SECI 1113

COMPUTATIONAL MATHEMATICS

ASSIGNMENT 5

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No.: ....

Date: ....

Assignment 5.

1. Newton Forward Difference : for 1980

k	(x) Year	(Y) Birth rate	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7	0.80	-6.20	3.70	5.20
1	1985	31.5	-5.40	-2.50	8.90	
2	1995	26.1	-7.90	6.40		
3	2005	18.2	-1.50			
4	2015	16.7				

since $x = 1980$ in between 1975 to 1985, use 1975 as it has highest forward-difference degree

$h = 1985 - 1975 = 10$.

$r = (1980 - 1975) \div 10 = 5 \div 10 = 0.5$.

$$P_4(1980) = 30.7 + 0.5(0.80) + \frac{(0.50)(0.50-1)}{2!}(-6.20) + \frac{(0.50)(0.50-1)(0.50-2)}{3!}(3.70) + \frac{(0.5)(0.5-1)(0.5-2)(0.50-3)}{4!}(5.20)$$

$= 30.7 + 0.4 + 0.775 + 0.23125 + (-0.203125)$

$= 31.903125$

≈ 31.9031 (4dp)

$\therefore y(1980) \approx P_4(1980) = 31.9031$

Newton Backward Difference : for 2010.

k	(x) year	(Y) Birth rate	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7				
1	1985	31.5	0.80			
2	1995	26.1	-5.40	-6.20		
3	2005	18.2	-7.90	-2.50	3.70	
4	2015	16.7	-1.50	6.40	8.90	5.20

Since $x = 2010$ in between 2005 to 2015, use 2015 as it has highest backward-difference degree.

$h = 1985 - 1975 = 10$

$r = (2010 - 2015) \div 10 = -5 \div 10 = -0.5$.

No.: .... Date: ....

$$P_4(2010) = 16.70 + (-0.5)(-1.50) + \frac{(-0.50)(-0.50+1)}{2!}(6.40) + \frac{(-0.50)(-0.50+1)(-0.50+2)}{3!}(8.90)$$

$$+ \frac{(-0.50)(-0.50+1)(-0.50+2)(-0.50+3)}{4!}(5.20)$$

$$= 16.70 + 0.75 + (-0.80) + (-0.55625) + (-0.203125)$$

$$= 15.890625$$

$$\approx 15.8906 \quad (4dp)$$

$$\therefore Y(2010) \approx P_4(2010) = 15.8906.$$

No.: .... Date: ....

2. a) $P(x) = a_0 + a_1 x$

$\therefore$ Matrix form = $\begin{bmatrix} S_0 & S_1 \\ S_1 & S_2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} V_0 \\ V_1 \end{bmatrix}$

k	x_k' year	x_k^0	x_k^2	f_k	$x_k^0 f_k$	$x_k' f_k$
0	1970	1	3880900	6.0	6.0	11820
1	1975	1	3900625	8.6	8.6	16985
2	1980	1	3920400	13.6	13.6	26928
3	1985	1	3940225	18.7	18.7	37119.5
4	1990	1	3960100	31.6	31.6	62884
5	1995	1	3980025	53.2	53.2	106134
6	2000	1	4000000	78.8	78.8	157600
7	2005	1	4020025	100.0	100.0	200500
8	2010	1	4040100	107.1	107.1	215271
Σ	17910	9	35642400	-	417.6	835241.5

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 17910 & 35642400 & 835241.5 \end{array}\right]$$

$$\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 0 & 1500 & 4217.5 \end{array}\right] \quad R_2 = R_2 - 1990 R_1$$

$$\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 0 & 1 & 2.81167 \end{array}\right] \quad R_2 = R_2 \div 1500$$

$$\left[\begin{array}{cc|c} 1 & 0 & -5548.8233 \\ 0 & 1 & 2.81167 \end{array}\right] \quad R_1 = [R_1 - 17910 R_2] \div 9$$

$\therefore a_0 = -5548.8233$ $\quad a_1 = 2.8117$ $\quad P(x) = -5548.8233 + 2.8117x$

b) $P(1979) = -5548.8233 + 2.8117(1979)$

$= 15.531.$

3.

k	0	1	2	3	4
x	1.5	2.5	4	5	6.5
y	0.455	1.615	3.750	5.333	7.857

Use Lagrange interpolating polynomials to estimate $y(3.0)$

$$P_n(x) = L_0(x)y_0 + L_1(x)y_1 + \ldots L_n(x)y_n = \sum_{i=0}^{n} L_i(x)y_i$$

$$P_4(x) = L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + L_4(x)y_4 = \sum_{i=0}^{4} L_i(x)y_i$$

$$L(x) = \prod_{\substack{j=0 \\ j\neq i}}^{4} \frac{(x - x_j)}{(x_1 - x_j)}$$

$$P_4(3.0) = \sum_{i=0}^{4} L_i(3.0)y_i \quad \text{with} \quad L_i(3.0) = \prod_{\substack{j=0 \\ j\neq i}}^{4} \frac{(3.0 - x_j)}{(x_1 - x_j)}$$

$$L_0(3.0) = \frac{(3.0 - x_1)(3.0 - x_2)(3.0 - x_3)(3.0 - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)}$$

$$= \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)} = -0.08$$

$$L_1(3.0) = \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)} = 0.7$$

$$L_2(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)} = 0.56$$

$$L_3(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)} = -0.2$$

$$L_4(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)} = 0.02$$

$$\sum_{i=0}^{4} L_i(3.0) = L_0(3.0) + L_1(3.0) + L_2(3.0) + L_3(3.0) + L_4(3.0)$$

$$= -0.08 + 0.7 + 0.56 + (-0.2) + (0.02)$$

$$= 1$$

$$P_4(3.0) = \sum_{i=0}^{4} l_i(3.0)y_i$$

$$= -0.08(0.455) + 0.7(1.615) + 0.56(3.750) - 0.2(5.333) + 0.02(7.857)$$

$$= 2.28464$$

④. $f(x)=2x^3+x^2-4$ $\quad f'(2.5)$ and $f''(2.5)$

ⓐ central three-point and five-point formula with $h=0.5$

(i) three point (1^{st} derivative)

$$f'(x)=\frac{f(x+h)-f(x-h)}{2h}$$

$$f'(2.5)=\frac{f(2.5+0.5)-f(2.5-0.5)}{2(0.5)}$$

$$=\frac{f(3.0)-f(2.0)}{2(0.5)}=\frac{59-16}{1}$$

$$=43$$

$\therefore 2^{nd}$ derivative

$$f''(x)=\frac{f(x-h)-2f(x)+f(x+h)}{h^2}$$

$$=\frac{f(2.5-0.5)-2f(2.5)+f(2.5+0.5)}{(0.5)^2}$$

$$=\frac{f(2.0)-2f(2.5)+f(3.0)}{0.25}$$

$$=\frac{16-2(33.5)+59}{0.25}$$

$$=32.$$

(ii) five-point

$$f'(x) = \frac{1}{12h}\left[f(x_1-2h) - 8f(x_1-h) + 8f(x_1+h) - f(x_1+2h)\right]$$

$$f'(2.5) = \frac{f(2.5-1) + 8f(2.5-0.5) + 8f(2.5+0.5) - f(2.5+1)}{12(0.5)}$$

$$\cancel{= f(1.5) + 8f(\ ) + 8}$$

$$= \frac{f(1.5) - 8f(2) + 8f(3) - f(3.5)}{6}$$

$$= \frac{5 - 8(16) + 8(59) - 94}{6}$$

$$\therefore f'(2.5) = 42.5$$

$$f''(x) = \frac{1}{12h^2}\left[-f(x_i-2h) + 16f(x_i-h) - 30f(x_i) + 16f(x_i+h) - f(x_i+2h)\right]$$

$$f''(2.5) = \frac{-f(\overset{1.5}{\cancel{2.5-1}}) + 16f(2.0) - 30f(2.5) + 16f(3.0) - f(3.5)}{12(0.5)^2}$$

$$= \frac{-5 + 16(16) - 30(33.5) + 16(59) - 94}{3}$$

$$= \cancel{\frac{96}{3}} = 32$$

$$\therefore f''(2.5) = 32$$

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(b). find the exact value of $f'(2.5)$ and $f''(2.5)$

$$f(x) = 2x^3 + x^2 - 4$$

$$f'(x) = 6x^2 + 2x$$

$$f'(2.5) = 6(2.5)^2 + 2(2.5)$$

$$= 42.5$$

$$f''(x) = 12x + 2$$

$$f''(2.5) = 12(2.5) + 2$$

$$= 32$$

(c) calculate error for each estimation

exact value - approximate value = error.

<u>three point 1st derivative</u>

$\therefore$ $42.50 - 43 = -0.50$.

<u>three-point 2nd derivative</u>

$\therefore$ $32 - 32 = 0$

<u>five-point 1st derivative</u>

$\therefore$ $42.50 - 42.50 = 0$.

<u>five-point 2nd derivative</u>

$\therefore$ $32 - 32 = 0$

No.: .... Date: ....

5. $\int_0^3 \frac{2}{1+5x^2}\,dx$.

a) Trapezoidal rule $N=6$.

$N=6 = \frac{3-0}{h}$ $h=0.5$.

i	x_i	$f(x)=\frac{2}{1+5x^2}$	
0	0.0	2	
1	0.5		0.8889
2	1.0		0.3333
3	1.5		0.1633
4	2.0		0.0952
5	2.5		0.0620
6	3.0	0.0435	
	Total	2.0435	1.5427.

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \int_0^3 f(x)\,dx$$

$$= \frac{0.5}{2}\left[2.0435 + 2(1.5427)\right]$$

$$= 0.25\,(5.1289)$$

$$= 1.2822$$

b) Simpson's 1/3 formula with $N=6$

$h=0.5$.

i	x_i	$f(x)=\frac{2}{1+5x^2}$		
0	0.0	2		
1	0.5		0.8889	
2	1.0			0.3333
3	1.5		0.1633	
4	2.0			0.0952
5	2.5		0.0620	
6	3.0	0.0435		
	Total	2.0435	1.1142	0.4285

No.: .... Date: ....

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{0.5}{3}\left[2.0435 + 4(1.1142) + 2(0.4285)\right]$$

$$= 0.16667\,(7.3573)$$

$$= 1.2262$$

c) Simpson's 3/8 formula with $h = 0.25$

i	x_i	$f(x) = \frac{2}{1+5x^2}$		
0	0.00	2		
1	0.25		1.5238	
2	0.50		0.8889	
3	0.75			0.5246
4	1.00		0.3333	
5	1.25		0.2270	
6	1.50			0.1633
7	1.75		0.1226	
8	2.00		0.0952	
9	2.25			0.0760
10	2.50		0.0620	
11	2.75		0.0515	
12	3.00	0.0435.		
Total		2.0435	3.3043	0.7639.

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{3(0.25)}{8}\left[2.0435 + 3(3.3043) + 2(0.7639)\right]$$

$$= \frac{3}{32}\,(13.4842)$$

$$= 1.2641$$

No.: .... Date: ....

d) Romberg until R_{33}

i	$h_i = \frac{3-0}{2^{i-1}}$	$R_{i,1}$	$R_{i,2}$	$R_{i,3}$
1	$\frac{3}{1} = 3$	3.06525		
2	$\frac{3}{2} = 1.5$	1.777575	1.34835	
3	$\frac{3}{4} = 0.75$	1.3392375	1.193125	1.1827767

$$R_{1,1} = \frac{3}{2}(f_0 + f_1)$$
$$= \frac{3}{2}(2 + 0.0435)$$
$$= 3.06525$$

$$R_{2,1} = \frac{1}{2}[3.06525 + 3(0.1633)]$$
$$= \frac{1}{2}(3.55515)$$
$$= 1.777575$$

$$R_{3,1} = \frac{1}{2}[1.777575 + 1.5(0.5246 + 0.0760)]$$
$$= \frac{1}{2}[2.678475]$$
$$= 1.3392375$$

$$R_{2,2} = \frac{4(1.777575) - 3.06525}{4-1}$$
$$= 1.34835$$

$$R_{3,2} = \frac{4(1.3392375) - 1.777575}{4-1}$$
$$= 1.193125$$

$$R_{3,3} = \frac{4^2(1.193125) - 1.34835}{4^2 - 1}$$
$$= 1.1827767$$
$$\approx 1.1828$$