Exercise 3

1. Let R be the relation from X = {1, 2, 3, 4} to Y= {1, 3, 5, 7} defined by xRy if and only if $x \in X$, $y \in Y$ and $x + 3y \leq 12$.

   a) List the elements of the set R.
   b) Find the domain of R.
   c) Find the range of R.

2. Let $A=\{a, b, c\}$ and $R: A \to A$. Draw the digraph representing the following properties:

   a) Symmetric and reflexive but not transitive
   b) Reflexive but not antisymmetric

3. Write a matrix representing an irreflexive relation R on a set $A = \{a, b, c\}$.

4. Let $B=\{d, e, f, g\}$ and R be the relation on B that has the matrix M_R.

$$M_R = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 1 \end{bmatrix}$$

   a) List in-degree and out-degree of all vertices.
   b) Determine whether the relation R on the set B represented by M_R is an equivalence relation. Explain.

5. Here are two function f: {1, 2, 3}→{10, 11, 12,13} and g: {10,11,12,13}→{4, 5, 6} whose rules are given in Table 1.

Table 1

x	1	2	3
$f(x)$	11	13	10

x	10	11	12	13
$g(x)$	4	5	4	6

   a) What is f(1)?
   b) What is g(11)?
   c) Draw arrow diagrams for f and g.
   d) Which of these compositions can be defined: *gof, gog, fog, or fof*?
   e) For any of the compositions above that are defined, give the domain and codomain, and draw the arrow diagram.

6. Let, f: **R** $\rightarrow$**R** be the function with rule $f(x) = 5x-7$.

   a) Show that f is one-to-one
   b) Find the inverse of f
      (note: **R** is the set of real numbers)

7. Let $X=\{-1, 0, 1\}$ and $Y=\{-2, 0, 2\}$. For each $x \in X$, define functions f: $X \rightarrow Y$ and g: $X \rightarrow Y$ by:

   $$f(x) = x^2 - x$$

   $$g(x) = 2x$$

   Determine if f and g are one-to-one, onto Y, and/or bijection.