of the same variety
   b) If there is no restriction
   c) If there must be at least six kaya-filled donuts

a) $${}^{20}C_{12} = \frac{20!}{12!\,(20-12)!} = \frac{20!}{12!\,18!} = 125,970$$

b) $n = 20$
$r = 12$

$$C(20+12-1,\ 12) = \frac{(20+12-1)!}{12!\,(20-1)!} = \frac{31!}{12!\,19!} = 141120525$$

c) $n = 20$
$r = 6$

$$C(20+6-1,\ 6) = \frac{(20+6-1)!}{6!\,(20-1)!} = \frac{25!}{6!\,19!} = 177,100$$

3. At a corporate dinner, five people including the president and vice president are sitting around a circular table
   a) How many ways for these people to be seated around the table
   b) If the president and vice president should be seated next to each other
   c) If exactly one person seated between president and vice president

a) $(5-1)! = 4! = 24$

b) $(4-1)! = 3!$

$\therefore \; 3! \times 2! = 12$

c) $(3-1)! \times {}^{2}C_{1} \times {}^{3}C_{1}$

$= 2! \times 2 \times 3$

$= 12$

4. Thirteen people on a softball team show up for a game
   a) how many ways are there to choose 10 players to take the filed
   b) How many ways are there to assign 10 position by selecting players from the 13 people who show up
   c) of the 13 people show up, three are women. How many ways are there to choose 10 players to take the field if at least one player must be a woman

a) $^{13}C_{10} = \dfrac{13!}{10!\,(13-10)!}$

$= \dfrac{13!}{10!\,3!}$

$= 286$

b) $^{13}P_{10} = \dfrac{13!}{(13-10)!}$

$= \dfrac{13!}{3!}$

$= 1,037,836,800$

c) $^{3}C_{1} \times {}^{10}C_{9} = 30$

$^{3}C_{2} \times {}^{10}C_{8} = 135$

$^{3}C_{3} \times {}^{10}C_{7} = 120$

$\therefore \; 30 + 135 + 120 = 285$

5. Show that if there are 30 students in a class, then at least two have last name begin with the same letter.

pigeon = 30

pigeonhole = 26

$\left\lceil \dfrac{30}{26} \right\rceil = \lceil 1.15 \rceil = 2$

since the pigeonholes < pigeon , so atleast 2 of the students have last name begin with the same letter.

6. What is the minimum number of students, each of whom comes from one of the 13 states, who must be enrolled in a university to guarantee that are at least100 come from the same state.

$\frac{n}{13} = 99$

$n = 1287$

$\therefore$ 1287 + 1 = 1288 students are considered such that there is atleast one student from each state, then there is at least one state from which 100 students might have come.

$\therefore$ 1288 students

7. Suppose that there are nine students in a discrete mathematics class at a small college
   a) show that the class must have at least five male students or five female students
   b) show that the class must have at least three male students or five female students

a) pairing :

(F,M) : (0,9), (1,8), (2,7), (3,6), (4,5), (5,4), (6,3), (7,2), (8,1), (9,0)

The number of male students is less than 5

The number of female students is more than 5 and vice versa

Pigeon = 5 students

Pigeonhole = 2

$$\left\lceil \frac{9}{2} \right\rceil = \lceil 4.5 \rceil = 5$$

b) pairing :

(F,M) : (0,9), (1,8), (2,7), (3,6), (4,5), (5,4), (6,3), (7,2), (8,1), (9,0)

F = female

M = male

when $(M \leq 3)$ then $(F > 5)$

when $(M > 3)$ then $(F \leq 5)$