SECI 1113

COMPUTATIONAL MATHEMATICS

ASSIGNMENT 3

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ASSIGNMENT 3 - CHAPTER 4

Question 1

$T : R^2 \to R^2$ $\quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$

$$T(0) = T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = \begin{bmatrix} 2(0) - (0) \\ (0) + 3(0) \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$T(-v) = -T(v)$

$$T\left(\begin{bmatrix} -x \\ -y \end{bmatrix}\right) = \begin{bmatrix} -2x + y \\ -x - 3y \end{bmatrix}$$

$$= -\begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$$

$$= -T(v)$$

$\therefore$ $T(0) = 0$, $T(-v) = -T(v)$. Hence, T is a linear transformation

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2. Determine whether the function T: R → R by T([z]) [x+z+1] is a linear transformation between vector space.

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ 2xy \\ x+z+1 \end{bmatrix}$$

① $T(\bar{u}+\bar{v}) = T(\bar{u}) + T(\bar{v})$

$$T(\bar{u}+\bar{v}) = \begin{bmatrix} \bar{u}_1 + \bar{v}_1 \\ \bar{u}_2 + \bar{v}_2 \\ \bar{u}_3 + \bar{v}_3 \end{bmatrix}$$

$$= \begin{bmatrix} (\bar{u}_1+\bar{v}_1) + (\bar{u}_2+\bar{v}_2) - (\bar{u}_3+\bar{v}_3) \\ 2(\bar{u}_1+\bar{v}_1)(\bar{u}_2+\bar{v}_2) \\ (\bar{u}_1+\bar{v}_1) + (\bar{u}_3+\bar{v}_3) + 1 \end{bmatrix} = \begin{bmatrix} \bar{u}_1+\bar{v}_1 + \bar{u}_2+\bar{v}_2 - \bar{u}_3 - \bar{v}_3 \\ (2\bar{u}_1+2\bar{v}_1)(2\bar{u}_2+\bar{v}_2) \\ \bar{u}_1+\bar{v}_1 + \bar{u}_3+\bar{v}_3+1 \end{bmatrix}$$

$$T(\bar{u}) = \begin{bmatrix} \bar{u}_1 \\ \bar{u}_2 \\ \bar{u}_3 \end{bmatrix} = \begin{bmatrix} \bar{u}_1 + \bar{u}_2 - \bar{u}_3 \\ 2(\bar{u}_1)(\bar{u}_2) \\ \bar{u}_1 + \bar{u}_3 + 1 \end{bmatrix} \qquad T(\bar{v}) = \begin{bmatrix} \bar{v}_1 + \bar{v}_2 - \bar{v}_3 \\ 2(\bar{v}_1)(\bar{v}_2) \\ v_1 + v_3 + 1 \end{bmatrix}$$

$$T(\bar{u}) + T(\bar{v}) = \begin{bmatrix} \bar{u}_1 + \bar{u}_2 - \bar{u}_3 + \bar{v}_1 + \bar{v}_2 - \bar{v}_3 \\ (2\bar{u}_1 + 2\bar{u}_2)(2\bar{v}_1 + 2\bar{v}_2) \\ u_1 + \bar{u}_3 + \bar{v}_1 + \bar{v}_2 + 2 \end{bmatrix}$$

$$T(\bar{u}+\bar{v}) \neq T(\bar{u}) + T(\bar{v})$$

② $T(c\bar{u}) = \begin{bmatrix} c\bar{u}_1 \\ c\bar{u}_2 \\ c\bar{u}_3 \end{bmatrix} = \begin{bmatrix} c(c\bar{u}_1 + \bar{u}_2 - \bar{u}_3) \\ c(2\bar{u}_1)(2\bar{u}_2) \\ c(c\bar{u}_1 + \bar{u}_3 + 1) \end{bmatrix} = \begin{bmatrix} cu_1 + c\bar{u}_2 - c\bar{u}_3 \\ (2c\bar{u}_1)(2c\bar{u}_2) \\ cu_1 + cu_3 + c \end{bmatrix}$

$$= c\begin{bmatrix} u_1 + \bar{u}_2 - \bar{u}_3 \\ 2(\bar{u}_1)(\bar{u}_2) \\ \bar{u}_1 + \bar{u}_3 + 1 \end{bmatrix}$$

$T: R^3 \to R^3$ by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ 2xy \\ x+z+1 \end{bmatrix}$ is not linear transformation

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3. a. $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$T(e_1) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$ $\quad T(e_2) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ $\quad T(e_3) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ $\quad = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$ $\quad = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$

b. $T(3e_1 - 4e_2 + 6e_3) = 3T(e_1) - 4T(e_2) + 6T(e_3)$

$$= 3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} + \begin{bmatrix} 18 \\ 18 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 20 \\ 3 \end{bmatrix}$$

Question 4

$T: R^2 \to R^2$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ -2x + 4y \end{bmatrix}$$

vector $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$

$$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ -2x + 4y \end{bmatrix}$$

$$= \begin{bmatrix} 1 - 2(3) \\ -2(1) + 4(3) \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ 10 \end{bmatrix}$$

$\therefore$ In order to checked whether v is in $N(T)$, $T(v)$ ~~must~~ is equal to ~~zero~~ 0.
Since $T(v) \neq 0$, hence v is not in $N(T)$

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4. Define a linear operator $T: R^2 \to R^2$ by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ -2x+4y \end{bmatrix}$.
Determine whether the vector $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is in $N(T)$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ -2x+4y \end{bmatrix}$$

$$v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$$

if $N(T)$, $T(v)=0$.

$$\begin{bmatrix} 1-2(3) \\ -2(1)+4(3) \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}$$

$T(v) \neq 0$

So, v is not vector for $N(T)$.

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Question 5.

$T : R^3 \to R^3$, basis, $B = \{v_1, v_2, v_3\}$

$$T(v_1) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad T(v_2) = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \quad T(v_3) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

(a)

$$c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 5 & -3 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] = \left[\begin{array}{ccc|c} 1 & 0 & 1 & 2 \\ 0 & 1 & -3/5 & -3/5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3/5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 = 2$, $c_2 = -3/5$, $c_3 = 0$

$\therefore$ Hence, $w \in R(T)$

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b. $c_1\begin{bmatrix}-1\\2\\1\end{bmatrix} + c_2\begin{bmatrix}0\\5\\0\end{bmatrix} + c_3\begin{bmatrix}-1\\-1\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$\left[\begin{array}{ccc|c}-1 & 0 & -1 & 0\\ 2 & 5 & -1 & 0\\ 1 & 0 & 2 & 0\end{array}\right]$$

$\downarrow R_1 = -R_1$

$$\left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\ 2 & 5 & -1 & 0\\ 1 & 0 & 2 & 0\end{array}\right]$$

$\downarrow R_2 = R_2 - 2R_1 \quad R_3 = R_3 - R_1$

$$\left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\ 0 & 5 & -3 & 0\\ 0 & 0 & 1 & 0\end{array}\right]$$

$\downarrow R_2 = R_2 \div 5$

$$\left[\begin{array}{ccc|c}1 & 0 & 1 & 0\\ 0 & 1 & -3/5 & 0\\ 0 & 0 & 1 & 0\end{array}\right]$$

$\downarrow R_1 = R_1 - R_3 \quad R_2 = R_2 + \frac{3}{5}R_3$

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{array}\right]$$

since leading 1 is in columns 1, 2 and 3,

therefore the basis for $R(T)$ is

$$R(T) = \text{span}\left\{\begin{bmatrix}-1\\2\\1\end{bmatrix}, \begin{bmatrix}0\\5\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\2\end{bmatrix}\right\}$$

c. $\begin{bmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & 1\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

- No basis for $N(T)$
- $\dim(N(T)) = 0$