Chapter 4

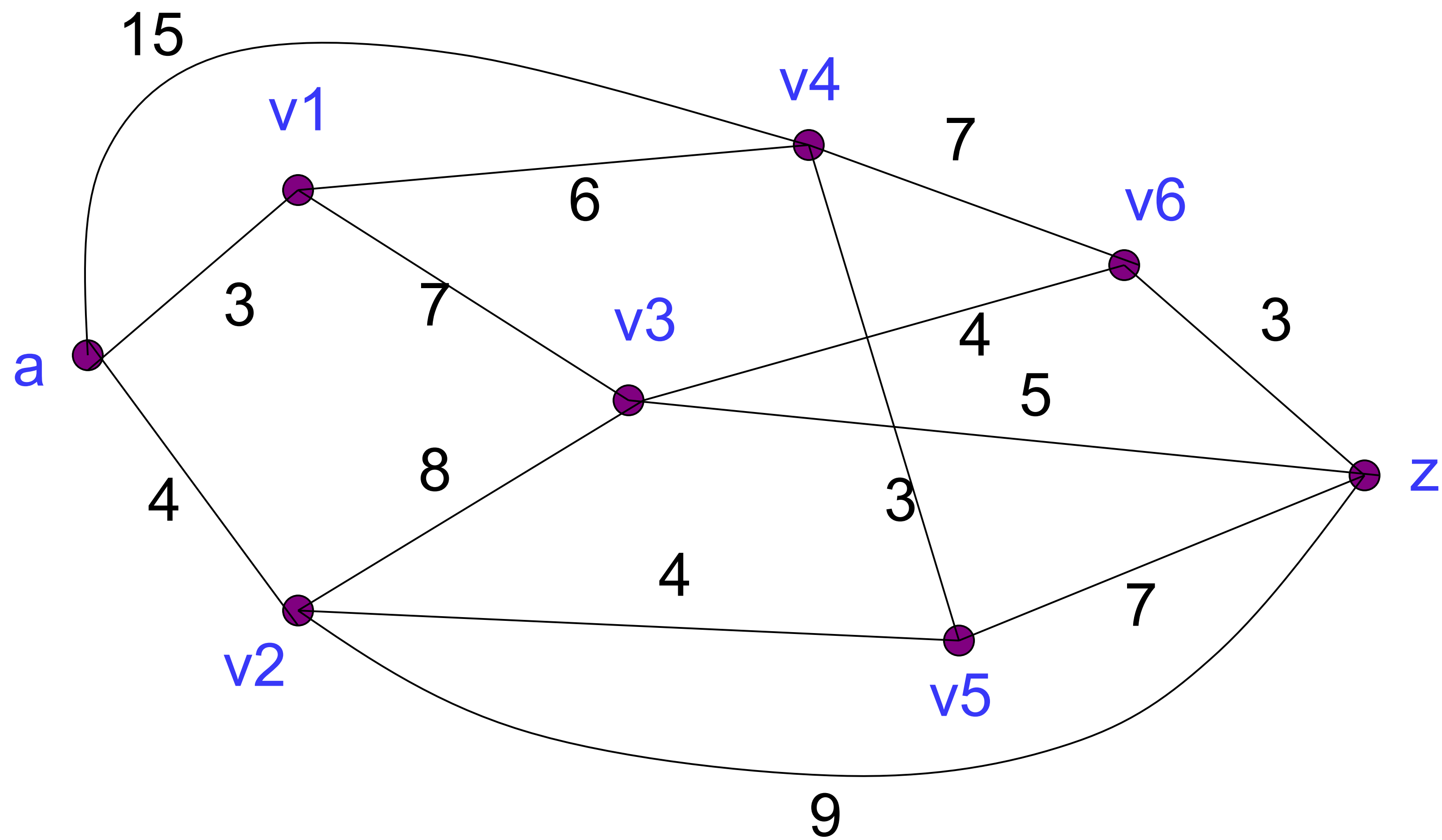
Shortest Path Problem

(Part 2)

Shortest Path Problems

- Let G be a weighted graph.
- Let u and v be two vertices in G, and let P be a path in G from u to v.
- The length of path P, written $L(P)$, is the sum of the weights of all the edges on path P.
- A shortest path from a vertex to another vertex is a path with the shortest length between the vertices.

example



Dijkstra’s Shortest Path Algorithm

1. $S := \emptyset$
2. $N := V$
3. For all vertices, $u \in V$, $u \neq a$, $L(u) := \infty$
4. $L(a) := 0$

Dijkstra's Shortest Path Algorithm

5. While $z \notin S$ do,

   5.a Let $v \in N$ be such that

   $L(v) = \min\{L(u) \mid u \in N\}$

   5.b $S := S \cup \{v\}$

   5.c $N := N - \{v\}$

Dijkstra's Shortest Path Algorithm

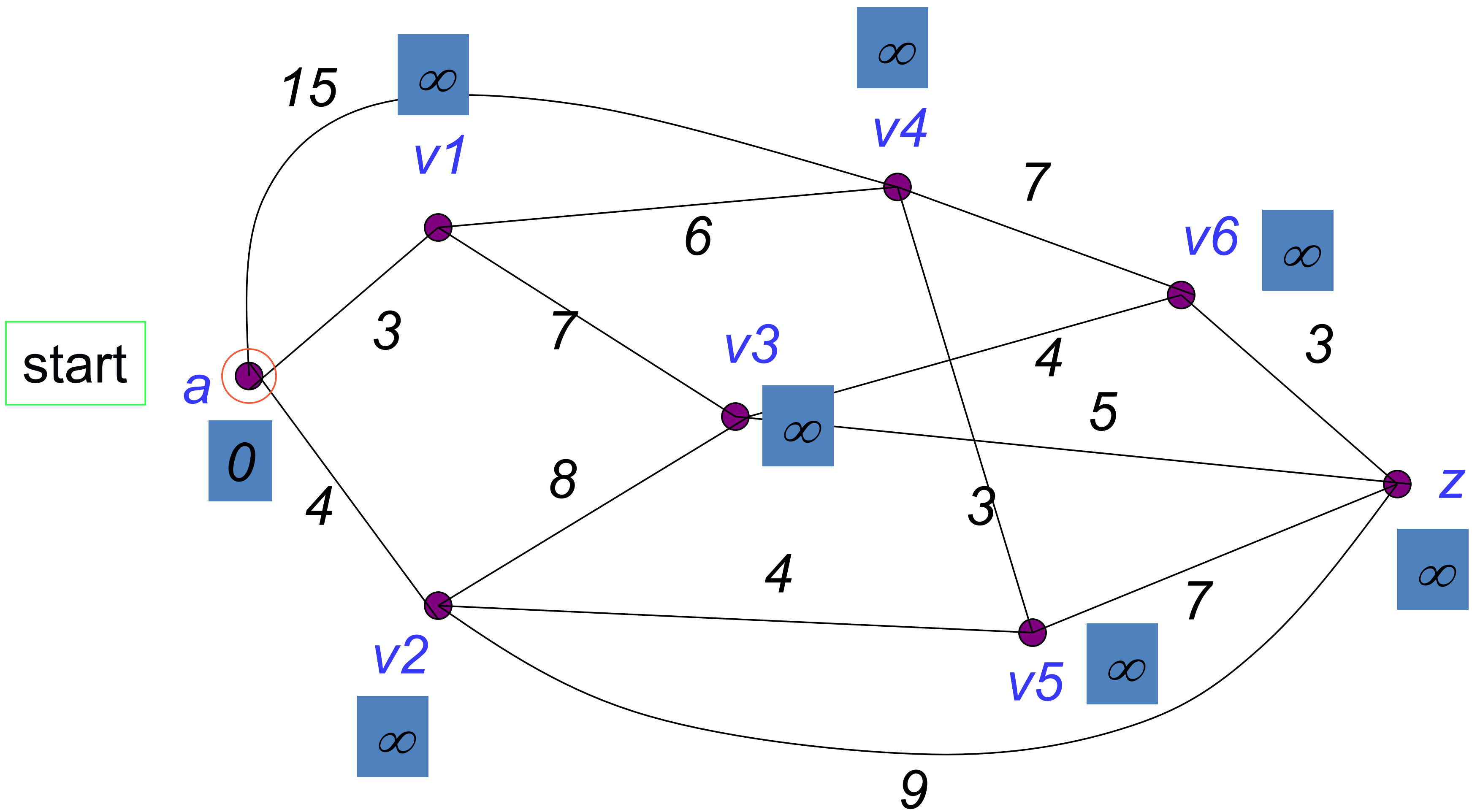
5.d For all $w \in N$ such that there is an edge from v to w

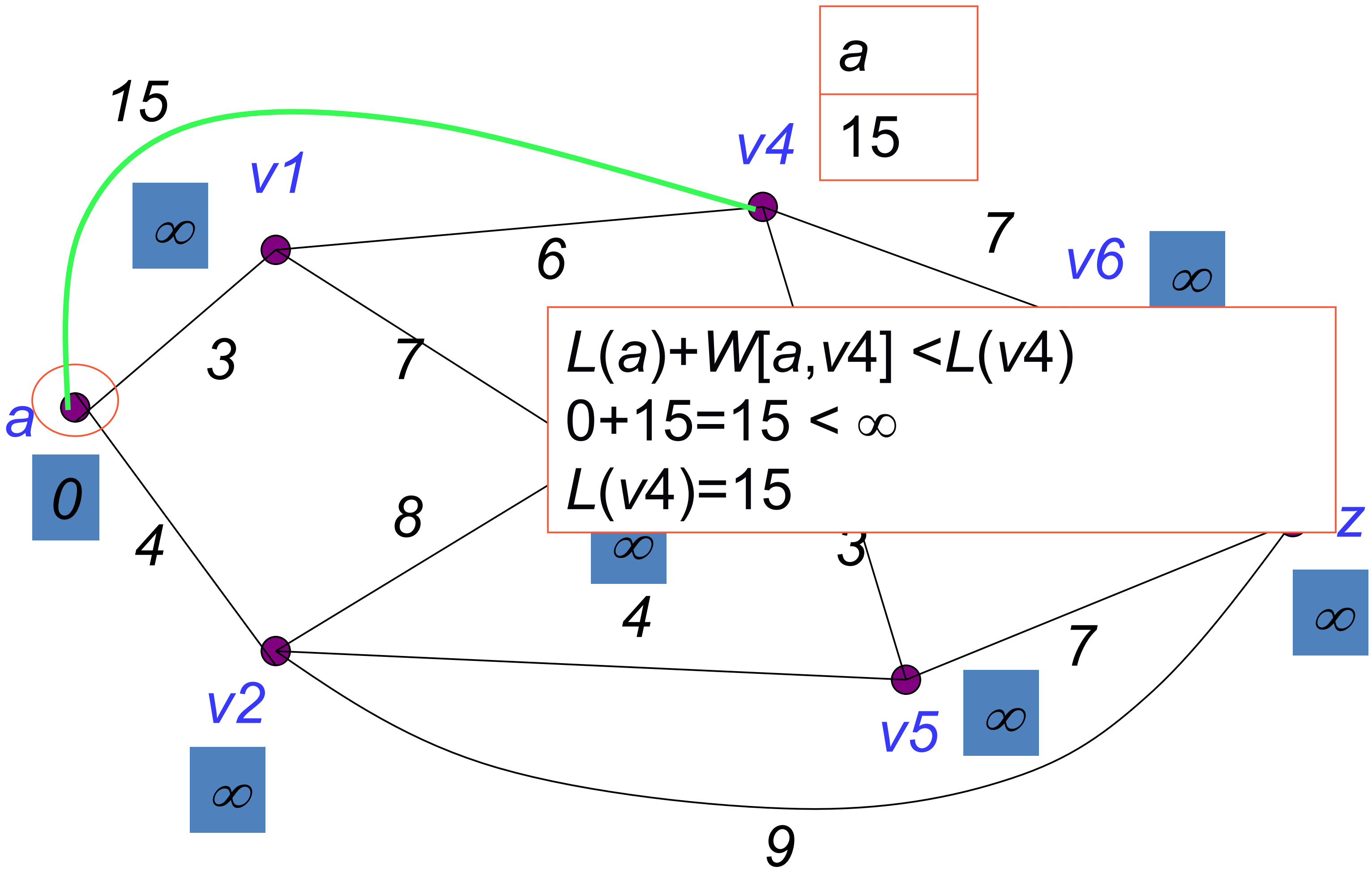
5.d.1 if $L(v)+W[v,w] < L(w)$ then

$$L(w)=L(v)+W[v,w]$$

$S=\varnothing$

$N= \{a,v1,v2,v3,v4,v5,v6,z\}$

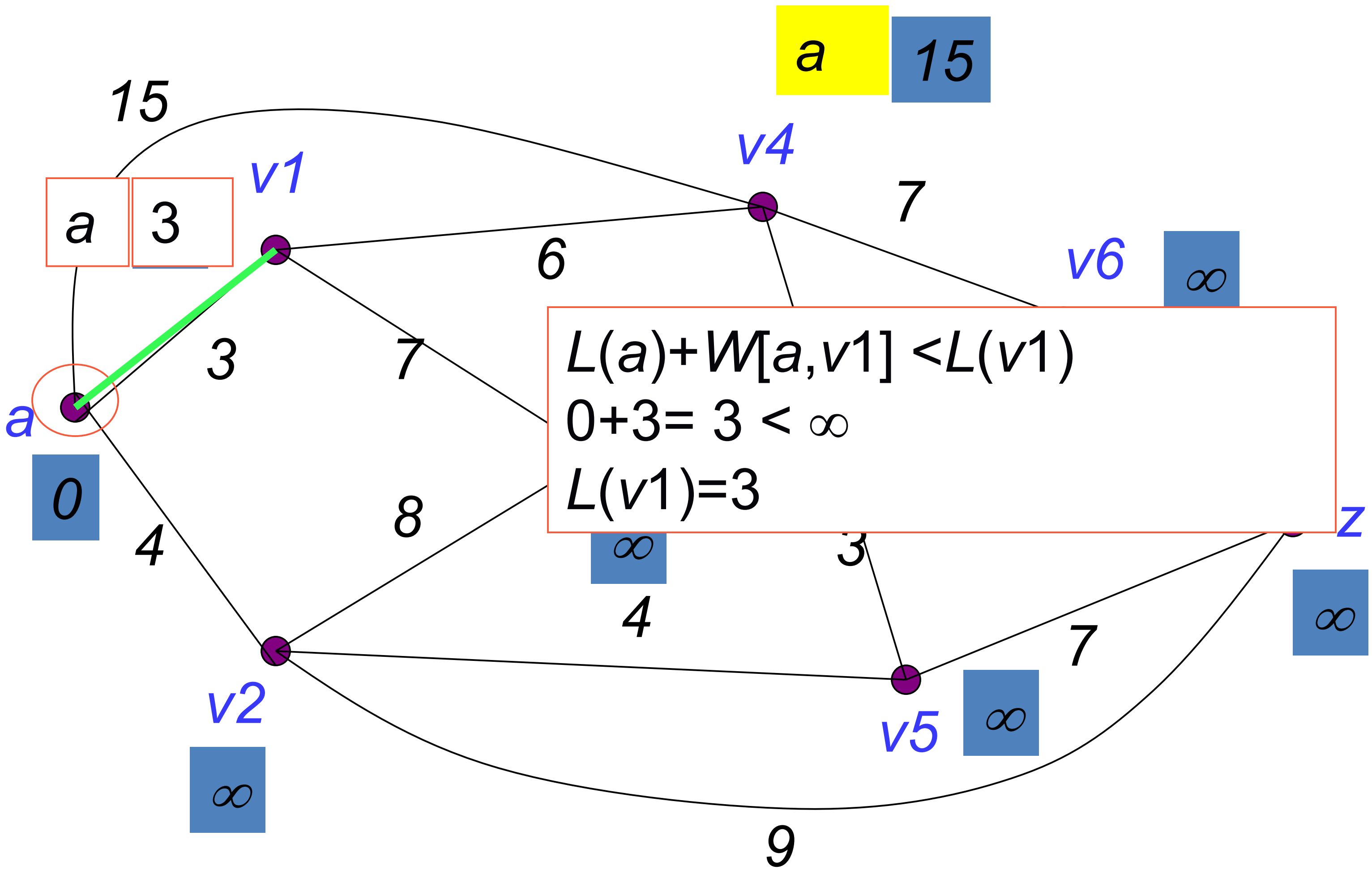




$L(a)+W[a,v4] < L(v4)$

$0+15=15 < \infty$

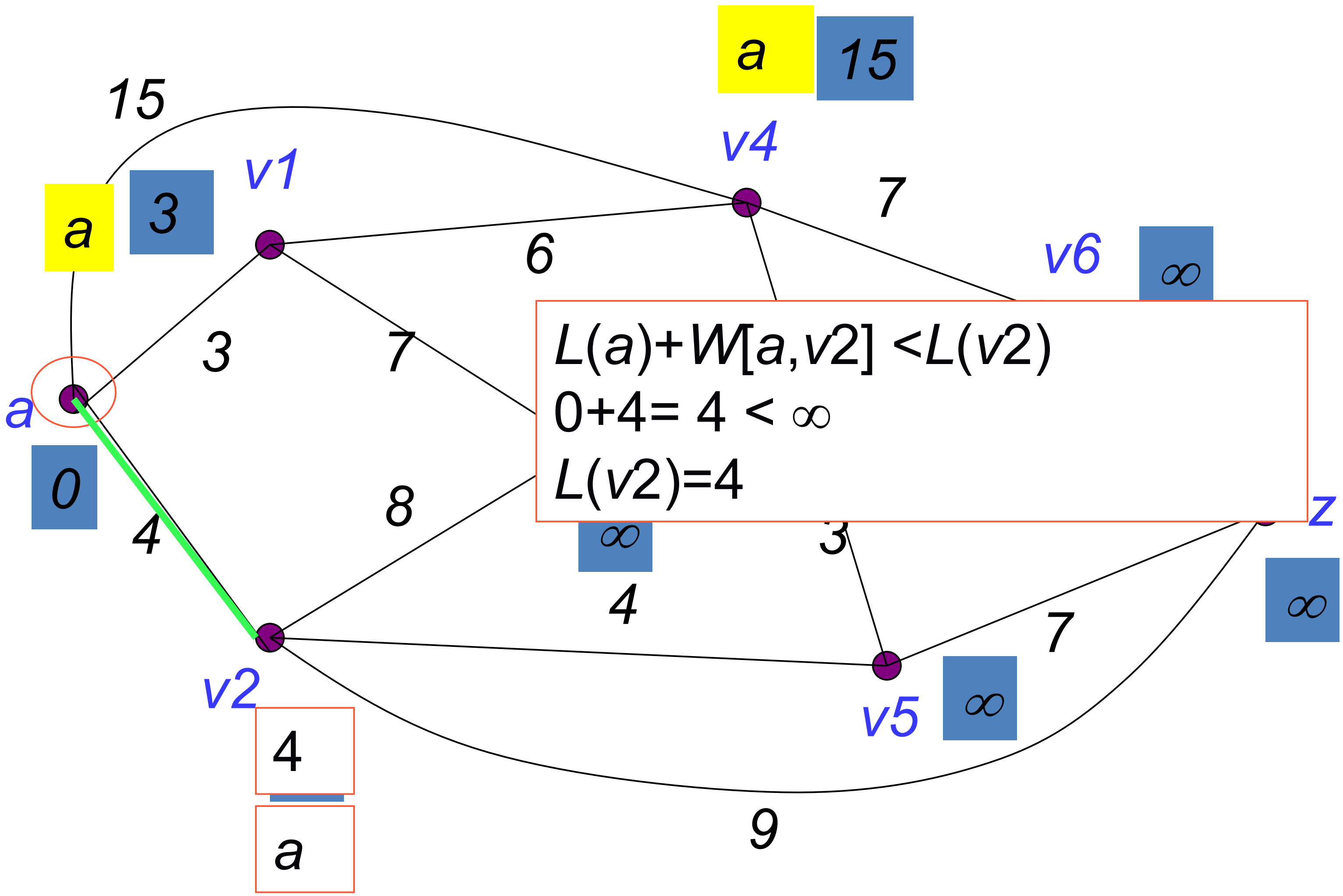
$L(v4)=15$



$L(a)+W[a,v1] < L(v1)$

$0+3= 3 < \infty$

$L(v1)=3$



$L(a)+W[a,v2] < L(v2)$

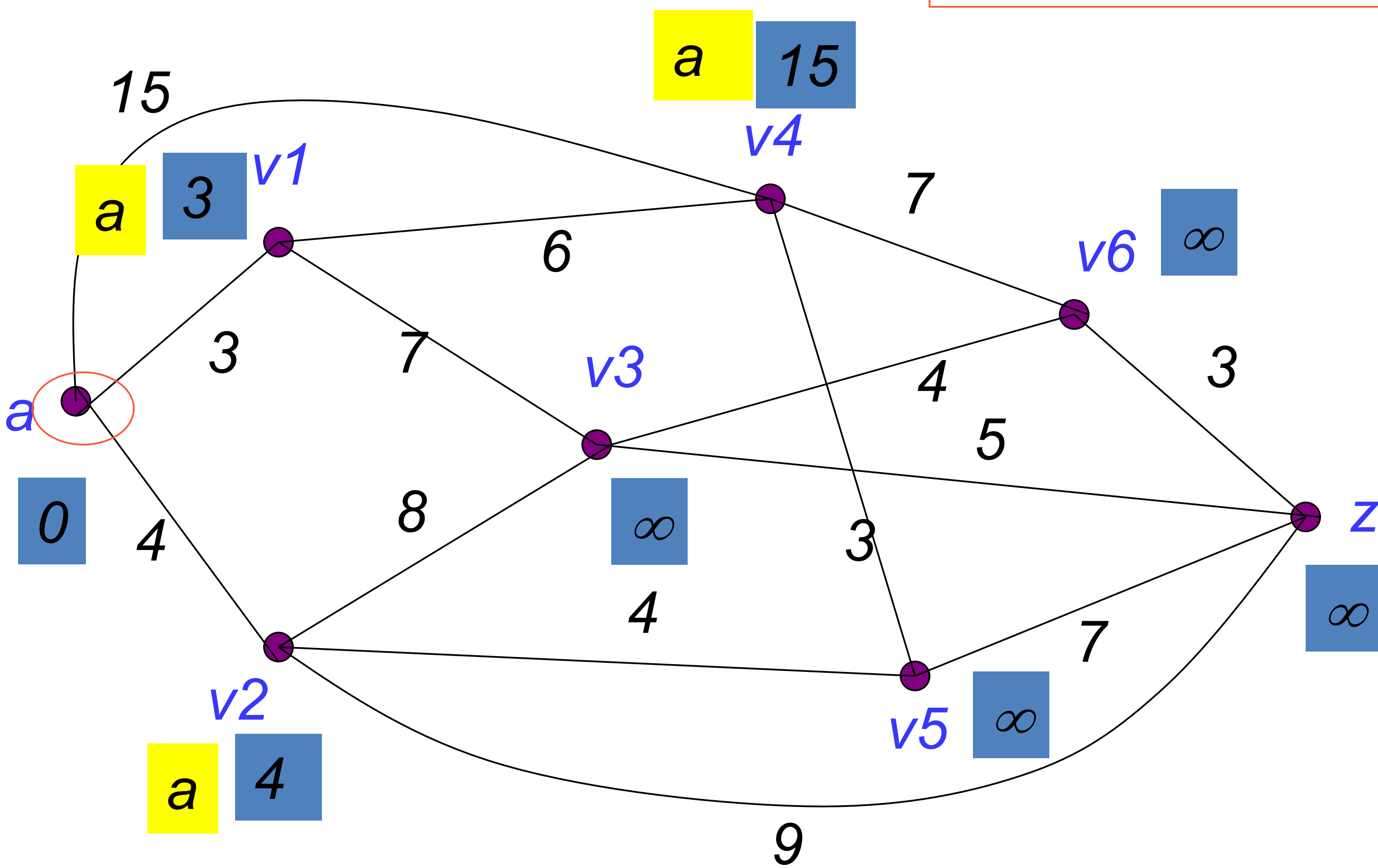
$0+4= 4 < \infty$

$L(v2)=4$

$S=\{a\}$

$N= \{v1,v2,v3,v4,v5$

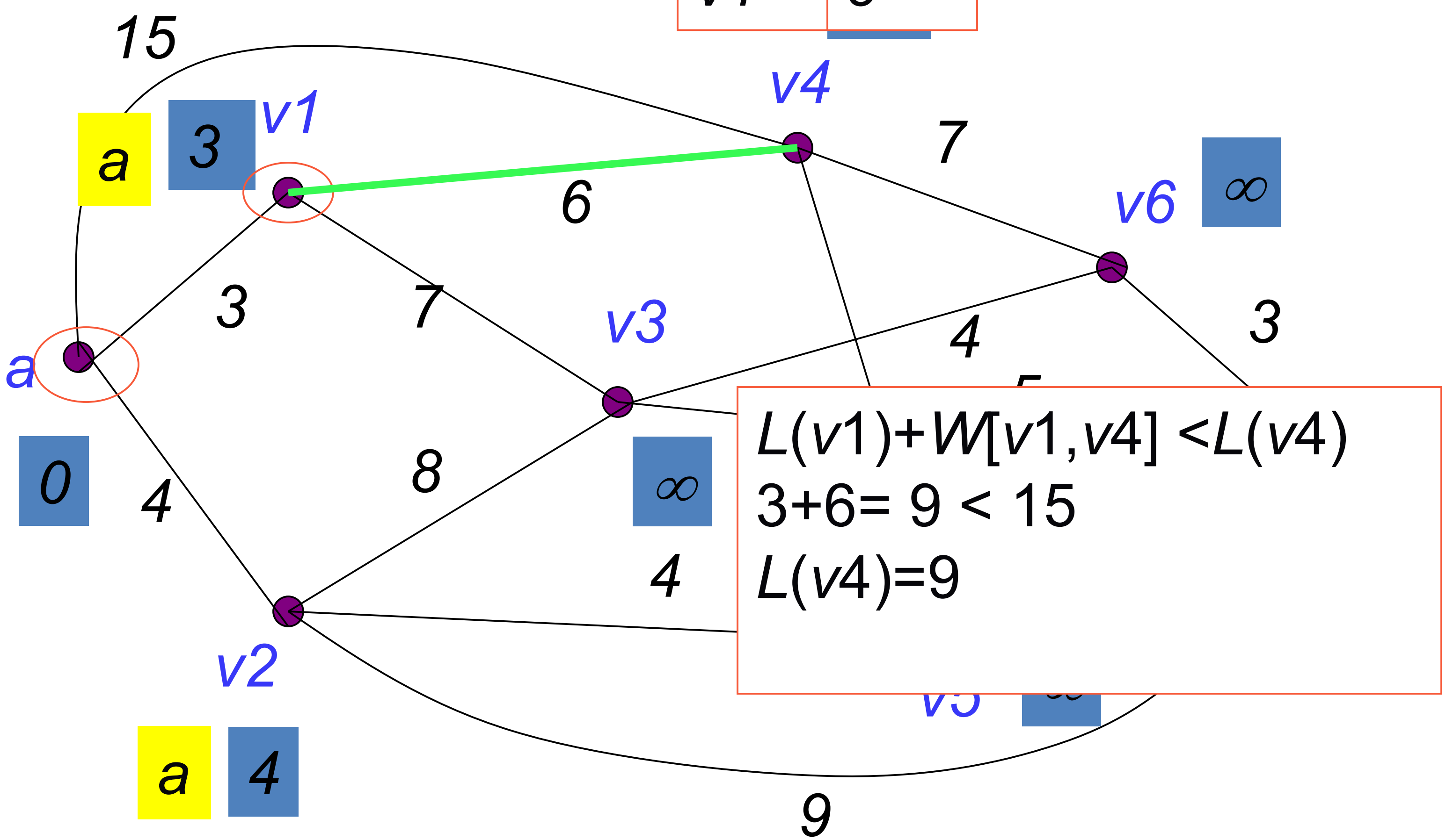
choose $v1$
because $L(v1)=3$
$= \min\{L(u)|u \in N\}$



$S=\{a,v1\}$

$N= \{v2,v3,v4,v5,v6,z\}$

$v1$	9



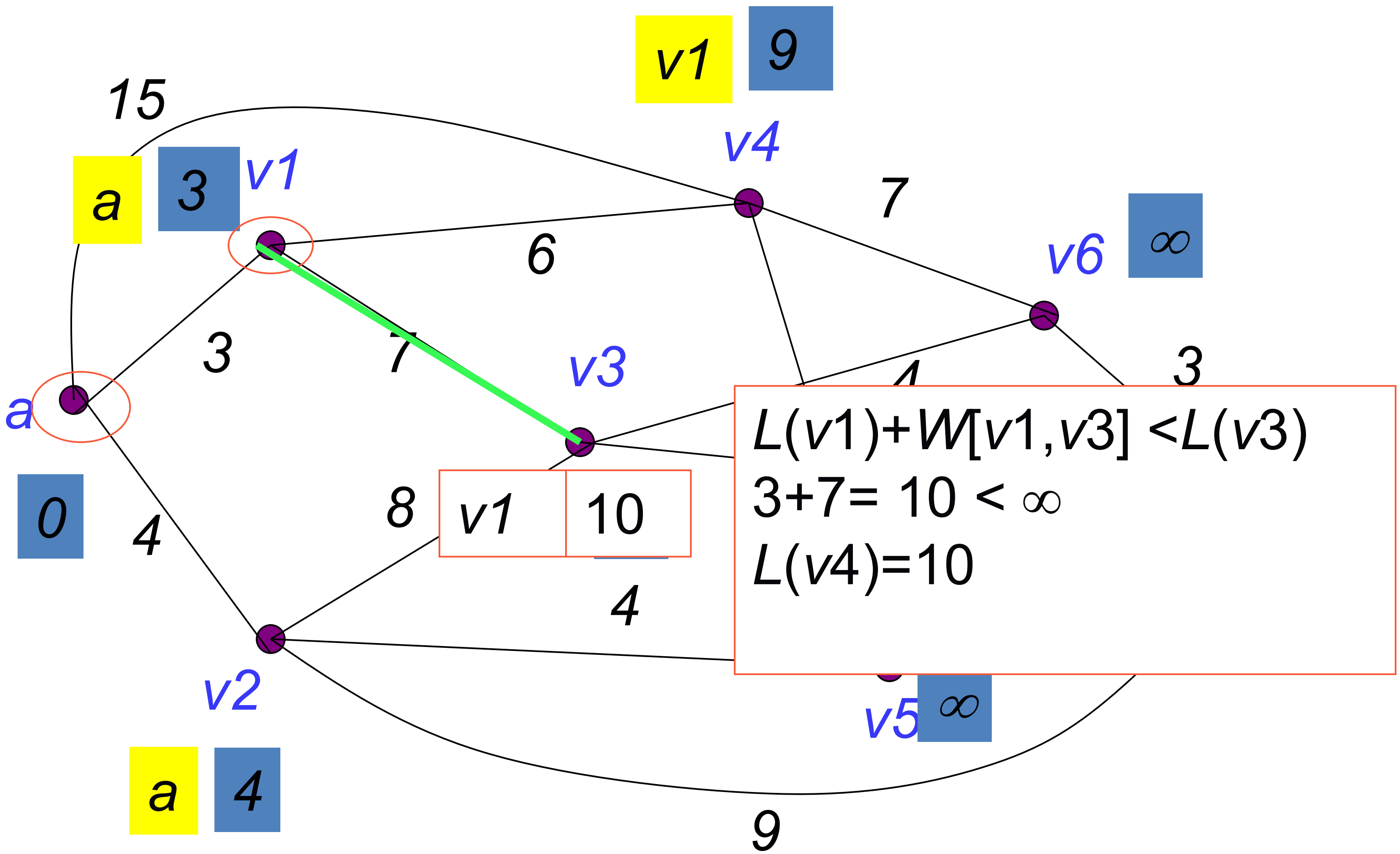
$L(v1)+W[v1,v4] <L(v4)$

3+6= 9 < 15

$L(v4)$=9

$S=\{a, v1\}$

$N=\{v2, v3, v4, v5, v6, z\}$



$L(v1)+W[v1,v3] < L(v3)$

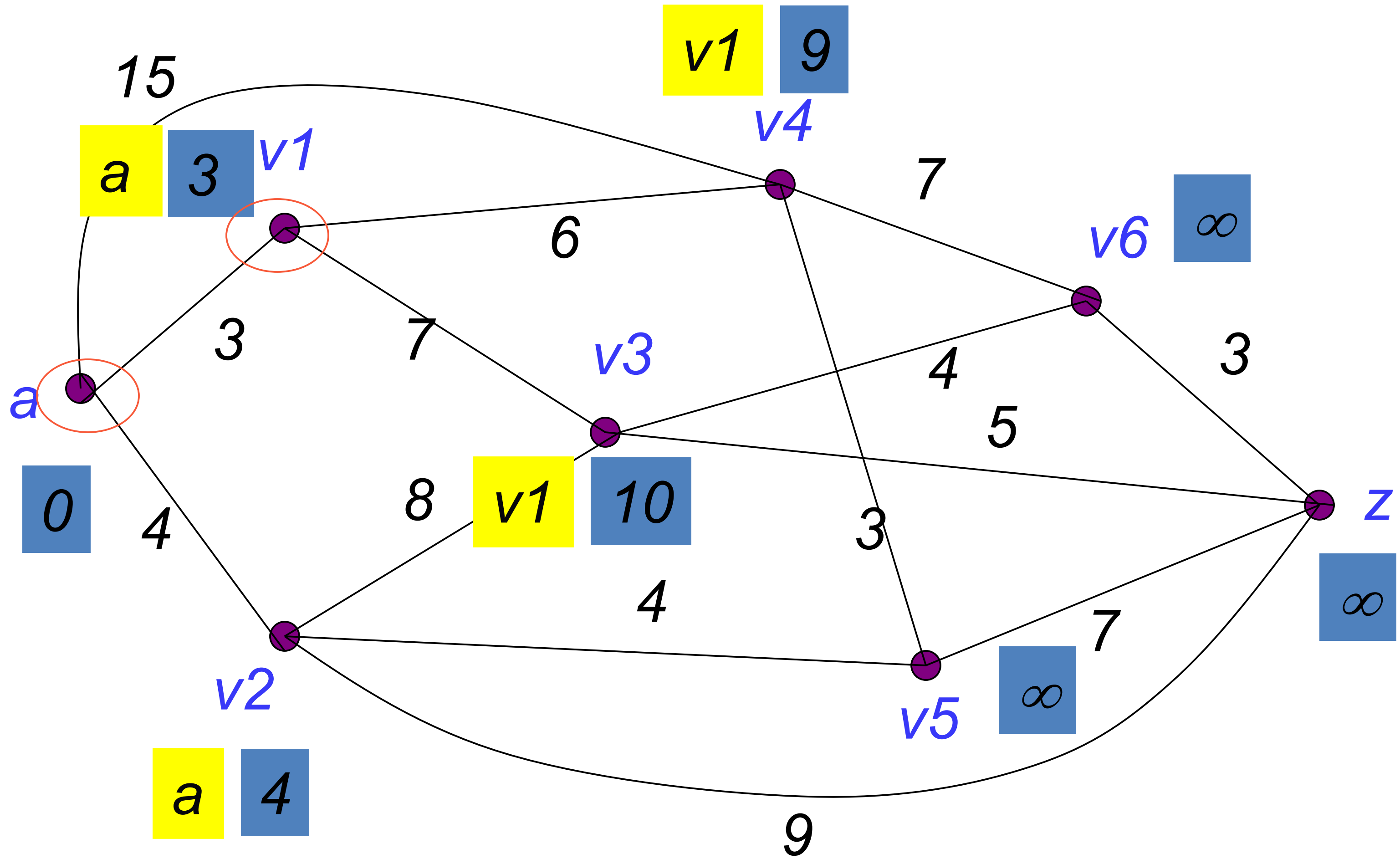
$3+7= 10 < \infty$

$L(v4)=10$

$S=\{a, v1\}$

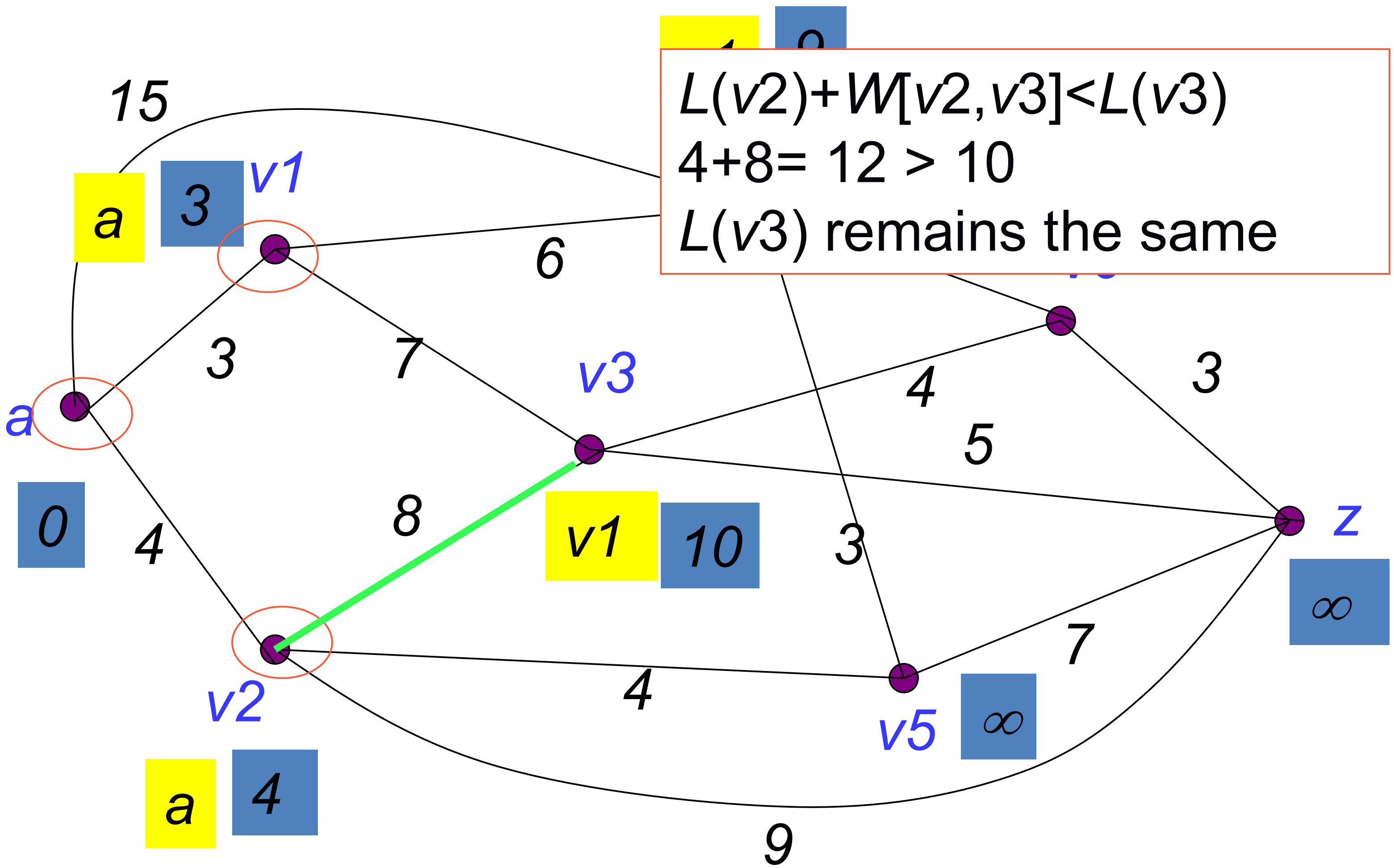
$N= \{v2,v3,v4,v5,v6,z\}$

choose $v2$ because $L(v2)=4 = \min\{L(u)|u \in N\}$



$S=\{a, v1, v2\}$

$N= \{v3,v4,v5,v6,z\}$



$L(v2)+W[v2,v3]<L(v3)$

$4+8= 12 > 10$

$L(v3)$ remains the same

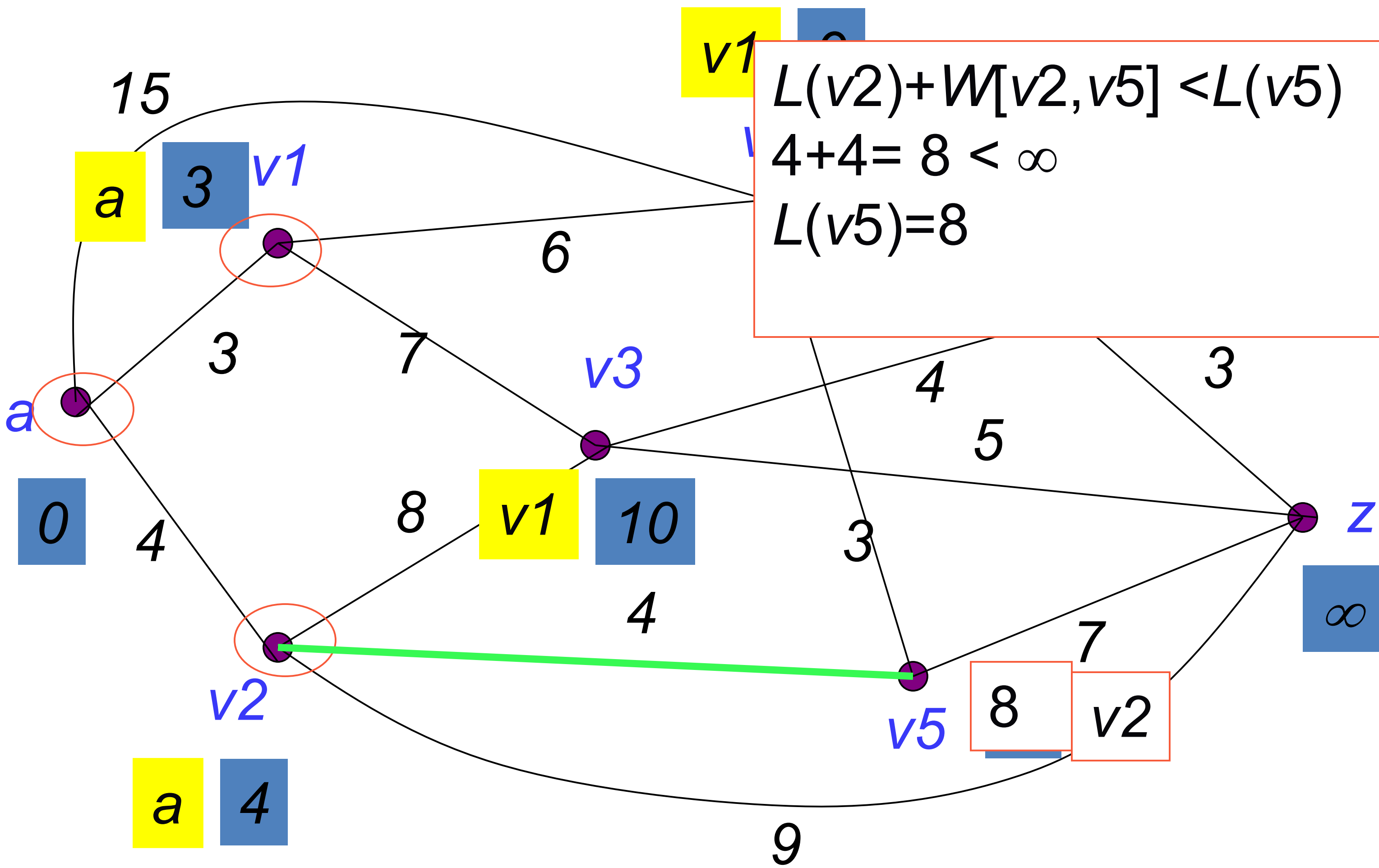
$S=\{a, v1, v2\}$

$N=\{v3,v4,v5,v6,z\}$

$L(v2)+W[v2,v5] < L(v5)$

$4+4= 8 < \infty$

$L(v5)=8$



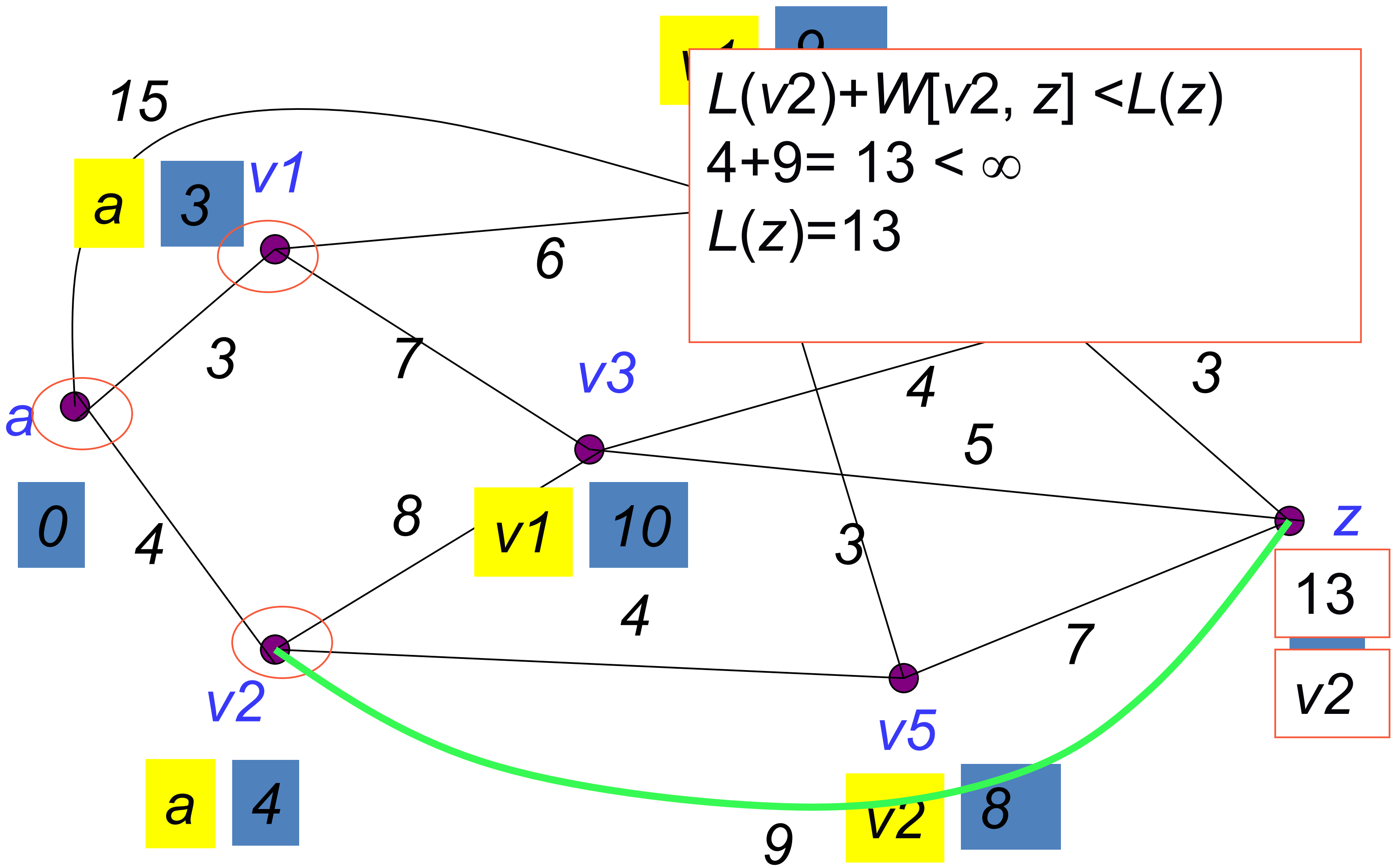
$S=\{a, v1, v2\}$

$N=\{v3,v4,v5,v6,z\}$

$L(v2)+W[v2, z] < L(z)$

$4+9= 13 < \infty$

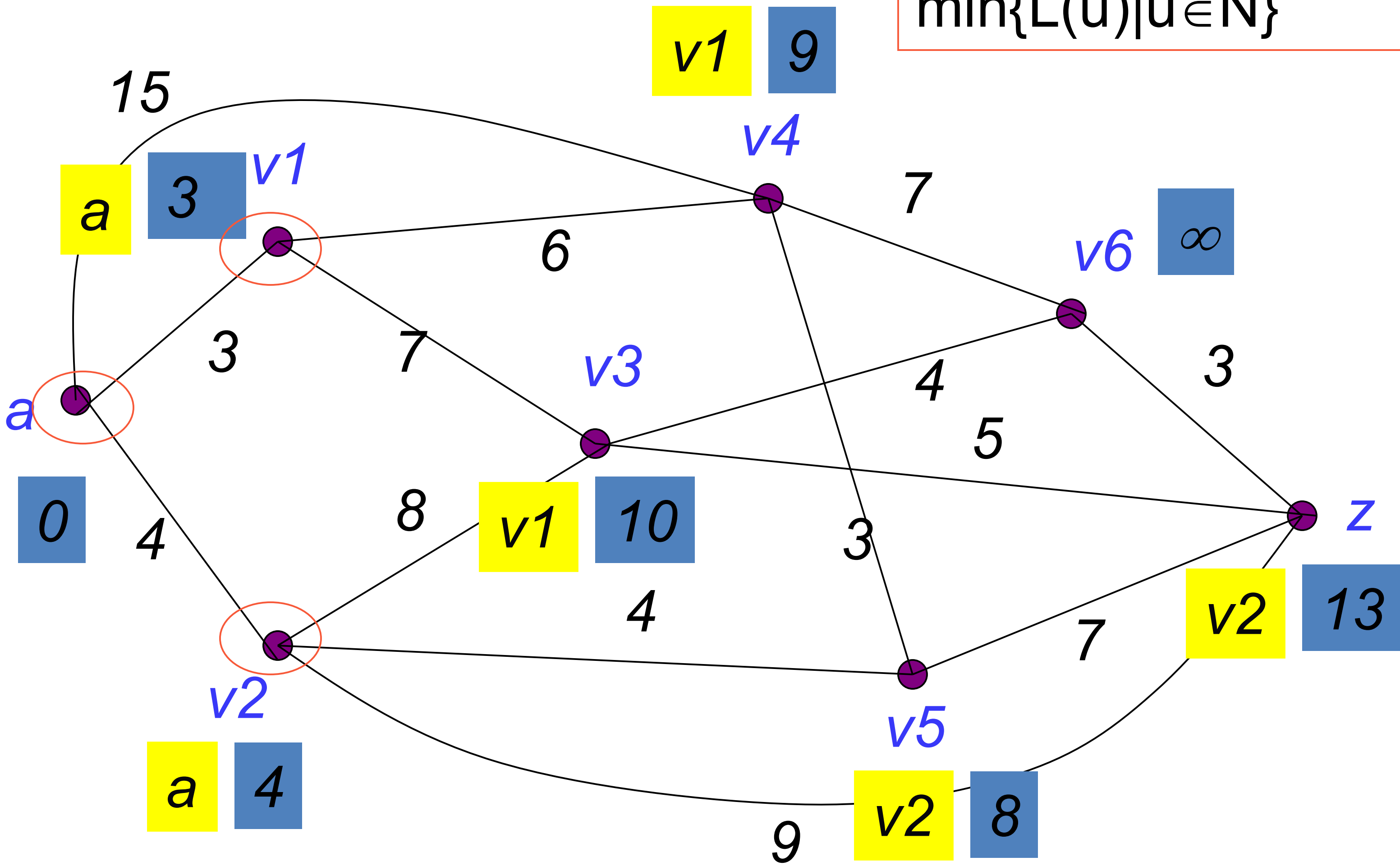
$L(z)=13$



$S=\{a, v1, v2\}$

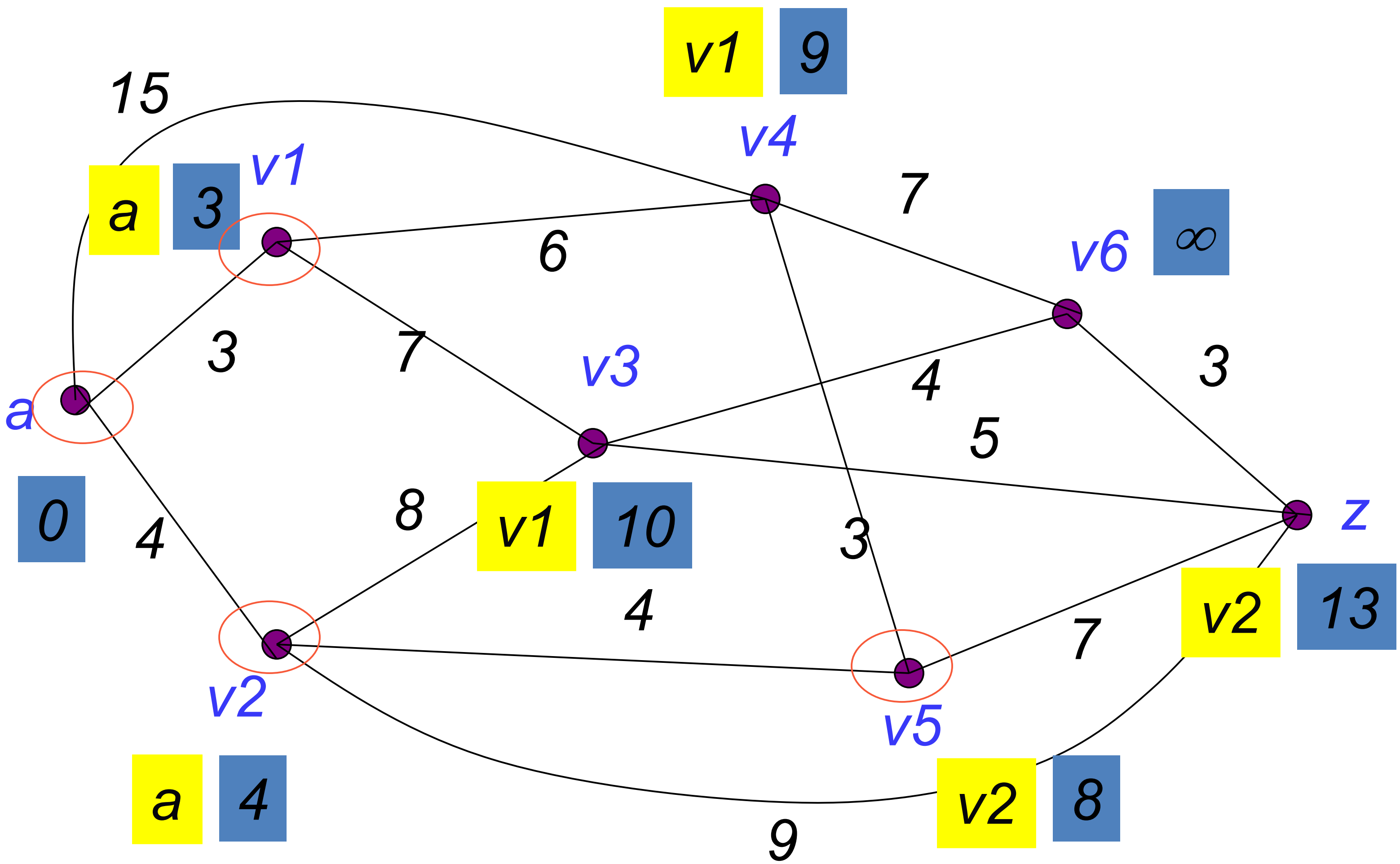
$N=\{v3, v4, v5, v6, z\}$

choose *v*5 because $L(v5)=8=\min\{L(u) \mid u \in N\}$



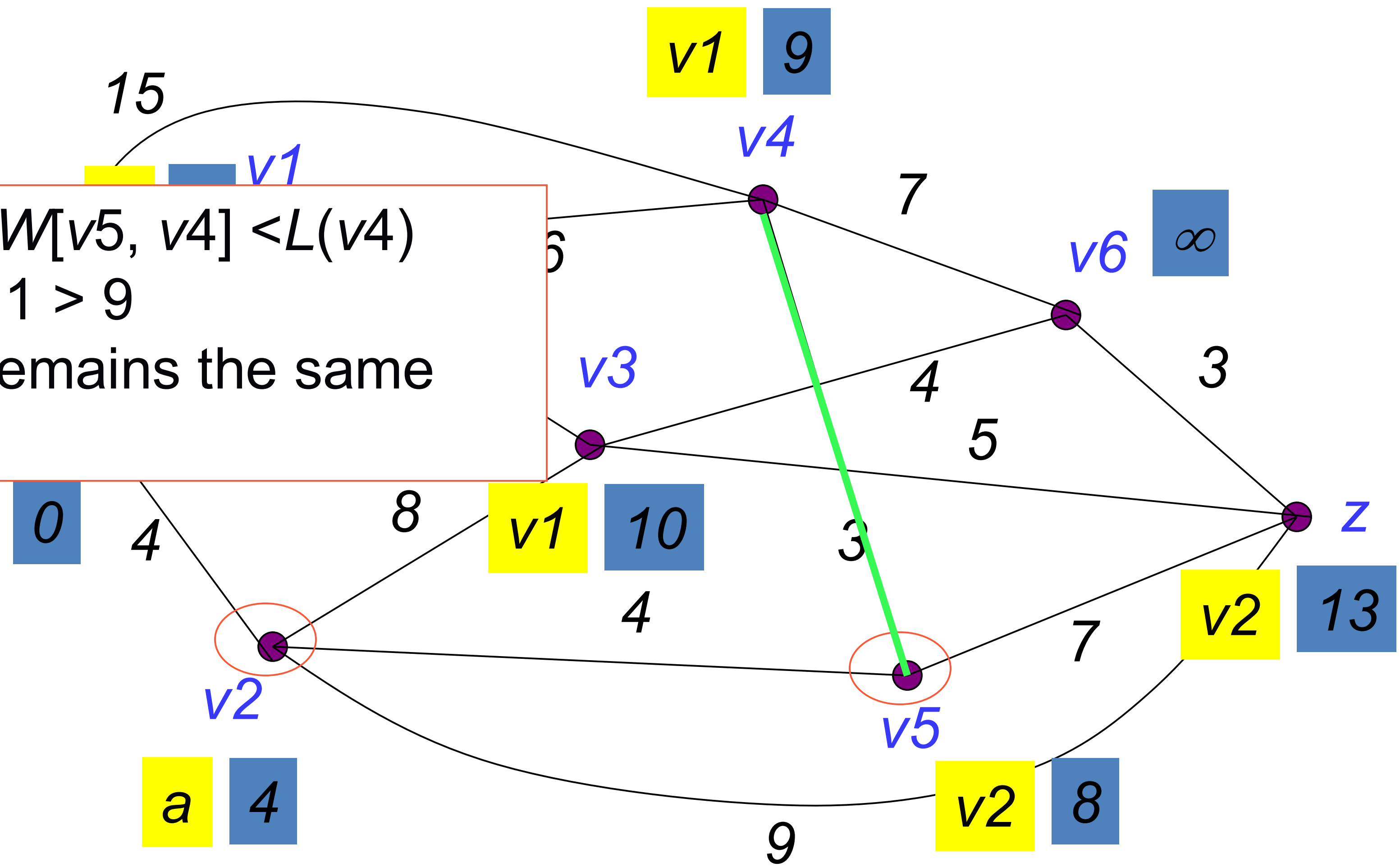
$S=\{a,v1,v2,v5\}$

$N=\{v3,v4,v6,z\}$



$S=\{a,v1,v2,v5\}$

$N=\{v3,v4,v6,z\}$



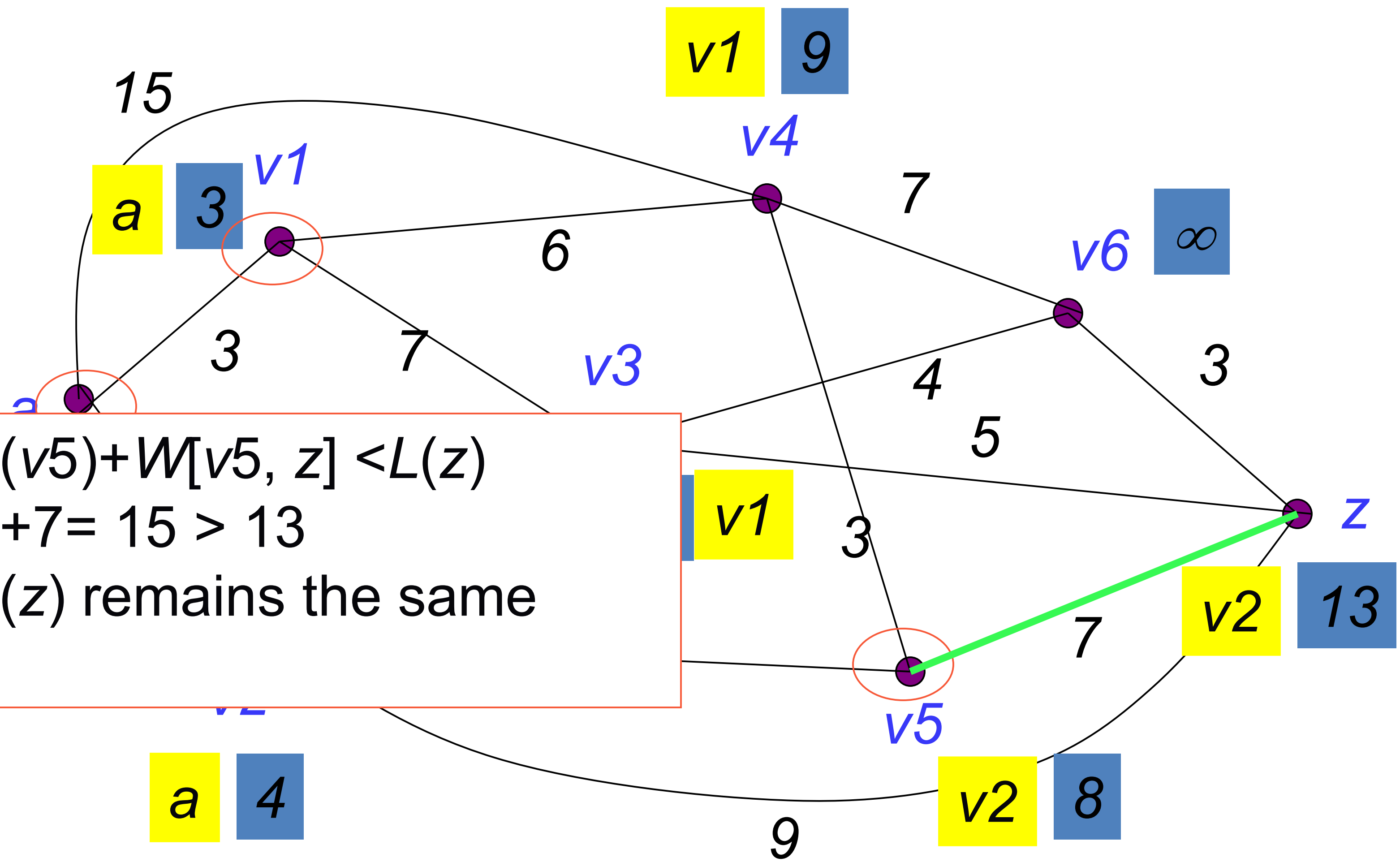
$L(v5)+W[v5, v4] < L(v4)$

$8+3= 11 > 9$

$L(v4)$ remains the same

$S=\{a, v1, v2, v5\}$

$N=\{v3, v4, v6, z\}$



$L(v5)+W[v5, z] < L(z)$

$8+7= 15 > 13$

$L(z)$ remains the same

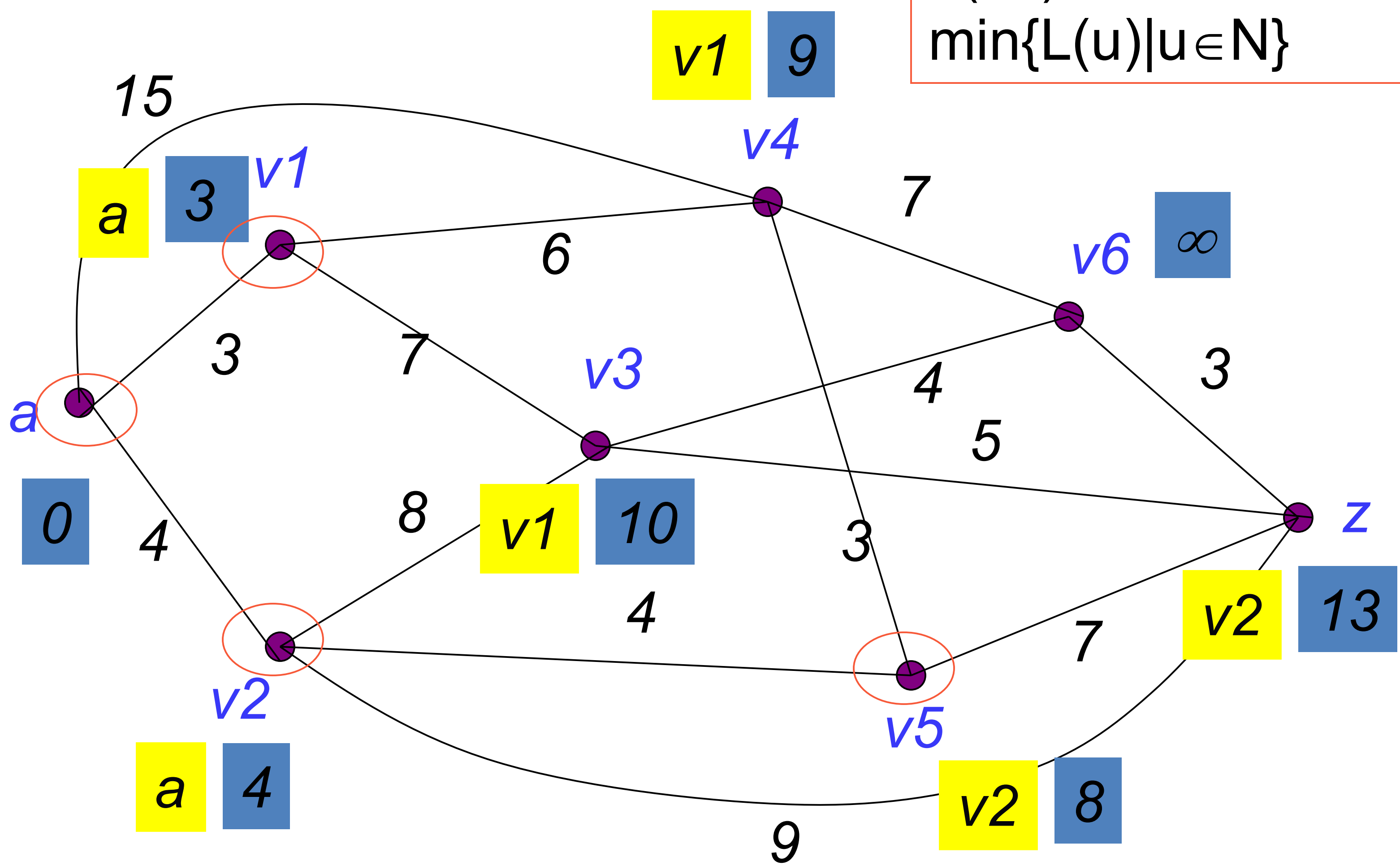
$S=\{a,v1,v2,v5\}$

$N= \{v3,v4,v6,z\}$

choose $v4$ because

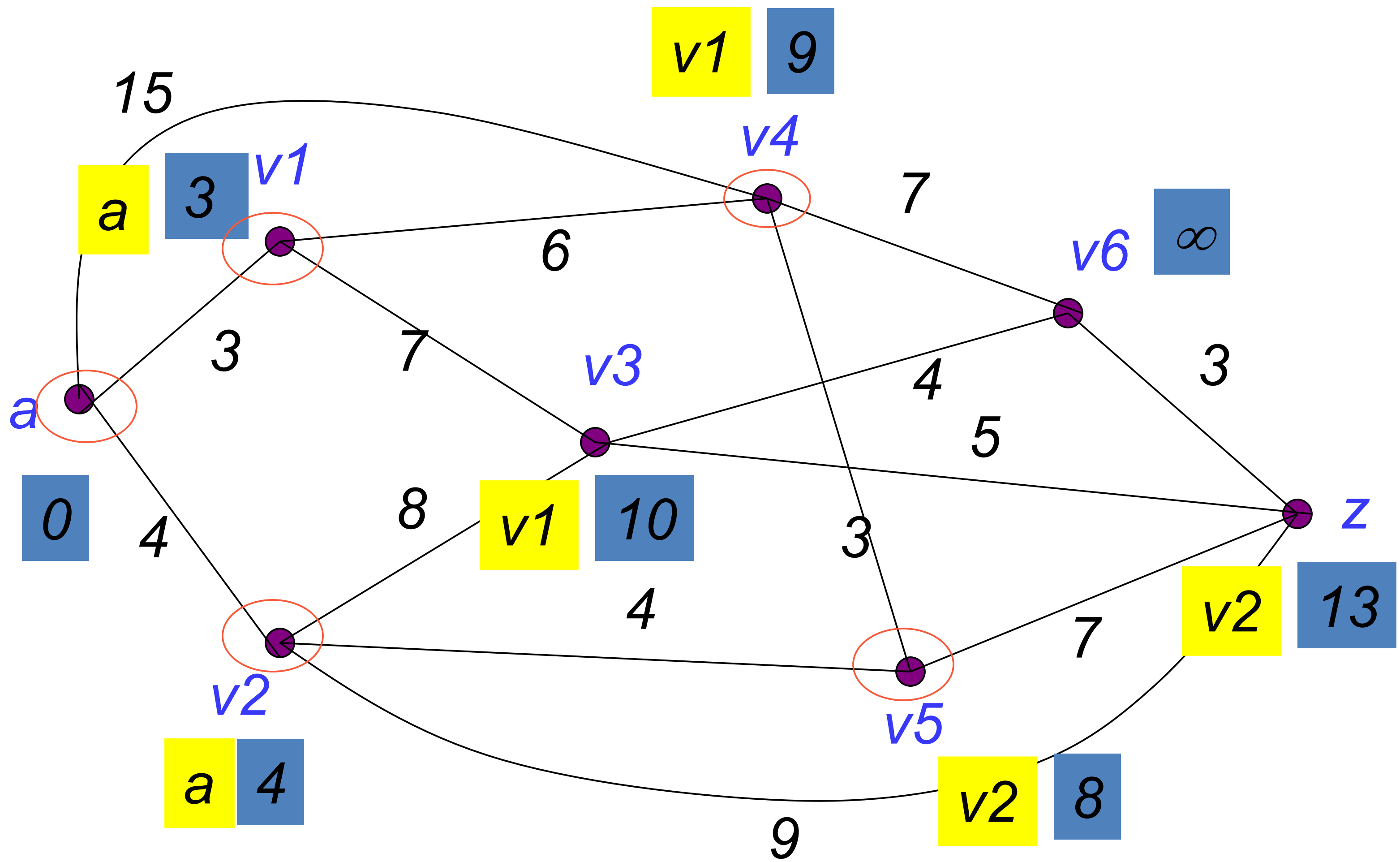
$L(v4)=9 =$

$\min\{L(u)|u\in N\}$



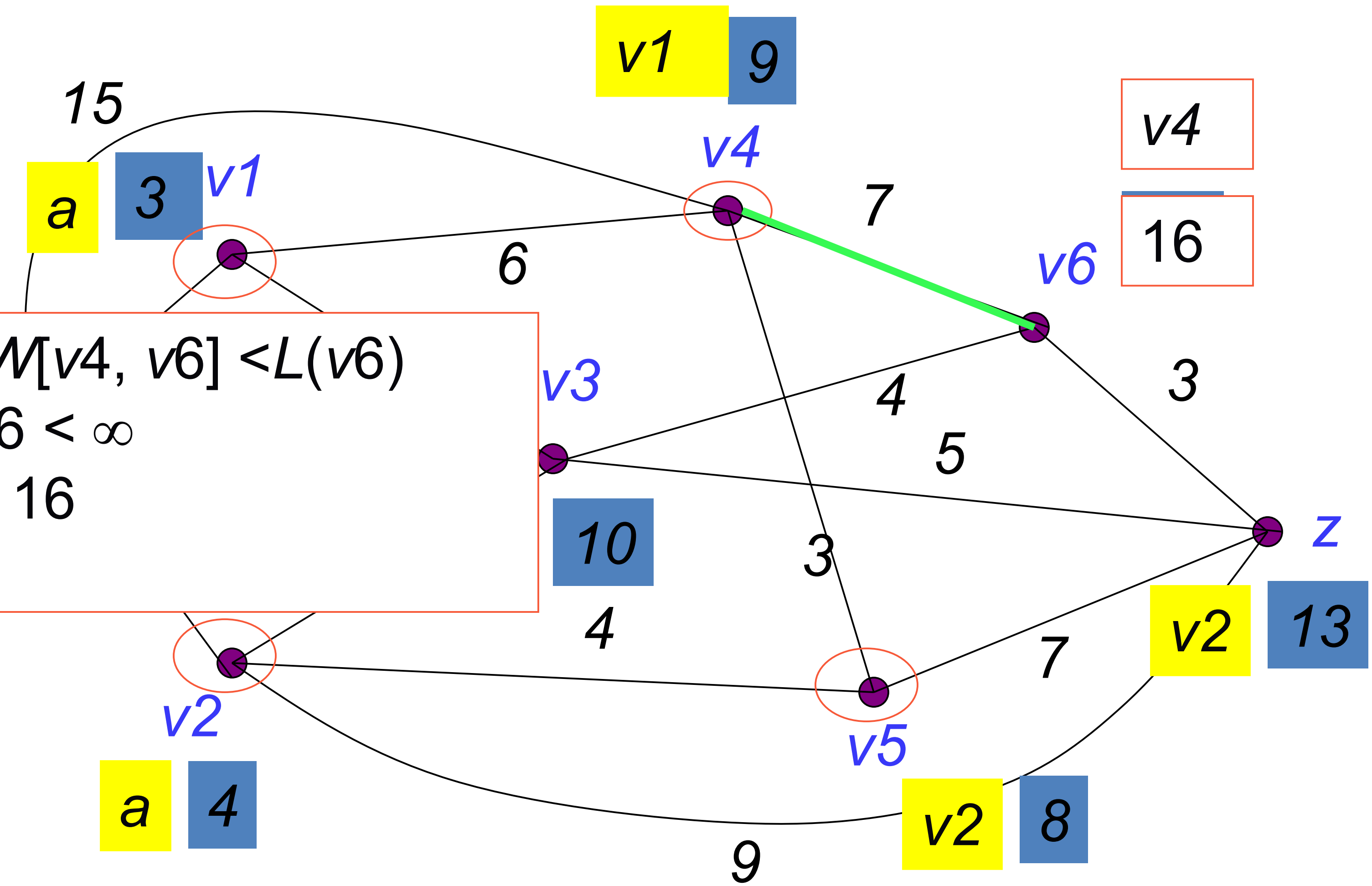
$S=\{a,v1,v2, v4, v5\}$

$N= \{v3, v6, z\}$



$S=\{a, v1, v2, v4, v5\}$

$N=\{v3, v6, z\}$



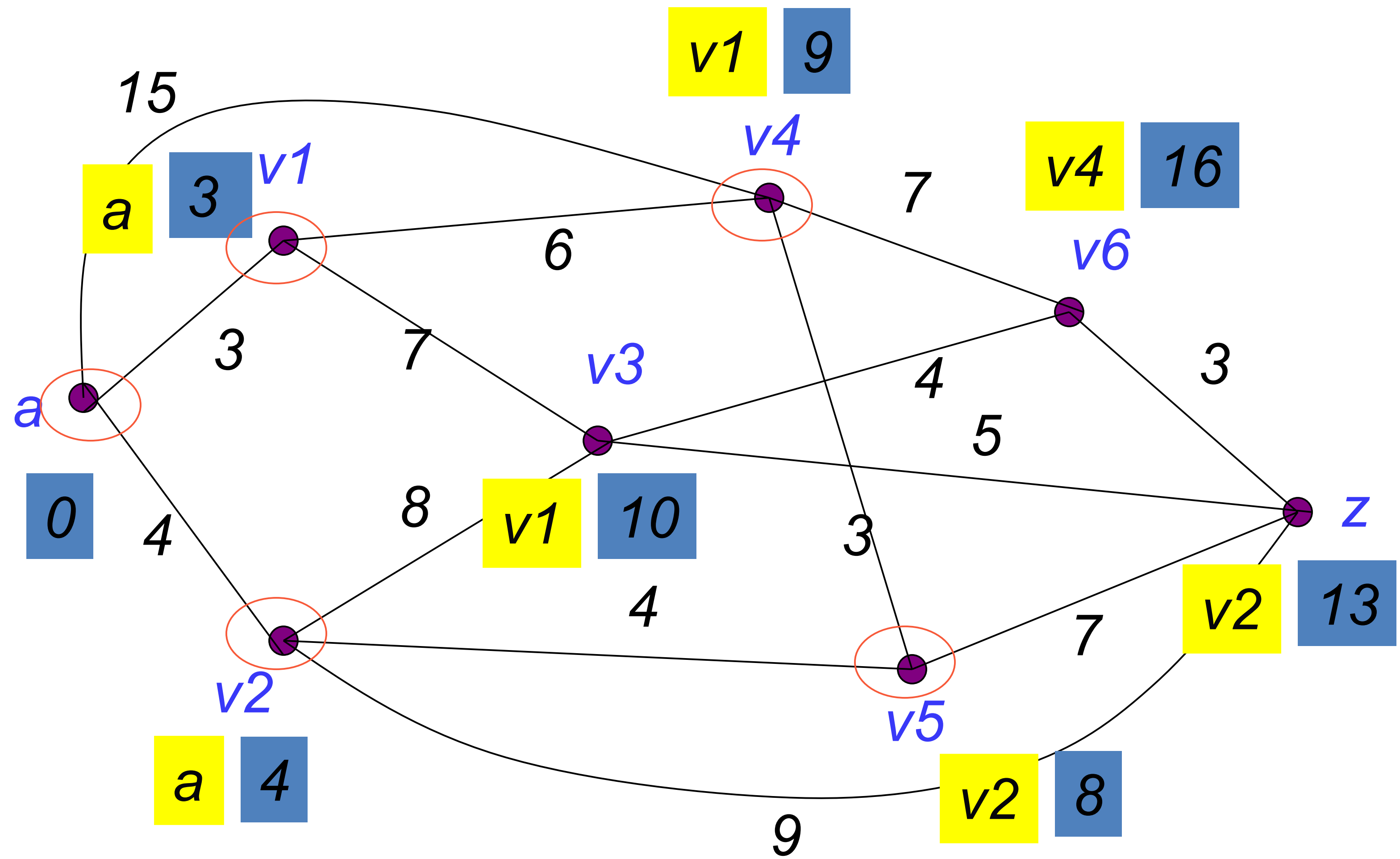
$L(v4)+W[v4, v6] < L(v6)$

$9+7= 16 < \infty$

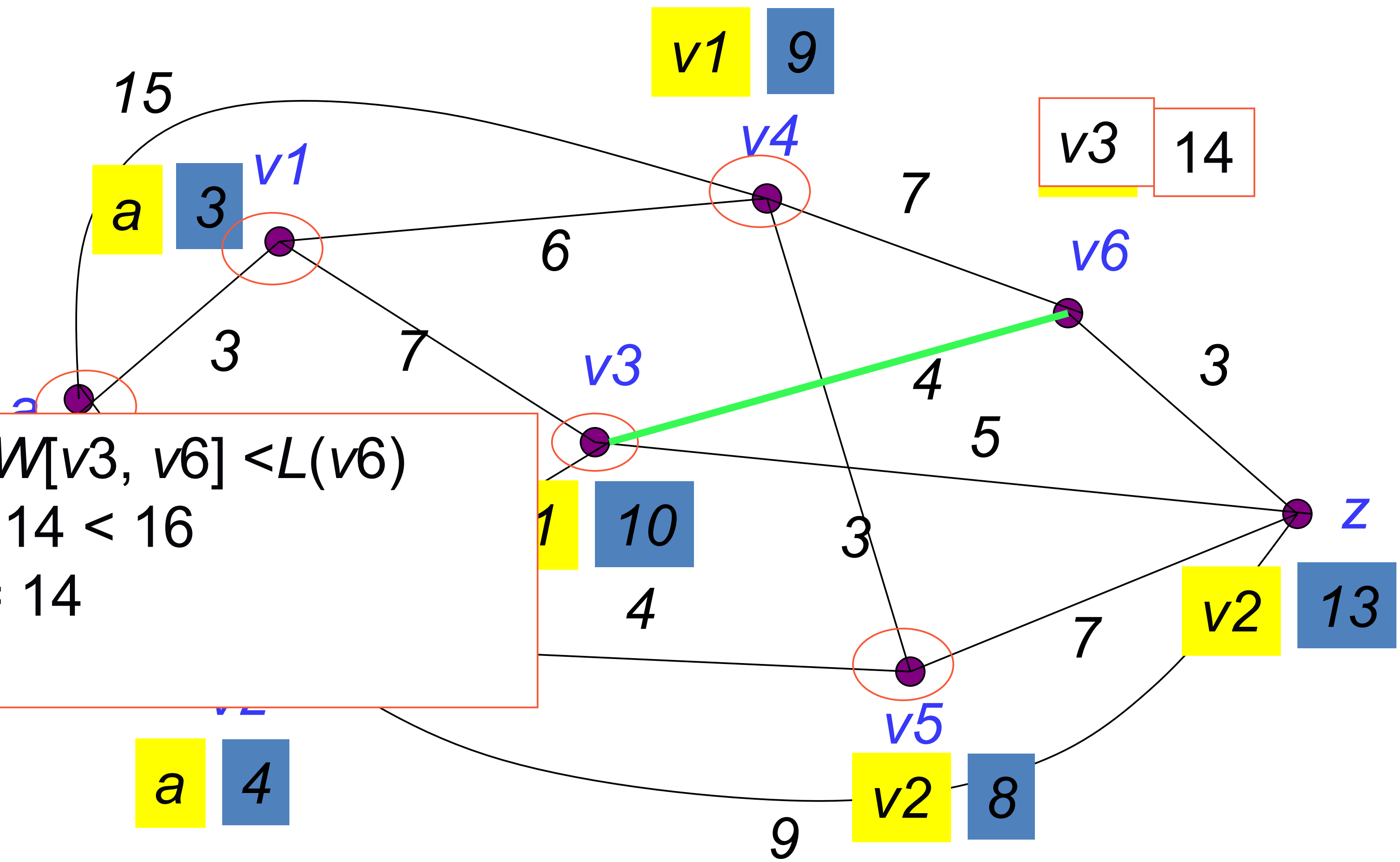
$L(v6) = 16$

$S=\{a,v1,v2, v4,v5\}$
$N= \{v3,v6,z\}$

choose $v3$
because $L(v3)=10$
$= \min\{L(u)|u\in N\}$



$S=\{a,v1,v2, v3, v4,v5\}$
$N= \{v6,z\}$



$L(v3)+W[v3, v6] < L(v6)$
$10+4= 14 < 16$
$L(v6) = 14$

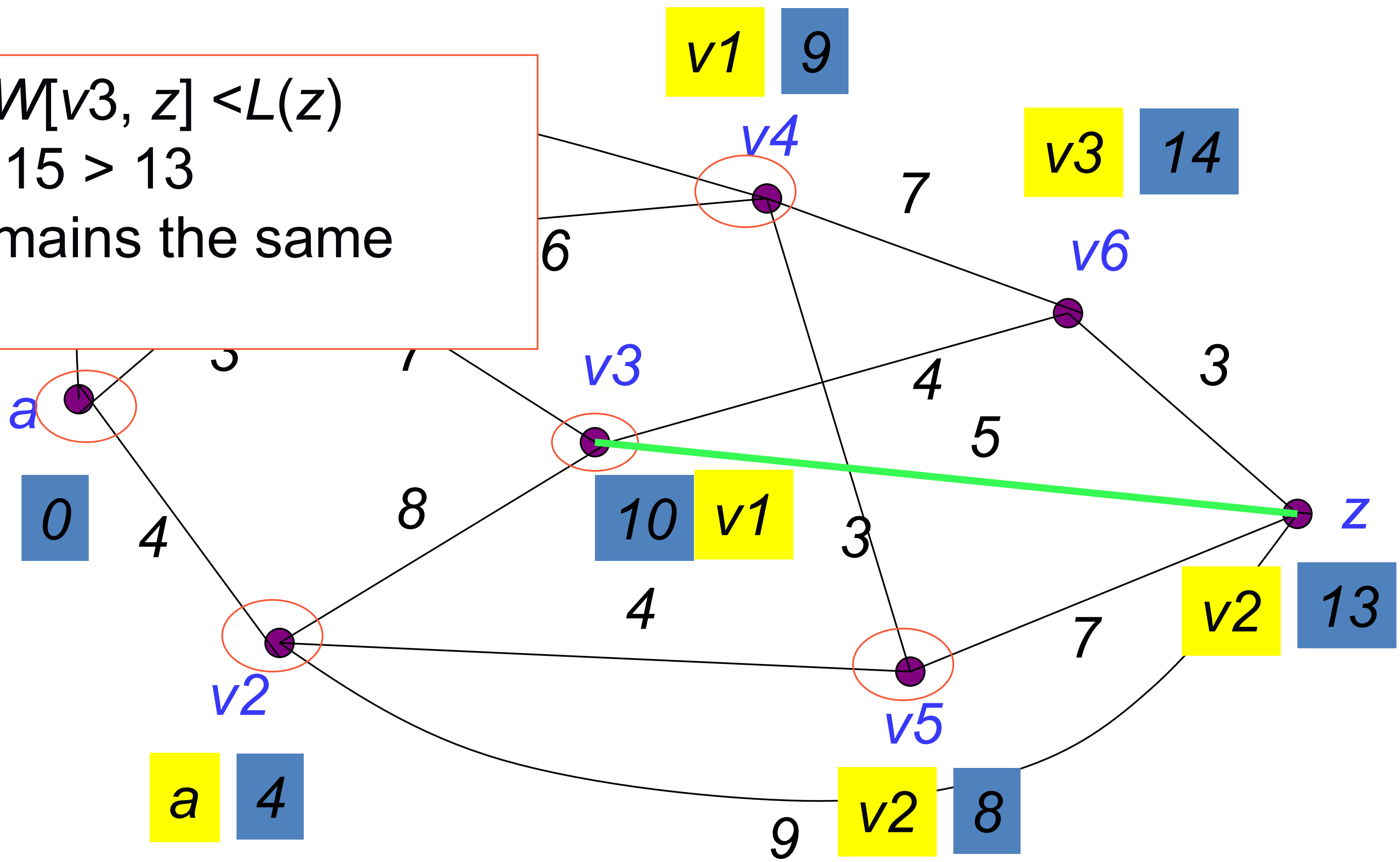
$S=\{a, v1, v2, v3, v4, v5\}$

$N=\{v6, z\}$

$L(v3)+W[v3, z] < L(z)$

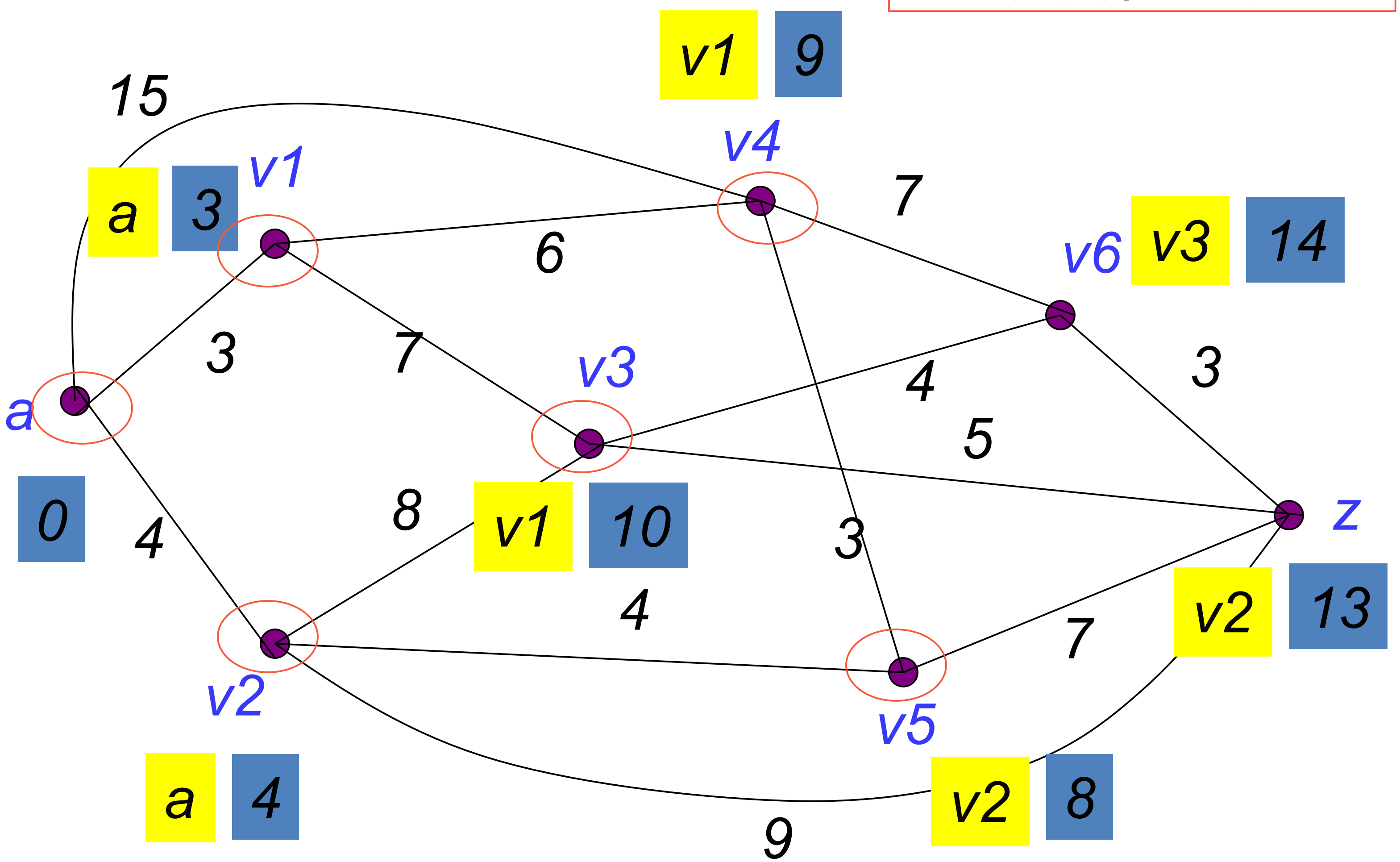
10+5= 15 > 13

$L(z)$ remains the same



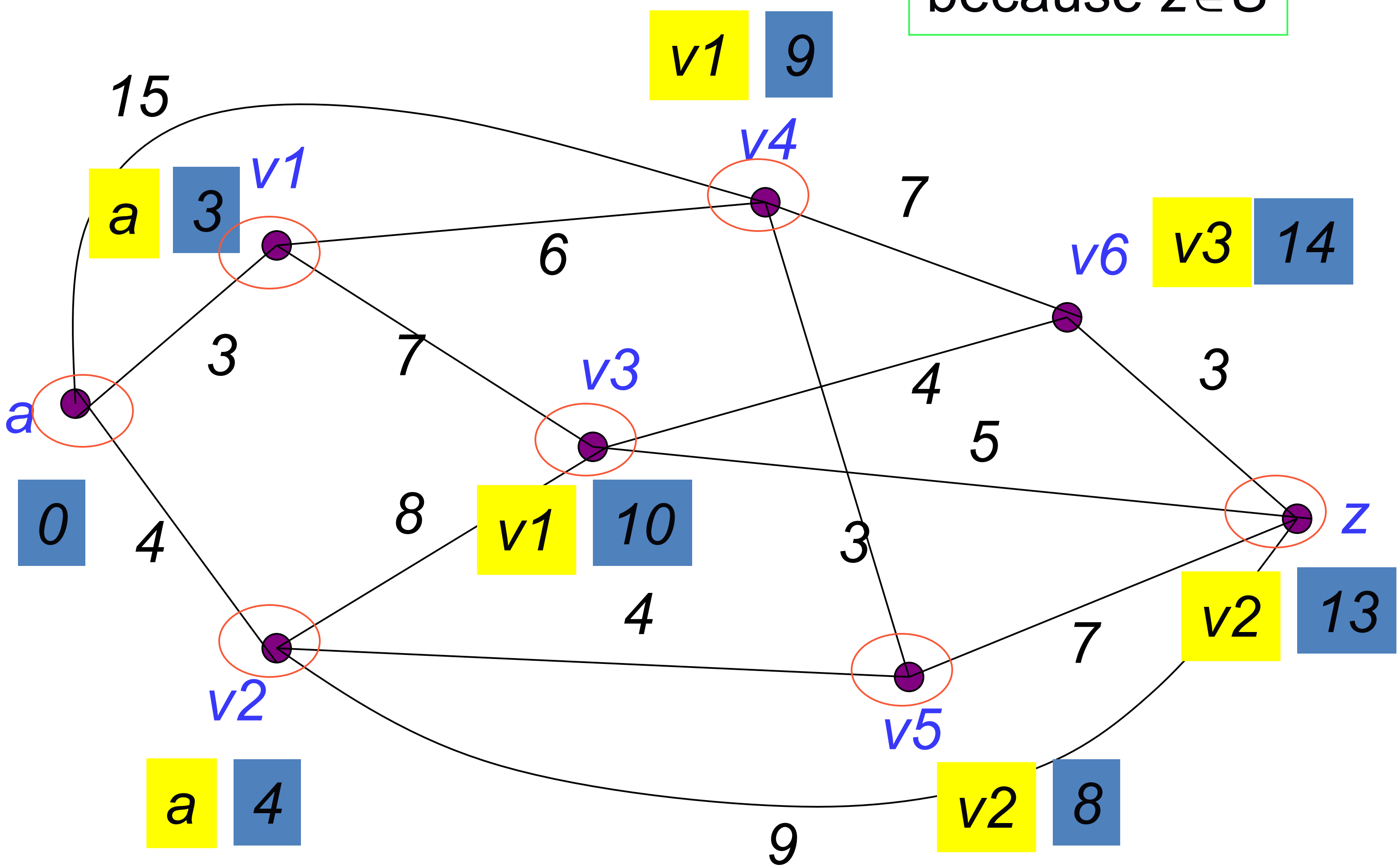
$S=\{a,v1,v2,v3,v4,v5\}$
$N=\{v6,z\}$

choose z because
$L(z)=13=\min\{L(u)|u\in N\}$



$S=\{a, v1, v2, v3, v4, v5, z\}$
$N=\{v6\}$

The loop terminates because $z \in S$



Shortest path from *a* to *z*

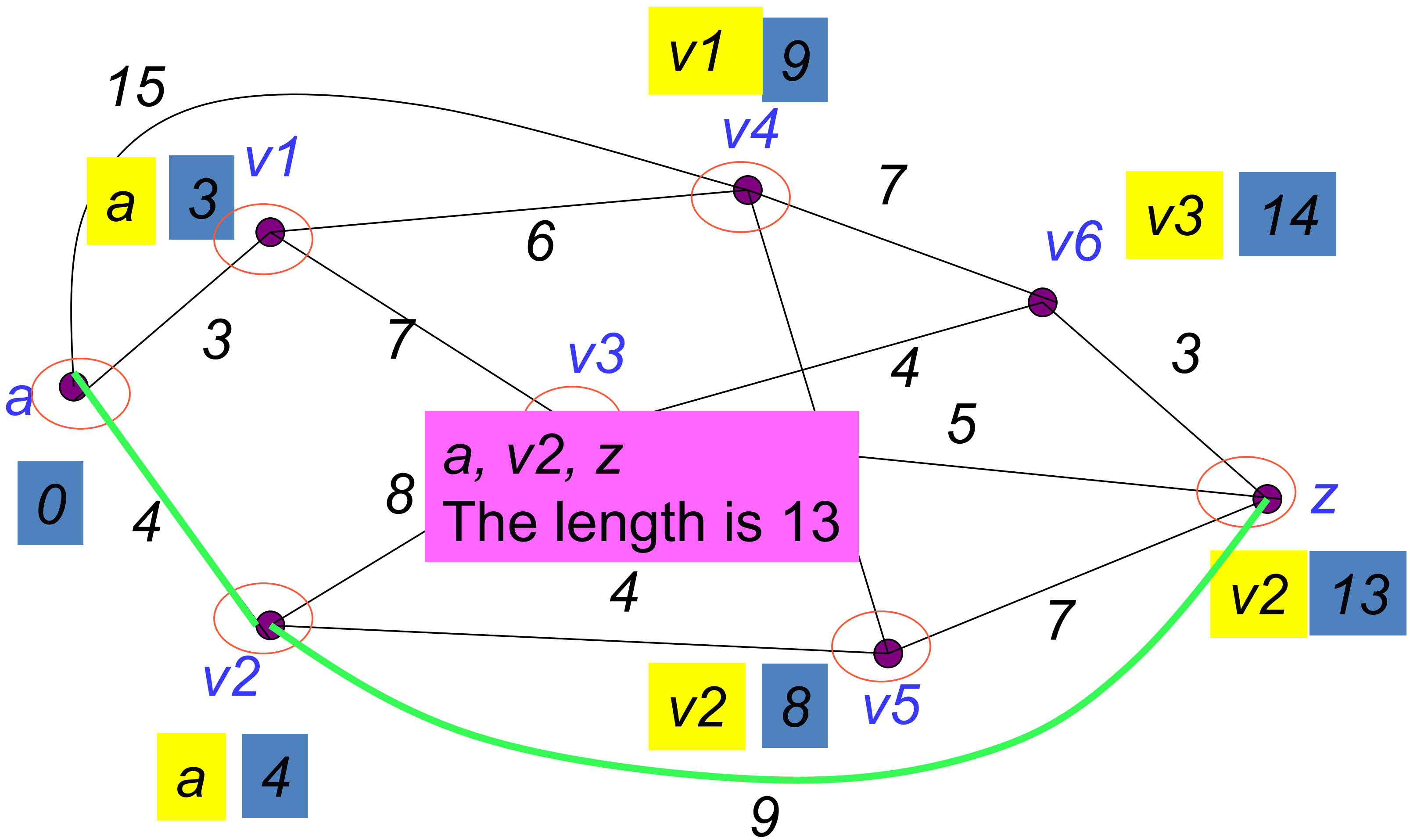
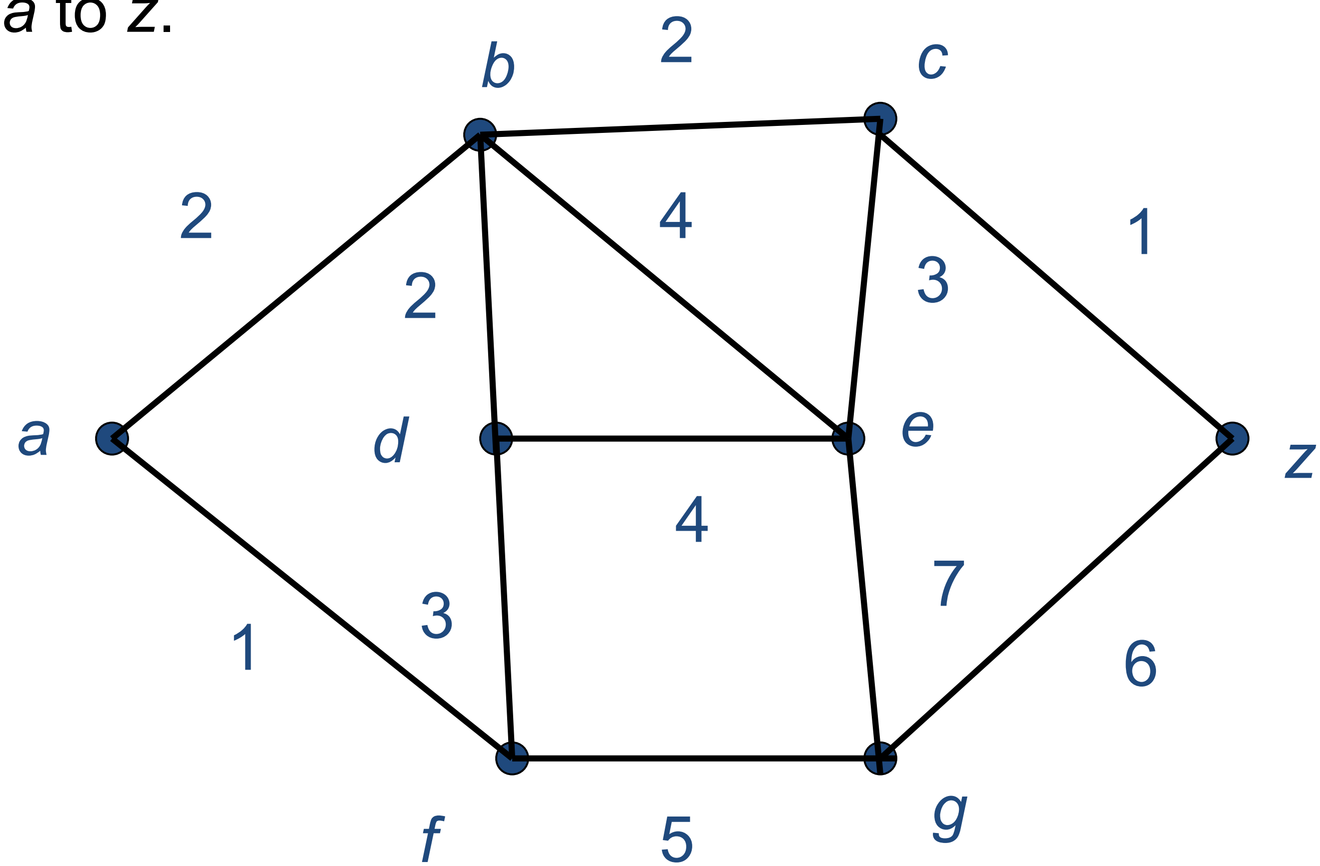


Table – Djikstra Algorithm

No.	S	N	L(a)	$L(V_1)$	$L(V_2)$	$L(V_3)$	$L(V_4)$	$L(V_5)$	$L(V_6)$	L(z)
0	{ }	$\{a, V_1, V_2, V_3, V_4, V_5, V_6, z\}$	0	∞	∞	∞	∞	∞	∞	∞
1	$\{a\}$	$\{V_1, V_2, V_3, V_4, V_5, V_6, z\}$		3	4	∞	15	∞	∞	∞
2	$\{a, V_1\}$	$\{V_2, V_3, V_4, V_5, V_6, z\}$		**3**	4	10	9	∞	∞	∞
3	$\{a, V_1, V_2\}$	$\{V_3, V_4, V_5, V_6, z\}$			**4**	10	9	8	∞	13
4	$\{a, V_1, V_2, V_5\}$	$\{V_3, V_4, V_6, z\}$				10	9	**8**	∞	13
5	$\{a, V_1, V_2, V_5, V_4\}$	$\{V_6, z\}$				10	**9**		16	13
6	$\{a, V_1, V_2, V_5, V_4, V_3,\}$	$\{V_6, z\}$				**10**			14	13
7	$\{a, V_1, V_2, V_5, V_4, V_3, z\}$	$\{V_6\}$							14	**13**

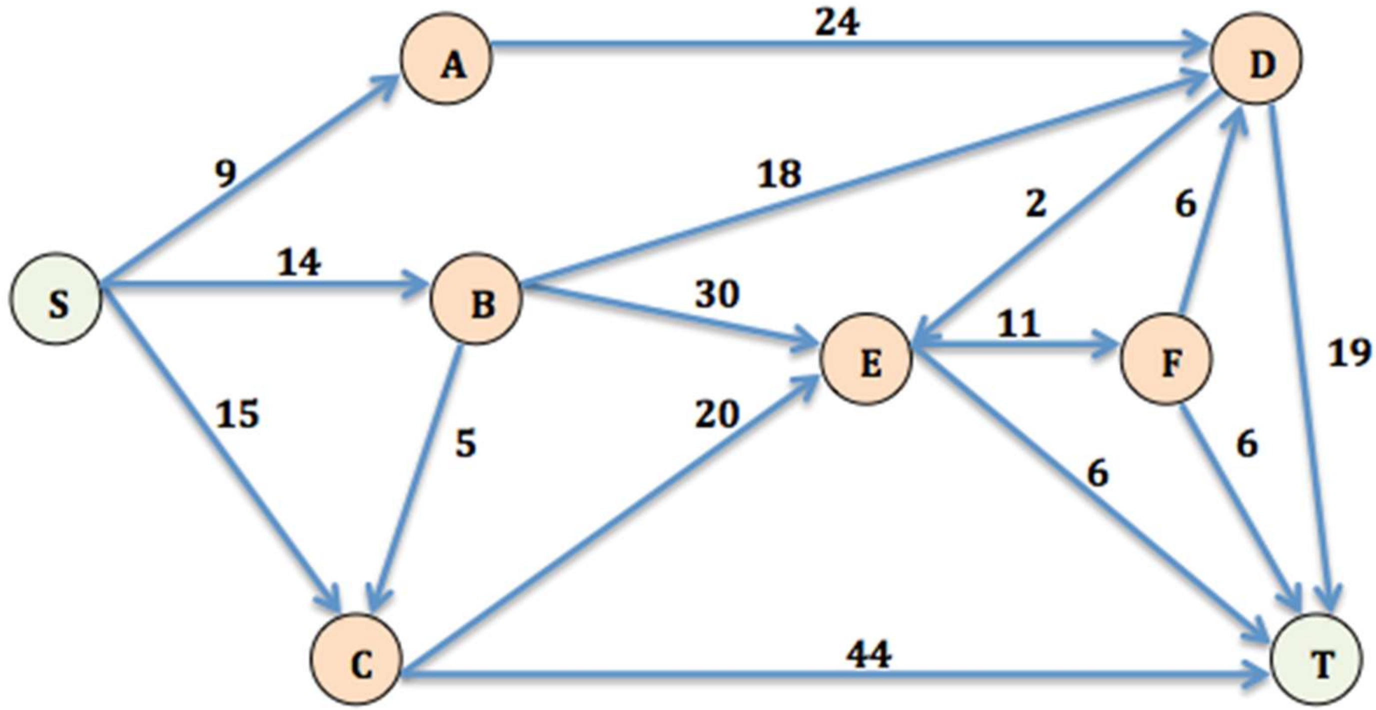
exercise

Use Dijkstra's algorithm to find the length of a shortest path from *a* to *z*.



exercise

Q: Given a weighted digraph, find the shortest path from S to T, using Djik



Note: Weights are arbitrary numbers (i.e., not necessarily distances).

Exercise Past Year 2015/2016

The network in Figure 5 gives the distances in miles between pairs of cities A, B, …, and H.

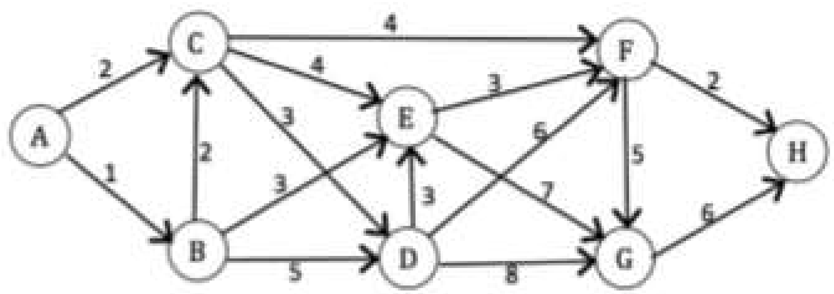


Figure 5

a) Based on Dijkstra's algorithm, complete Table 1 to find the shortest path from city A to city H. (Note: Copy Table 1 into your answer booklet).

(8 marks)

Table 1

Iteration	S	N	L(A)	L(B)	L(C)	L(D)	L(E)	L(F)	L(G)	L(H)
0										
1										
2										
3										
4										
5										
6										
7										

b) State the minimum distance and the shortest path from city 1 to city 8.

(2 marks)