**SECI 1113**

**COMPUTATIONAL MATHEMATICS**

**ASSIGNMENT 4**

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Assignment 4

Question 1

a) $|\varepsilon| = |\alpha - \bar{\alpha}|$

$2 = 60 - \bar{a}$

$\bar{\alpha} = 58$

$\varepsilon_r = \left|\frac{a - \bar{\alpha}}{\alpha}\right|$

$\varepsilon_r = \left|\frac{60 - 58}{60}\right|$

$\varepsilon_r = \frac{1}{30}$

$\varepsilon_r = 3.33\%$

b)

$f'(x) \approx \frac{f(x+h) - f(x)}{h}$

$= \frac{f(x+0.1) - f(x)}{0.1}$

$= \frac{x + 0.1 - x}{0.1}$

$f'(x) = 1$

$f'(2) = 1$

$f(2) = 2$

$|\varepsilon| = |\alpha - \bar{\alpha}|$

$|\varepsilon| = |2 - 1|$

$|\varepsilon| = 1$

c) exact solution $= 6.232 + 7.0321 + 5.43 - 3.385$

$= 15.3091$

$|\varepsilon| \leq 0.0005 + 0.00005 + 0.005 + 0.0005 = 0.00605$

Therefore, the result is between

$15.3091 - 0.00605$ and $15.3091 + 0.00605$

$15.30305$ and $15.31515$

Both of these values are same if rounded to one decimal point.

Therefore, the best approximate value is 15.3 (1 decimal point).

Question 2 :

$x^6 - x - 1 = 0$ $\quad \varepsilon = 0.001$ $\quad [1, 2] \Rightarrow f(a)f(b) < 0$ ∴, have a root between 1 and 2.

let $a=1$ , $b=2$

$f(a) = f(1) = 1^6 - 1 - 1$ $\quad f(b) = f(2) = 2^6 - 2 - 1$

$= -1$ (-ve) $\quad = 61.$ (+ve)

$$P_1 = \frac{2+1}{2} = 1.5.$$

$f(P_1) = f(1.5) = (1.5)^6 - 1.5 - 1$ $\quad$ since $f(p_1)$ is positive, therefore let $b = p_1$

$= 8.89$ (+ve)

$$[1, 1.5]$$

$$P_2 = \frac{1.5+1}{2} = 1.25.$$

$f(p_2) = f(1.25) = (1.25)^6 - 1.25 - 1$ $\quad$ since $f(p_2)$ is positive, therefore let $b = p_2$.

$= 1.565$

$$[1, 1.25]$$

$$P_3 = \frac{1+1.25}{2} = 1.125.$$

$f(p_3) = f(1.125) = (1.125)^6 - 1.125 - 1$ $\quad$ since $f(p_3)$ is negative, therefore let $a = p_3$

$= -0.0977$

$$[1.125, 1.25]$$

$$P_4 = \frac{1.125+1.25}{2} = 1.188.$$

$f(P_4) = f(1.188) = (1.188)^6 - 1.188 - 1$ $\quad$ since $f(p_4)$ is positive, therefore let $b = p_4$

$= 0.62324$

$$[1.125, 1.188]$$

$$P_5 = \frac{1.125+1.188}{2} = 1.156$$

$f(P_5) = f(1.156) = (1.156)^6 - 1.156 - 1$ $\quad$ since $f(p_5)$ is positive, therefore let $b = p_5$.

$= 0.23042$

$$[1.125, 1.156]$$

$$P_6 = \frac{1.125+1.156}{2} = 1.140$$

$f(P_6) = f(1.140) = (1.140)^6 - 1.140 - 1$ $\quad$ since $f(p_6)$ is positive, therefore let $b = p_6$

$= 0.05497$

$$[1.125, 1.140]$$

No.

Date

$$[1.125,\ 1.140]$$

$$P_7 = \frac{1.125+1.140}{2}$$

$$= 1.132$$

$f(P_7) = f(1.132) = (1.132)^6 - 1.132 - 1$ since $f(P_7)$ is negative, therefore let $a = 1.132$.

$= -0.02784$

$$[1.132,\ 1.140]$$

$$P_8 = \frac{1.132+1.140}{2}$$

$$= 1.136.$$

$f(P_8) = f(1.136) = (1.136)^6 - 1.136 - 1$ since $f(P_8)$ is positive, therefore let $b = 1.136$

$= 0.013166$

$$[1.132,\ 1.136]$$

$$P_9 = \frac{1.132+1.136}{2}$$

$$= 1.134.$$

$f(P_9) = f(1.134) = (1.134)^6 - 1.134 - 1$ since $f(P_9)$ is negative, therefore let $a = 1.134$

$= -0.0074366$

$$[1.134,\ 1.136]$$

$$P_{10} = \frac{1.134+1.136}{2}$$

$$= 1.135.$$

$f(P_{10}) = f(1.135) = (1.135)^6 - 1.135 - 1$ since $f(P_{10})$ is positive, therefore let $b = 1.135$

$= 0.0028398$

$$[1.134,\ 1.135]$$

$$P_{11} = \frac{1.134+1.135}{2}$$

$$= 1.134$$

$f(P_{11}) = f(1.134) = -0.0074366.$ let $a = 1.134$

$$[1.134,\ 1.135]$$

$$P_{12} = \frac{1.134+1.135}{2}$$

$$= 1.134$$

since $|P_{12} - P_{11}| = |0| = 0 < 0.001$.

$\therefore$ The root for $x^6 - x - 1 = 0$ is 1.13 in 2 decimal place.

$f(x) = \cos(x) + 2\sin(x) + x^2$

(a) secant method $x_0 = 0$ ; $x_1 = -0.1$

formula = $x_{i+2} = \dfrac{x_i f(x_{i+1}) - x_{i+1} f(x_i)}{f(x_{i+1}) - f(x_i)}$

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$

$f(x_0) = f(0) = \cos(0) + 2\sin(0) + (0)^2$

$= 1$

$f(x_1) = f(1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$

$= \cancel{0.805344}$

$= 0.8053$

$x^2 = \dfrac{0(0.8053) - (-0.1)(1)}{0.8053 - 1} = \dfrac{0.1}{-0.1947} = -0.5136.$

$\approx -0.51$

| $n$ | $x_{n-1}$ | $x_n$ | $x_{n+1}$ | $\lvert f(x_{n+1}) \rvert$ | $\lvert x_{n+1} - x_n \rvert$ |
|---|---|---|---|---|---|
| 1 | 0.0 | -0.1 | -0.5136 | 0.1522 | 0.4136. |
| 2 | -0.1 | -0.5136 | -0.6100 | 0.0457 | 0.0964 |
| 3 | -0.5136 | -0.6100 | -0.6514 | 0.0065 | 0.0414 |
| 4 | -0.6100 | -0.6514 | -0.6582 | 0.0013 | 0.0068 |
| 5 | -0.6514 | -0.6582 | -0.6598 | 0.0006 | 0.0016 |
| 6 | -0.6582 | -0.6598 | -0.6595 | 0.0002 | 0.0003 |

~~$f(x_{n+1})$~~

0.0002 , 0.0003 < 0.0005.

so, our proximation ~~out~~ to root is -0.6598.

b) Newton's method $x_0 = -0.1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

~~= -0.1~~

$$f(x_0) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$$

$$= 0.8053$$

$$f'(x_1) = -\sin x + 2\cos x + 2x$$

$$f'(x_0) = -\sin(-0.1) + 2\cos(-0.1) + 2(-0.1)$$

$$= 1.88984$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = -0.1 - \frac{(0.8053)}{1.8898}$$

$$x_1 = -0.5261$$

$$\approx -0.53$$

Newton's Method

| i | $p_i$ | $f(p_i)$ | $f'(p_i)$ |
|---|---|---|---|
| 0 | -0.1 | 0.8053 | 1.8898 |
| 1 | -0.526 | 0.1373 | 1.1797 |
| 2 | -0.6424 | 0.01510 | 0.915639 |
| 3 | -0.6589 | 0.0003 | |

$0.0003 < \cancel{0.001}\ 0.005$

at $p = p_3 = -0.6589$

So, the secant method only require one function input which is $f(x)$ and two initial guess, $x_0$ and $x_1$

but in Newton's method requires two function input which if $f(x)$ and $f'(x)$ and one initial guess which is $x_0$.

Question 4.

$$\frac{1}{x} = 1.37$$

$$\frac{1}{x} - 1.37 = 0$$

$$f(x) = \frac{1}{x} - 1.37$$

i ) $f(x_n) = \frac{1}{x_n} - 1.37$

$$f'(x_n) = -\frac{1}{x_n^2}$$

$$y - f(x_n) = f'(x_n)(x_{n+1} - x_n)$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \left(\frac{1/x_n - 1.37}{-1/x_n^2}\right)$$

$$= x_n - (-x_n + 1.37x_n^2)$$

$$= 2x_n - 1.37x_n^2$$

$$= x_n(2 - 1.37x_n) \qquad \text{if } \alpha = 1.37$$

$$= x_n(2 - \alpha x_n)$$

ii) $x_{n+1} = x_n(2 - 1.37x_n)$

$\varepsilon = 0.000000005$

$x_1 = 0.75(2 - 1.37(0.75))$

$= 0.7293750000$

$f(x_0) = \frac{1}{0.75} - 1.37$

$= -0.0366666667$

$f'(x_0) = -\frac{1}{0.75^2}$

$= -1.777777778$

$x_2 = 0.7293750000(2 - 1.37(0.7293750000))$

$= 0.729926590$

$f(x_1) = \frac{1}{0.729926500} - 1.37$

$= 0.000000952$

$f'(x_1) = -\frac{1}{0.729926500^2}$

$= -1.876902609$

$x_3 = 0.729926590(2 - 1.37(0.729926590))$

$= 0.729927007$

$f(x_2) = \frac{1}{0.729926590} - 1.37$

$= 0.000000783$

$f'(x_2) = -\frac{1}{0.729926590^2}$

$= -1.876902146$

Date

Question 5 :

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$S - I\lambda$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\lambda$$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$P(\lambda) = \det(S - I\lambda)$

$$= (1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 1-\lambda \end{vmatrix} + (0)\begin{vmatrix} -1 & 2-\lambda \\ 0 & -1 \end{vmatrix}$$

$$= (1-\lambda)(2-3\lambda+\lambda^2-1) + 1(-1+\lambda-0) + 0$$

$$= (1-\lambda)(\lambda^2-3\lambda+1) - 1 + \lambda$$

$$= \lambda^2 - 3\lambda \cancel{+1} - \lambda^3 + 3\lambda^2 \cancel{-\lambda} \cancel{-1} + \cancel{\lambda}$$

$$= -\lambda^3 + 4\lambda^2 - 3\lambda$$

let $-\lambda^3 + 4\lambda^2 - 3\lambda = 0$

$\lambda(\lambda-1)(\lambda-3) = 0$

$\therefore$ Eigen value $= \lambda=0,\ \lambda=1,\ \lambda=3$

$\lambda=0 \quad \lambda=1 \quad \lambda=3.$

let $\lambda = 0$ $\quad \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - 0\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] \quad R_2 = R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] \quad R_3 = R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$

$V_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$x - y = 0$

$y - z = 0 \qquad y - t = 0 \qquad x - t = 0$

$z = t \qquad y = t \qquad x = t.$

No.

Date

let $\lambda = 1$, $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - 1\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$

$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R_3 = R_3 - R_1,\ R_2 = -R_2} \begin{bmatrix} 0 & -1 & 0 \\ 1 & -1 & 1 \\ 0 & 0 & 0 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_1} \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t.$

$x - y + z = 0$ $\quad V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$z = t$ $\quad x - 0 + t = 0$

$-y = 0 \quad y = 0 \quad x = -t$

let $\lambda = 3$, $\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix}$

$\begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_2 = -R_2,\ R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 1 \\ -2 & -1 & 0 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_2 = 2R_1 + R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_3 = R_3 + R_2} \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}$

$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} t$

$V_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$x + y + z = 0$

$z = t \quad y + 2t = 0 \quad x - 2t + t = 0$

$y + 2z = 0 \quad y = -2t \quad x = t$

Eigen value $= \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

6. Gerschgorin's Circle Theorem

$$r_i = \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}| \qquad B_i = |\lambda - a_{ii}| \leq r_i$$

$$B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$$

$= |\lambda - (-18)| < |20| + |-6|$

$= |\lambda + 18| < 26$

$8 < \lambda < 44$

$\lambda \in (8.44)$

$= |\lambda - 13| < |30| + |-30|$

$= |\lambda - 13| < 60$

$= 47 \lambda < 60.$

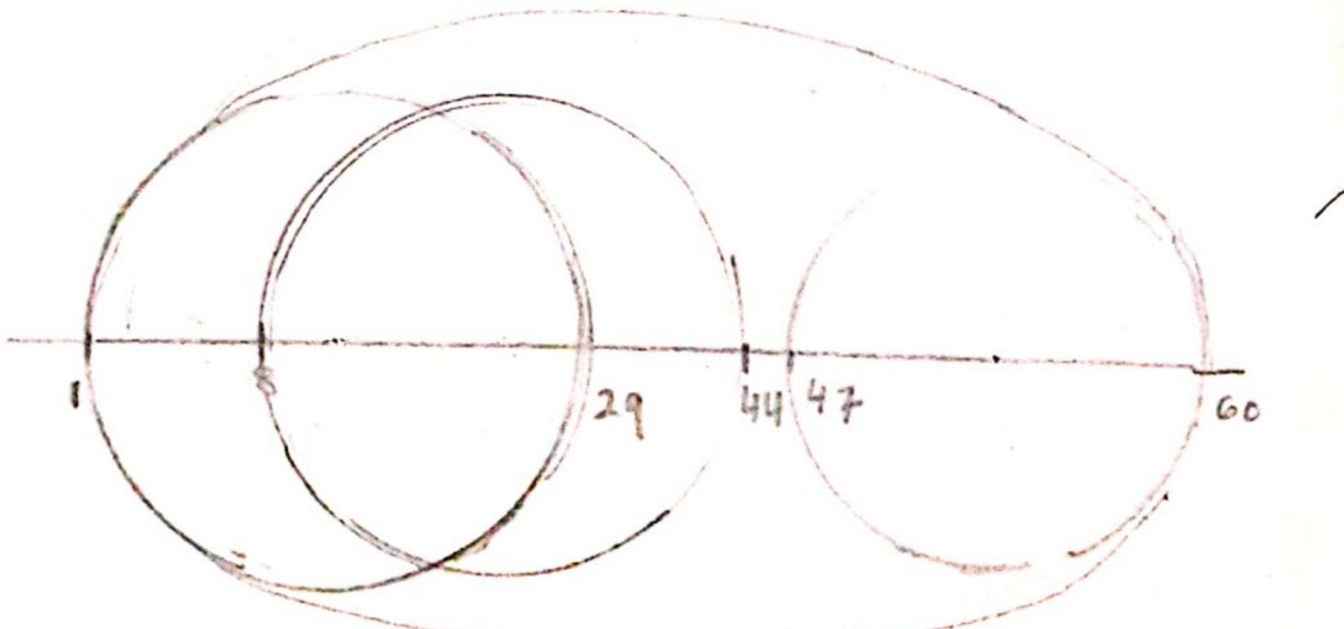
$= \lambda \in (47, 60)$

$= |\lambda - 14| < |-12| + |-3|$

$= |\lambda - 14| < 15$

$= 1 < \lambda < 29$

$= \lambda \in (1, 29)$

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(b) $\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$

<u>1st iteration</u> $\quad \varepsilon : 0.03$

$$V = AV^{(0)} : \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} : \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} \quad m_1 = 13$$

$$V^{(1)} = V/m_1 = \frac{1}{13}\begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} = \begin{bmatrix} -0.462 \\ -0.231 \\ 1 \end{bmatrix}$$

<u>2nd iteration</u>

$$V = AV^{(1)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \cancel{\begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix}} \begin{bmatrix} -0.462 \\ -0.231 \\ 1 \end{bmatrix} = \begin{bmatrix} -2.308 \\ -0.692 \\ 6.077 \end{bmatrix} \quad m_2 = 6.077$$

$$\frac{1}{6.077}\begin{bmatrix} -2.308 \\ -0.692 \\ 6.077 \end{bmatrix} = \begin{bmatrix} -0.38 \\ -0.114 \\ 1 \end{bmatrix} \qquad |V^2 - V^1| > \varepsilon : \begin{bmatrix} -0.38 \\ -0.144 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.462 \\ -0.231 \\ 1 \end{bmatrix} : \begin{bmatrix} 0.082 \\ 0.087 \\ 0 \end{bmatrix}$$

<u>3rd iteration</u>

$$AV^{(2)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.38 \\ -0.114 \\ 1 \end{bmatrix} = \begin{bmatrix} -1.443 \\ -0.038 \\ 5.025 \end{bmatrix} \quad m_3 = 5.025$$

$$V_3 = \frac{1}{5.025}\begin{bmatrix} -1.443 \\ -0.038 \\ 5.025 \end{bmatrix} = \begin{bmatrix} -0.287 \\ -0.008 \\ 1 \end{bmatrix} \qquad |V^3 - V^2| > \varepsilon \begin{bmatrix} -0.287 \\ -0.008 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.38 \\ -0.144 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.093 \\ 0.106 \\ 0 \end{bmatrix}$$

<u>4nd iteration</u>

$$AV^{(3)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.287 \\ -0.008 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.982 \\ 0.34 \\ 4.612 \end{bmatrix} \quad m_4 = 4.612$$

$$V^4 = \frac{1}{4.612}\begin{bmatrix} -0.982 \\ 0.34 \\ 4.612 \end{bmatrix} = \begin{bmatrix} -0.213 \\ 0.074 \\ 1 \end{bmatrix} \qquad |V^4 - V^3| = \begin{bmatrix} -0.213 \\ 0.074 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.287 \\ -0.008 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.074 \\ 0.082 \\ 0 \end{bmatrix}$$

5th iteration

$$AV^{(4)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.213 \\ 0.074 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.691 \\ 0.588 \\ 4.398 \end{bmatrix} \qquad m^5 = 4.398.$$

$$V^5 = \frac{1}{4.398} \begin{bmatrix} -0.691 \\ 0.588 \\ 4.398 \end{bmatrix} = \begin{bmatrix} \cancel{-0.517}\,0.157 \\ 0.134 \\ 1 \end{bmatrix} \qquad \|V^6 - V^4\| = \begin{bmatrix} -0.157 \\ 0.134 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.213 \\ 0.074 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.056 \\ 0.06 \\ 0 \end{bmatrix}$$

6th iteration

$$AV^{(5)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.157 \\ 0.134 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.495 \\ 0.759 \\ 4.272 \end{bmatrix} \qquad m^6 = 4.272$$

$$V^6 = \frac{1}{4.272} \begin{bmatrix} -0.495 \\ 0.759 \\ 4.272 \end{bmatrix} = \begin{bmatrix} -0.116 \\ 0.178 \\ 1 \end{bmatrix} \qquad \|V^6 - V^5\| = \begin{bmatrix} -0.116 \\ 0.178 \\ 1 \end{bmatrix} - \begin{bmatrix} -0.157 \\ 0.134 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.041 \\ 0.313 \\ 0 \end{bmatrix}$$

7th iteration

$$AV^6 = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.116 \\ 0.178 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.359 \\ 0.879 \\ 4.191 \end{bmatrix} \qquad m^7 = 4.191$$

$$V^7 = \frac{1}{4191} \begin{bmatrix} -0.359 \\ 0.874 \\ 4.191 \end{bmatrix} = \begin{bmatrix} -0.086 \\ 0.21 \\ 1 \end{bmatrix} \qquad \|V^7 - V^6\| = \begin{bmatrix} -0.086 \\ 0.21 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.116 \\ 0.178 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.03 \\ 0.032 \\ 0 \end{bmatrix}$$

since $\|V^7 - V^6\| < \varepsilon = 0.05$, then an legenvalue is $\lambda \approx m^7 = 4.191$

and the corresponding eigenvector is $V_1 \approx V^7(-0.086, 0.21, 1)$

(b) Smallest eigenvalue ($\lambda_3$) and corresponding eigenvector of matrix B.

A shifted $v = \lambda$ shifted $v$

$B - A - \lambda_1 I = A - 3I$

dominant $= \lambda_1 = 4.191$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4.191 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -22.19\cancel{3} & 20 & -6 \\ -12 & 9.809 & -3 \\ 30 & -30 & 8.809 \end{bmatrix}$$

let $v^{(0)} = [0,0,1]^T \quad \varepsilon = 0.05$

$$v = B v^{(0)} = \begin{bmatrix} -22.19 & 20 & -6 \\ -12 & 9.809 & -3 \\ 30 & -30 & 8.809 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 8.809 \end{bmatrix} \quad m_1 = -6.$$

$$v^1 = \frac{v}{m} = \frac{1}{-6} \begin{bmatrix} -6 \\ -3 \\ 8.809 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.5 \\ -1.468 \end{bmatrix}$$

| k | $(V^k)^T$ | | | $(BV^k)^T$ | | | $m_{k+1}$ |
|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 1 | -6 | -3 | 8.809 | m = -6 |
| 1 | 1 | 0.5 | -1.468 | -3.382 | -2.6915 | 2.0684 | -3.382 |
| 2 | 1 | 0.796 | -0.612 | -2.598 | -2.356 | 0.7289 | -2.598 |
| 3 | 1 | ~~0.9079~~ 0.907 | -0.281 | -2.364 | -2.2602 | 0.3147 | -2.364 |
| 4 | 1 | 0.956 | -0.133 | -2.272 | -2.2236 | 0.1484 | -2.272 |
| 5 | 1 | 0.97~~87~~ | -0.065 | -2.26 | -2.2216 | 0.1174 | -2.26 |
| 6 | 1 | 0.983 | -0.05 | | | | |

$\| v^6 - v^5 \| < \varepsilon = 0.05$

$\lambda_3 \approx m_6 = -2.26$

$$V_1 \approx V_6 = \begin{bmatrix} 1 \\ 0.983 \\ -0.05 \end{bmatrix}$$

The smallest eigenvalue for B is $\lambda_3 = \lambda^* + \lambda_1 = -2.26 + 4.191$
$= 1.931$
~~= 1.9~~

corresponding eigenvector $V_3$: $[1.0, 0.983, -0.05]$

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(c) $\sum_{i=1}^{3} \lambda_i = \sum_{i=1}^{3} a_{ii}$

$$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33}$$
$$= -18 + 14 + 13$$
$$= 9$$

Therefore $\lambda_2 = 9 - 1.931 - 4.191$
$$= 2.878.$$

so, intermediate eigenvalue is $2.878$ #