ASSIGNMENT 2

GROUP (2) MEMBERS

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1. Determine whether the relation R on set Z (set of integer number) is reflexive, irreflexive, symmetric, asymmetric, antisymmetric, or transitive.

$$a\,R\,b \text{ if and only if } |a-b| = 2$$

$R = \{ \ldots (-3,-1), (-1,-3), (-1,1), (1,-1), (0,-2), (-2,0), (0,2), (2,0), (1,3), (3,1) \ldots \}$

$\therefore$ Relation R on set $\mathbb{Z}$ is irreflexive because $(a,a) \notin R$, $\forall a : a \in \mathbb{Z}$

Relation R on set $\mathbb{Z}$ is symmetric because $\forall a,b \in R$ $(a,b) \in R \rightarrow (b,a) \in R$

2. Given a relation, R on $A = \{a, b, c, d\}$ on as follows:

$$R = \{(a,a),(a,b),(a,d),(b,b),(b,c),(c,c),(c,d),(d,a),(d,d)\}$$

Show the matrix of relation, M_R and determine whether the relation, R is an equivalence relation.

$$M_R = \begin{array}{c|c} & \begin{matrix} a & b & c & d \end{matrix} \\ \begin{matrix} a \\ b \\ c \\ d \end{matrix} & \begin{bmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

equivalence relation
- reflexive
- symmetric
- transitive

$\therefore$ R is reflexive because $(a,a) \in R$ for every $a \in A$.

$\therefore$ R is not symmetric $(a,b) \in R$, $(b,a) \notin R$

$\therefore$ R is not transitive (b,c) and $(c,d) \in R$ but $(b,d) \notin R$

$\therefore$ R is not an equivalence

3. Let $f(x,y) = (2x - y, x - 2y)$; $(x,y) \in \mathbf{R} \times \mathbf{R}$, ($\mathbf{R}$ is set of real numbers.)

a) Show that f is one to one.
b) Find f^{-1}

a) $f(x_1, y_1) = f(x_2, y_2)$

$(2x_1 - y_1, x_1 - 2y_1) = (2x_2 - y_2, x_2 - 2y_2)$

$2x_1 - y_1 = 2x_2 - y_2$ —— 1

$x_1 - 2y_1 = x_2 - 2y_2$ —— 2

From 2

$x_1 = x_2 - 2y_2 + 2y_1$ — 3

3 into 1

$2(x_2 - 2y_2 + 2y_1) - y_1 = 2x_2 - y_2$

$2x_2 - 4y_2 + 4y_1 - y_1 = 2x_2 - y_2$

$-3y_2 + 3y_1 = 0$

$3y_1 = 3y_2$

$\boxed{y_1 = y_2}$

$y_1 = y_2$ into 1

$2x_1 - y_2 = 2x_2 - y_2$

$2x_1 = 2x_2$

$\boxed{x_1 = x_2}$

$\therefore (x_1, y_1) = (x_2, y_2)$

$f(x,y)$ is one-to-one shown

b) Let $f(x,y) = (X, Y)$

$(2x - y,\ x - 2y) = (X, Y)$

$2x - y = X$ — ①

$x - 2y = Y$ — ②

From ②

$x = Y + 2y$ — ③

③ into ①

$2(Y + 2y) - y = X$

$2Y + 4y - y = X$

$2Y + 3y = X$

$y = \dfrac{X - 2Y}{3}$ — ④

④ into ③

$x = Y + 2\left(\dfrac{X - 2Y}{3}\right)$

$x = Y + \left(\dfrac{2X - 4Y}{3}\right)$

$x = \dfrac{3Y + 2X - 4Y}{3}$

$x = \dfrac{2X - Y}{3}$

$\therefore\ x = \dfrac{2X - Y}{3}, \quad y = \dfrac{X - 2Y}{3}$

$(x, y) = \left(\dfrac{2X - Y}{3}, \dfrac{X - 2Y}{3}\right)$

$\therefore\ f^{-1}(X, Y) = (x, y) = \left(\dfrac{2X - Y}{3}, \dfrac{X - 2Y}{3}\right)$

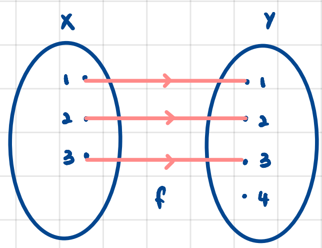
$\therefore\ f^{-1}(x, y) = \left(\dfrac{2X - Y}{3}, \dfrac{X - 2Y}{3}\right)$ *

4. Let a set $X = \{1, 2, 3\}$, $Y = \{1, 2, 3, 4\}$ and $W = \{1,2\}$.

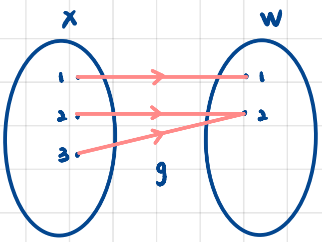
a) Draw the arrow diagram to define function $f: X \to Y$ that is one-to-one but not onto.

b) List the three ordered pairs to define function $g: X \to W$ that is onto but not one-to-one.

a)



b)



$g: \{(1,1), (2,2), (3,2)\}$

5. Function f and g are defined by formulas as shown below.

$$f(x) = x^3 \quad \text{and} \quad g(x) = x - 1, \text{ for all real number } x.$$

i) Find $g \circ f$ and $f \circ g$.
ii) Determine whether $g \circ f$ equals $f \circ g$.

i) $gof = g[f(x)]$ $\quad fog = f[g(x)] = f(x-1)$
$= g(x^3)$ $\quad = x^3 - 1$
$= (x-1)^3$

ii) $gof \neq fog$
$(x-1)^3 \neq x^3 - 1$
$\therefore$ gof does not equal fog

6. Let $A = \{0,1\}$. Give a recurrence relation for the strings of length n in A * that do not contain 01. Note: A * is the set of all string over A

$A = \{0,1\}$

$a_1 = \{0\}$ or $\{1\}$
$\therefore a_1 = 2$

$a_2 = \{00, 10, 11\}$ where $01 \notin a_2$
$a_2 = 3$

$a_3 = \{000, 100, 110, 111\}$ where $001, 010 \notin a_3$
$a_3 = 4$

$\therefore a_1 = 2$
$a_2 = 3$
$a_3 = 4$

$\therefore$ Hence, the recurrence relation is :
$a_n = a_{n-1} + 1, \quad n \geq 2$

7. A game is played by moving a marker ahead either 2 or 3 steps on a linear path. Let c_n be the number of different ways a path of length n can be covered. Given,

$c_n = c_{n-2} + c_{n-3}$, $c_1 = 0$, $c_2 = 1$, $c_3 = 1$

Write a recursive algorithm to compute c_n.

```
C(n) {
        if (n = 1)
            return 0

        if (n = 2 or n = 3)
            return 1

        return C(n-2) + C(n-3)

}
```