

ASSIGNMENT 1

1. Let set $U = \{n \mid n \in \text{whole numbers}, 10 \leq n \leq 30\}$;

set $B = \{b \mid b \in \text{even numbers}\}$;

set $C = \{c \mid c \in \text{natural numbers}, c > 10 \text{ and } c < 30\}$; $10 < c < 30$

$B \subseteq C$;

$C \subseteq U$.

Answer all the following questions:

- Write down all the possible proper subsets for C and find $|C|$.
- Write down all the possible proper subsets for B and find $|B|$.
- Construct a Venn diagram based on the given sets.
- Find cardinality of symmetric difference of set B and set C.

1) a) $C = \{ \emptyset, \{11\}, \{12\}, \{13\}, \{14\}, \{15\}, \{16\}, \{17\}, \{18\}, \{19\}, \{20\}, \{21\}, \{22\}, \{23\}, \{24\}, \{25\}, \{26\}, \{27\}, \{28\}, \{29\} \}$

b) $B = \{ \emptyset, \{10\}, \{12\}, \{14\}, \{16\}, \{18\}, \{20\}, \{22\}, \{24\}, \{26\}, \{28\}, \{30\} \}$

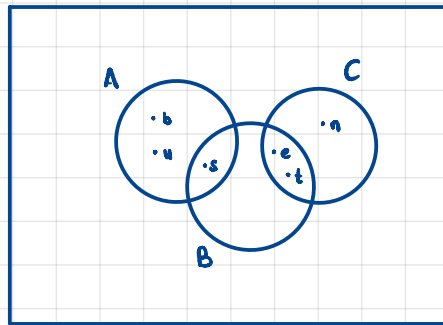
2. Let set $A = \{s, u, b\}$;

set $B = \{s, e, t\}$;

set $C = \{n, e, t\}$.

Answer all the following questions:

- Find $|P(A)|$.
- Find $A \cup B \cap C$
- Find $A \cap B \cup C$
- Find $A - B$
- Find $B \times C$



$$2) a) |P(A)| = 2^3 \\ = 8$$

$$b) A \cup B \cap C = \{s, u, b, e, t\} \cap \{n, e, t\} \\ = \{e, t\}$$

$$c) A \cap B \cup C = \{s\} \cup \{s, n, e, t\} \\ = \{s, n, e, t\}$$

$$d) B' = \{b, u, n\} \\ A - B = A \cap B' \\ = \{s, u, b\} \cap \{b, u, n\} \\ = \{b, u\}$$

$$e) B \times C = \{(s, n), (s, e), (s, t), (e, n), (e, e), (e, t), (t, n), (t, e), (t, t)\}$$

3. Which of the following statements are propositions?

State true or false.

- The discrete structure implements set theory, relations, and functions to solve computer science problems.
- In computer science, the Boolean data type defines $0 = \text{false}$ and $1 = \text{true}$.
- Let A and B be the subsets of U . $A \cup (A \cap B) = A$ can be proved by distributive, idempotent, and commutative laws.
- $a^2 - 2a + 1 = 0$
- $a^2 - b^2 = 0$

- 3) a) TRUE
b) TRUE
c) TRUE
d) FALSE
e) FALSE

4. Construct a truth table for each of the following conditional statements.

a. $(p \rightarrow q) \wedge (\neg p \leftrightarrow \neg q)$

b. $(p \leftrightarrow q) \vee (\neg p \rightarrow \neg q)$

4) a)

p	q	$(p \rightarrow q)$	$(\neg p \leftrightarrow \neg q)$	$(p \rightarrow q) \wedge (\neg p \leftrightarrow \neg q)$
T	T	T	T	T
T	F	F	F	F
F	T	T	F	F
F	F	T	T	T

b)

p	q	$(p \leftrightarrow q)$	$(\neg p \rightarrow \neg q)$	$(p \leftrightarrow q) \vee (\neg p \rightarrow \neg q)$
T	T	T	T	T
T	F	F	T	T
F	T	F	F	F
F	F	T	T	T

5. Given, $A = \neg p \wedge (\neg q \vee \neg r)$ and $B = p \vee (q \wedge r)$. State whether $A \equiv B$ or not.

5)

p	q	r	$(\neg q \vee \neg r)$	A	p	q	r	$(q \wedge r)$	B
T	T	T	F	F	T	T	T	T	T
T	T	F	T	F	T	T	F	F	T
T	F	T	T	F	T	F	T	F	T
T	F	F	T	F	T	F	F	F	T
F	T	T	F	F	F	T	T	T	T
F	T	F	T	T	F	T	F	F	F
F	F	T	T	T	F	F	T	F	F
F	F	F	T	T	F	F	F	F	F

$\therefore A \neq B$

6. Given, $A = p \wedge (p \vee q)$ and $B = p \vee (p \wedge q)$. State whether $A \equiv B$ or not.

6)

p	q	$(p \vee q)$	A	p	q	$(p \wedge q)$	B
T	T	T	T	T	T	T	T
T	F	T	T	T	F	F	T
F	T	T	F	F	T	F	F
F	F	F	F	F	F	F	F

$\therefore A \equiv B$

7. Let $P(x)$, $Q(x)$, and $R(x)$ be the statements.

" x is a student," " x is smart," and " x is shy," respectively.

Express each of these statements using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of all people.

a. Some students are shy.

b. All smart people are not shy.

7) a) $P(x) = x \text{ is a student}$

$Q(x) = x \text{ is smart}$

$R(x) = x \text{ is shy}$

$\therefore \exists x (P(x) \wedge R(x))$

b) $\forall x (Q(x) \wedge R(x))$

8. Give direct proof to show a square of any negative numbers is positive.

$P(n) = n \text{ is a negative number}$

$Q(n) = n^2 \text{ is positive}$

$n = -2$

$\therefore \forall x (P(x) \rightarrow Q(x)) \text{ is true}$

$n^2 = (-2)^2 = 4$

$n^2 = 4 = \text{positive}$

9. Give a proof by contradiction to show if A and B are sets, then $A \cap (B \cap A') = \{\}$.

$A \cap (B \cap A') = \{\}$