SECI 1113

COMPUTATIONAL MATHEMATICS

ASSIGNMENT 2

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Question 1

$$v: \left\{ \begin{bmatrix} 1 + t \\ 2 - t \\ 3 + 2t \end{bmatrix} \middle| \; t \in R \right\}$$

$$\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + \begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 + 2t_2 \end{bmatrix} = \begin{bmatrix} 1 + (t_1 + t_2) \\ 2 - (t_1 + t_2) \\ 3 + (2t_1 + 2t_2) \end{bmatrix}$$

axiom 3 = $u + (v + w) = (u + v) + w$

$$u = \begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} \quad v = \begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 + 2t_2 \end{bmatrix} \quad w = \begin{bmatrix} 1 + t_3 \\ 2 - t_3 \\ 3 + 2t_3 \end{bmatrix} ; \; t \in R$$

$$u + (v + w) = \begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + \left(\begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 + 2t_2 \end{bmatrix} + \begin{bmatrix} 1 + t_3 \\ 2 - t_3 \\ 3 + 2t_3 \end{bmatrix} \right)$$

$$= \begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + \begin{bmatrix} 1 + (t_2 + t_3) \\ 2 - (t_2 + t_3) \\ 3 + (2t_2 + 2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + (t_1 + t_2 + t_3) \\ 2 - (t_2 + t_2 + t_3) \\ 3 + (2t_1 + 2t_2 + 2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + t_1 + t_2 \\ 2 - t_1 - t_2 \\ 3 + 2t_1 + 2t_2 \end{bmatrix} + \begin{bmatrix} 1 + t_3 \\ 2 - t_3 \\ 3 + 2t_3 \end{bmatrix}$$

$$(u + v) + w = \left(\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + \begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 + 2t_2 \end{bmatrix} \right) + \begin{bmatrix} 1 + t_3 \\ 2 - t_3 \\ 3 + 2t_3 \end{bmatrix}$$

so; $u + (v + w) = (u + v) + w$

axiom 3 is sattisfied

axiom 7 = $c(u+v) = cu + cv$

$$u = \begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} \quad v = \begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 - 2t_2 \end{bmatrix}$$

$$c\left(\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + \begin{bmatrix} 1 + t_2 \\ 2 - t_2 \\ 3 - 2t_2 \end{bmatrix}\right) = c\begin{bmatrix} 1 + (t_1 + t_2) \\ 2 - (t_1 + t_2) \\ 3 + (2t_1 + 2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1 + ct_1 + ct_2 \\ 2 - (ct_1 + ct_2) \\ 3 + (2ct_1 + 2ct_2) \end{bmatrix} = \begin{bmatrix} 1 + ct_1 \\ 2 - ct_1 \\ 3 + 2ct_1 \end{bmatrix} + \begin{bmatrix} 1 + ct_2 \\ 2 - ct_2 \\ 3 + 2ct_2 \end{bmatrix}.$$

axiom 7 is sattisfied ⁂

axiom 8 = $(c+d)u = cu + du$

$$u = \begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix}$$

$$(c+d)\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} = \begin{bmatrix} 1 + (c+d)t_1 \\ 2 - (c+d)t_1 \\ 3 + 2(c+d)t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + ct_1 \\ 2 - ct_1 \\ 3 + 2ct_1 \end{bmatrix} + \begin{bmatrix} 1 + dt_1 \\ 2 - dt_1 \\ 3 + 2dt_1 \end{bmatrix}$$

$$= c\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix} + d\begin{bmatrix} 1 + t_1 \\ 2 - t_1 \\ 3 + 2t_1 \end{bmatrix}$$

axiom 8 also sattisfied

so, V is a vector space ⁂

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Question 2

$$S = \left\{ \left\{ \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} \right\} \middle| \; a, b, c \in \mathbb{R} \right\}$$

a) Let $s_1 = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix}$, $s_2 = \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 + c_2 & a_2 - c_2 \end{bmatrix}$

$$s_1 + s_2 = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix} + \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 + c_2 & a_2 - c_2 \end{bmatrix}$$

$$\cancel{= ((a_1-b_1)+(a_2-b_2)) + (a_1+a_2) + ((b_1+c_1)+(b_2+c_2)) + ((a_1-c_1)+(a_2-c_2))}$$

$$= \begin{bmatrix} a_1 + a_2 - b_1 - b_2 & a_1 + a_2 \\ b_1 + b_2 + c_1 + c_2 & a_1 + a_2 - c_1 - c_2 \end{bmatrix} \text{ is in } S$$

(The subset S in closed under addition)

Let m be any real number

$$m \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} = \begin{bmatrix} m(a-b) & ma \\ m(b+c) & m(a-c) \end{bmatrix} \text{ is in } S$$

(The subset S in closed under multiplication)

$\therefore$ Hence S is a subspace of $M_{2\times 2}$

b) Let $a = 3$ $\quad a-b=5$

$a-c=3$ $\quad b+c=-2$

$a=3$ $\quad a-b=5$

$a-c=3$ $\quad 3-b=5$

$(3)-c=3$ $\quad b=-2$

$-c=0$

$c=0$ $\quad a=3, b=-2, c=-1$

$\therefore \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S

Question 3

$$S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} \right\}$$

(a)

$$c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} + c_5 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 2 & 3 & 2 & 6 & 4 & x_2 \\ 3 & -1 & -1 & 1 & 5 & x_3 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 5 & 0 & 6 & 10 & x_2 - 2x_1 \\ 0 & 2 & -4 & 1 & 14 & x_3 - 3x_1 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 1 & 0 & 6/5 & 2 & \frac{1}{5}x_2 - \frac{2}{5}x_1 \\ 0 & 0 & -4 & -7/5 & 10 & 7/5\, x_1 - 17/5\, x_2 + x_3 \end{array}\right]$$

$$= \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 1 & 0 & 6/5 & 2 & 1/5\, x_2 - 2/5\, x_1 \\ 0 & 0 & 1 & 7/20 & -5/2 & -1/5\, x_1 + 17/20\, x_2 - \frac{1}{4} x_3 \end{array}\right]$$

$\therefore$ Shows that a solution exists for all choice of x_1, x_2 and x_3. Thus, the ~~we~~ span $(S) = R^3$.

(b)

$$c_1\begin{bmatrix}1\\2\\3\end{bmatrix}+c_2\begin{bmatrix}-1\\3\\-1\end{bmatrix}+c_3\begin{bmatrix}1\\2\\-1\end{bmatrix}+c_4\begin{bmatrix}0\\6\\1\end{bmatrix}+c_5\begin{bmatrix}-3\\4\\5\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\2&3&2&6&4&0\\3&-1&-1&1&5&0\end{array}\right]$$

$$=\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&5&0&6&10&0\\0&2&-4&1&14&0\end{array}\right]=\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&1&0&6/5&2&0\\0&2&-4&1&14&0\end{array}\right]$$

$$=\left[\begin{array}{ccccc|c}1&0&-3&-7/5&7&0\\0&1&0&6/5&2&0\\0&0&-4&-7/5&10&0\end{array}\right]=\left[\begin{array}{ccccc|c}1&0&-3&-7/5&7&0\\0&1&0&6/5&2&0\\0&0&1&-7/20&-5/2&0\end{array}\right]$$

$$=\left[\begin{array}{ccccc|c}1&0&0&-49/20&25/4&0\\0&1&0&6/5&2&0\\0&0&1&-7/20&-5/2&0\end{array}\right]$$

∴ ~~Thus~~ Since it is not a trivial solution, S is not linearly independent. Thus, S is not the basis of R^3.

Question 4 :

4. a)

$$c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1&3&1\\1&-1&0\end{array}\right]$$

$R_2 = R_2 - R_1 \downarrow$

$$\left[\begin{array}{cc|c}1&3&1\\0&-4&-1\end{array}\right]$$

$-4c_2 = -1$

$c_2 = \frac{1}{4}$

$c_1 + 3c_2 = 1$

$c_1 + 3(\frac{1}{4}) = 1$

$c_1 = \frac{1}{4}$

$\therefore$ coordinates of $\begin{bmatrix}1\\0\end{bmatrix}=\begin{bmatrix}1/4\\1/4\end{bmatrix}$

b)

$$c_1\begin{bmatrix}1\\-1\end{bmatrix}+c_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1&0&1\\-1&2&1\end{array}\right]$$

$R_2 = R_2 + R_1$

$$\left[\begin{array}{cc|c}1&0&1\\0&2&2\end{array}\right]$$

$2c_2 = 2$ $\quad c_1 = 1$

$c_2 = 1$

$$d_1\begin{bmatrix}1\\-1\end{bmatrix}+d_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1&0&3\\-1&2&-1\end{array}\right]$$

$R_2 = R_2 + R_1$

$$\left[\begin{array}{cc|c}1&0&3\\0&2&2\end{array}\right]$$

$c_1 = 3$ $\quad 2c_2 = 2$

$c_2 = 1$

$\therefore$ Transition matrix from B_2 to B_1 =

$$[I]_{B_2}^{B_1}=\begin{bmatrix}1&3\\1&1\end{bmatrix}$$

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c) $[v]_{B_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$$[v]_{B_1} = [I]_{B_2}^{B_1} [v]_{B_2}$$

$$= \begin{bmatrix} 1 & 3 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$= \begin{bmatrix} (1)(2)+(3)(3) \\ (1)(2)+(3)(1) \end{bmatrix}$$

$$= \begin{bmatrix} 11 \\ 5 \end{bmatrix}$$