Chapter 1

SET THEORY

[Part 2: Operation on Set]

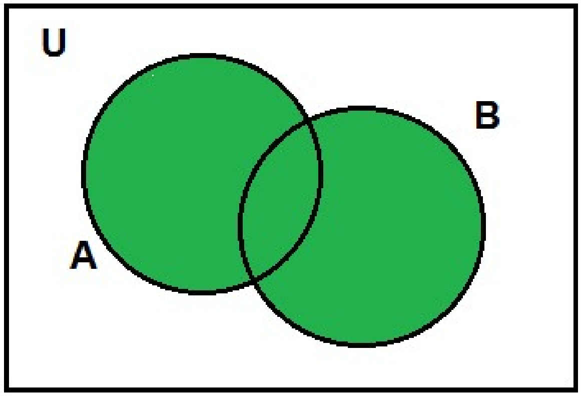
Union

- The union of two sets A and B, denoted by $A \cup B$, is defined to be the set

$$A \cup B = \{ x \mid x \in A \text{ or } x \in B\}$$

- The union consists of all elements belonging to either A or B (or both)

Venn diagram of $A \cup B$



Example

A={1, 2, 3, 4, 5}, B={2, 4, 6} and C={8, 9}

$A \cup B = \{1, 2, 3, 4, 5, 6\}$

$A \cup C = \{1, 2, 3, 4, 5, 8, 9\}$

$B \cup C = \{2, 4, 6, 8, 9\}$

$A \cup B \cup C = \{1, 2, 3, 4, 5, 6, 8, 9\}$

Union

If A and B are finite sets, the cardinality of $A \cup B$,

$$|A \cup B| = |A| + |B| - |A \cap B|$$

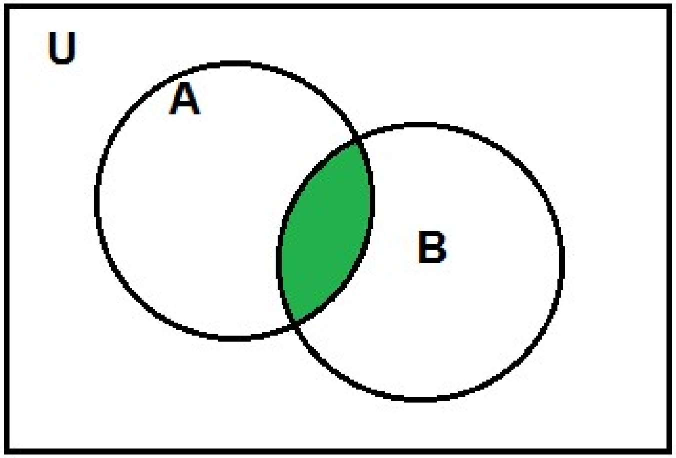
Intersection

- The intersection of two sets A and B, denoted by $A \cap B$, is defined to be the set

$$A \cap B = \{ x \mid x \in A \text{ and } x \in B \}$$

- The intersection consists of all elements belonging to both A and B.

Venn diagram of $A \cap B$



Example

$A=\{1, 2, 3, 4, 5, 6\}$, $B=\{2, 4, 6, 8, 10\}$ and $C=\{ 1, 2, 8, 10 \}$

$A \cap B = \{2, 4, 6\}$

$A \cap C = \{1, 2\}$

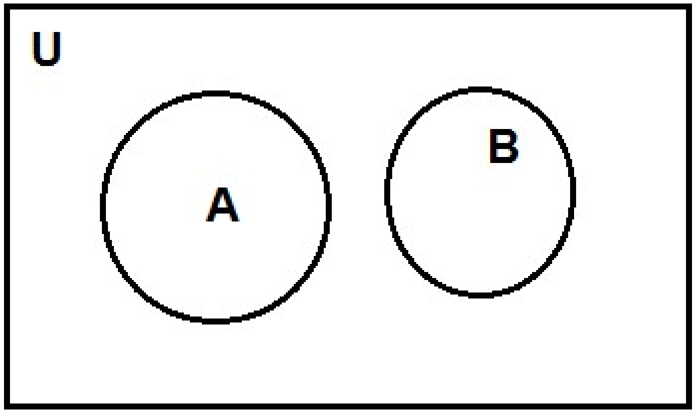
$C \cap B = \{2, 8, 10\}$

$A \cap B \cap C = \{2\}$

Disjoint

Two sets A and B are said to be disjoint if, $A \cap B = \varnothing$

Venn diagram, $A \cap B = \varnothing$



Example

$A = \{1, 3, 5, 7, 9, 11\}$, $B = \{2, 4, 6, 8, 10\}$

$A \cap B = \varnothing$

Difference

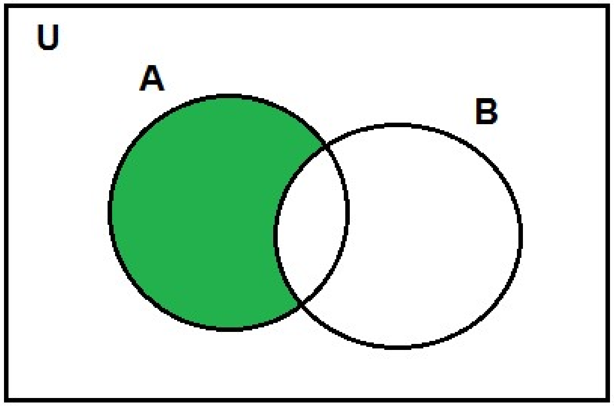
The set,

$$A - B = \{x \mid x \in A \text{ and } x \notin B\}$$

is called the difference.

The difference $A - B$ consists of all elements in A that are not in B.

Venn diagram of $A - B$



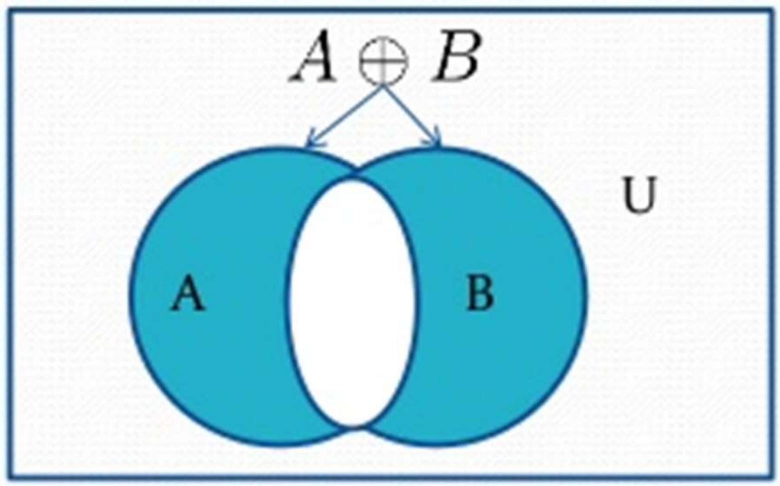
Example

$A = \{ 1, 2, 3, 4, 5, 6, 7, 8 \}$
$B = \{ 2, 4, 6, 8 \}$

$A - B = \{ 1, 3, 5, 7 \}$

Symmetric Difference

The symmetric difference of set *A* and set *B*, denoted by $A \oplus B$ is the set **$(A - B) \cup (B - A)$**



Venn Diagram

Example

$$U = \{0,1,2,3,4,5,6,7,8,9,10\}$$

$$A = \{1,2,3,4,5\};\ B = \{4,5,6,7,8\}$$

$$A \oplus B = (A - B) \cup (B - A) = \{1,2,3,6,7,8\}$$

B-A $= \{6,7,8\}$

$$A - B = \{1,2,3\}$$

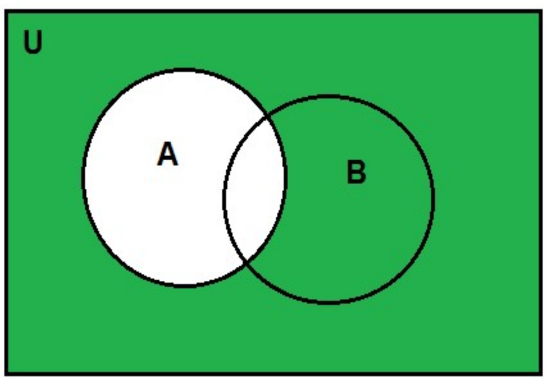
Complement

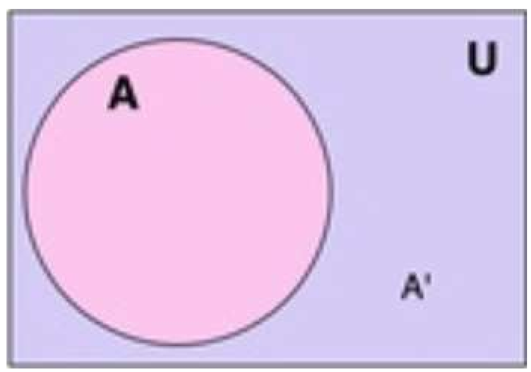
The complement of a set A with respect to a universal set U, denoted by A' is defined to be

$$A' = \{x \in U \mid x \notin A\}$$

$$A' = U - A$$

Venn diagram of *A'*





Example

Let U be a universal set,

$U = \{ 1, 2, 3, 4, 5, 6, 7 \}$

$A = \{ 2, 4, 6 \}$

$A' = U - A = \{ 1, 3, 5, 7 \}$

Exercise

Let,

$$U = \{ a, b, c, d, e, f , g, h, i, j, k, l, m \}$$
$$A = \{ a, c, f, m\}$$
$$B = \{ b, c, g, h, m \}$$

Find:

$A \cup B$, $A \cap B$, $| A \cup B|$, $A - B$ dan A'.

Exercise

Let the universe be the set $U=\{1, 2, 3, 4, \ldots\ldots, 10\}$.

Let $A=\{1, 4, 7, 10\}$, $B=\{1, 2, 3, 4, 5\}$ and $C=\{2, 4, 6, 8\}$.

List the elements of each set:

a) U'

b) $B' \cap (C - A)$

c) $B - A$

d) $(A \cup B) \cap (C - B)$

Set Identities (Properties of Set)

- **Commutative laws**

  $A \cap B = B \cap A, \quad A \cup B = B \cup A$

- **Associative laws**

  $A \cap (B \cap C) = (A \cap B) \cap C$

  $A \cup (B \cup C) = (A \cup B) \cup C$

- **Distributive laws**

  $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$

  $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

Set Identities (Properties of Set)

- **Absorption laws**

$$A \cup (A \cap B) = A$$

$$A \cap (A \cup B) = A$$

- **Idempotent laws**

$$A \cap A = A, \quad A \cup A = A$$

- **De Morgan's laws**

$$(A \cap B)' = A' \cup B'$$

$$(A \cup B)' = A' \cap B'$$

Set Identities (Properties of Set)

- **Complement laws**

$A \cap A' = \varnothing \qquad A \cup A' = U$

- **Double complement laws**

$(A')' = A$

- **Complement of U and $\varnothing$**

$\varnothing' = U \qquad U' = \varnothing$

Set Identities (Properties of Set)

- **Properties of universal set**

  $A \cup U = U \qquad A \cap U = A$

- **Set difference laws**

  $A - B = A \cap B'$

- **Identity laws**

  $A \cup \varnothing = A \qquad A \cap U = A$

- **Properties of empty set**

  $A \cup \varnothing = A \qquad A \cap \varnothing = \varnothing$

Example

- Let A, B and C denote the subsets of a set S and let C' denote a complement of C in S.
- If $A \cap C = B \cap C$ and $A \cap C' = B \cap C'$, then prove that $A = B$

$$
\begin{aligned}
A &= A \cap S \\
&= A \cap (C \cup C') \\
&= (A \cap C) \cup (A \cap C') && \textbf{(distributive)} \\
&= (B \cap C) \cup (B \cap C') && \textbf{(the given conditions)} \\
&= B \cap (C \cup C') && \textbf{(distributive)} \\
&= B \cap S \\
&= B
\end{aligned}
$$

Example

By referring to the properties of set operations, show that:

$$A - (A \cap B) = A - B$$

set difference
$A - B = A \cap B'$

$A - (A \cap B)$	$= A \cap (A \cap B)'$	[set difference laws]
	$= A \cap (A' \cup B')$	[De Morgan's laws]
	$= (A \cap A') \cup (A \cap B')$	[distributive laws]
	$= \varnothing \cup (A \cap B')$	[complement laws]
	$= (A \cap B') \cup \varnothing$	[commutative]
	$= A \cap B'$	[Identity laws]
	$= A - B$	

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Exercise

1) Let A, B and C be sets. Show that

$$(A \cup (B \cap C))' = A' \cap (B' \cup C')$$

2) Let A, B and C be sets such that

$$A \cap B = A \cap C \text{ and } A \cup B = A \cup C$$

Prove that $B = C$

Generalized Unions and Intersections

The ***union*** of a collection of sets is the set that contains those elements that are members of at least one set in the collection.

Notation:

$$\bigcup_{i=1}^{n} A_i = A_1 \cup A_2 \cup \ldots \cup A_n = \{x \in U \mid x \in A_i \text{ for at least one } i = 0,1,2,\ldots..,n\}$$

$$\bigcup_{i=1}^{\infty} A_i = A_1 \cup A_2 \cup \ldots \cup A_\infty = \{x \in U \mid x \in A_i \text{ for at least one nonnegative integer } i\}$$

Generalized Unions and Intersections

The ***intersection*** of a collection of sets is the set that contains those elements that are members of all the sets in the collection.

Notation:

$$\bigcap_{i=1}^{n} A_i = A_1 \cap A_2 \cap \ldots \cap A_n = \{x \in U \mid x \in A_i \text{ for all } i = 0,1,2,\ldots,n\}$$

$$\bigcap_{i=1}^{\infty} A_i = A_1 \cap A_2 \cap \ldots \cap A_{\infty} = \{x \in U \mid x \in A_i \text{ for all nonnegative integer } i\}$$

Example

For i = 1, 2, ……, let $A_i = \{i, i+1, i+2, \ldots\text{.}).$ Then,

$$\bigcup_{i=1}^{n} A_i = \bigcup_{i=1}^{n} \{i, i+1, i+2, \ldots\ldots\} = \{1,2,3,\ldots\},$$

and

$$\bigcap_{i=1}^{n} A_i = \bigcap_{i=1}^{n} \{i, i+1, i+2, \ldots\ldots\} = \{n, n+1, n+2, \ldots\} = A_n$$

Cartesian Product

- Let A and B be sets. An ordered pair of elements $a \in A$ dan $b \in B$ written (a, b) is a listing of the elements a and b in a specific order.

- The ordered pair (a, b) specifies that a is the first element and b is the second element.

- An ordered pair (a, b) is considered distinct from ordered pair (b, a), unless $a=b$.

Example $(1, 2) \neq (2, 1)$

Cartesian Product

- The Cartesian product of two sets A and B, written $A \times B$ is the set,

$$A \times B = \{(a,b) | \ a \in A, \ b \in B\}$$

- For any set A,

$$A \times \varnothing = \varnothing \times A = \varnothing$$

Example

$A = \{a, b\}$, $B = \{1, 2\}$.

$A \times B = \{(a, 1), (a, 2), (b, 1), (b, 2)\}$

$B \times A = \{(1, a), (1, b), (2, a), (2, b)\}$

Cartesian Product

- if $A \neq B$, then $A \times B \neq B \times A$.
- if $|A| = m$ and $|B| = n$, then $|A \times B| = mn$.

Example $A = \{1, 3\}$, $B = \{2, 4, 6\}$.

$A \times B = \{(1, 2), (1, 4), (1, 6), (3, 2), (3, 4), (3, 6)\}$

$B \times A = \{(2, 1), (2, 3), (4, 1), (4, 3), (6, 1), (6, 3)\}$

$A \neq B,\ A \times B \neq B \times A$

$|A| = 2$, $|B| = 3$, $|A \times B| = 2.3 = 6$.

Cartesian Product

- The Cartesian product of sets A_1, A_2, …., A_n is defined to be the set of all n-tuples

  $(a_1, a_2, \ldots a_n)$ where $a_i \in A_i$ for $i=1,\ldots,n$;

- It is denoted $A_1 \times A_2 \times \ldots. \times A_n$

  $|A_1 \times A_2 \times \ldots. \times A_n| = |A_1|.|A_2| \ldots. |A_n|$

Example

$A = \{a, b\}$, $B = \{1, 2\}$, $C = \{x, y\}$

$$A \times B \times C = \{(a,1,x),(a,1,y), (a,2,x), (a,2,y), (b,1,x), (b,1,y), (b,2,x), (b,2,y)\}$$

$$|A \times B \times C| = 2.\ 2.\ 2 = 8$$

Exercise

Let $A = \{w, x\}$, B={1, 2} and C={nm, ds, ps}.

1) Find $|A\times B|$, $|B\times C|$, $|A\times C|$, $|A\times B\times C|$, $|B\times C\times A|$, $|A\times B\times A\times C|$

2) Determine the following set,
   a) $A\times B$, $B\times C$, $A\times C$
   b) $A\times B\times C$
   c) $B\times C\times A$
   d) $A\times B\times A\times C$

Exercise

Let $X = \{1,2\}$, $Y=\{a\}$ and $Z=\{b,d\}$.

List the elements of each set.

a) $X \times Y$

b) $Y \times X$

c) $X \times Y \times Z$

d) $X \times Y \times Y$

e) $X \times X \times X$

f) $Y \times X \times Y \times Z$

Thank You

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