SECI 1113

COMPUTATIONAL MATHEMATICS

ASSIGNMENT 1

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Assignment 1 : (Chapter 1 & 2)

Question 1 :

points : $(1,-1)$ and $(2,1)$

$2x - y = 3$ $\quad\quad$ $x + 3y = 5$

For point $(1,-1)$,

$2(1) - (-1) = 2 + 1$ $\quad\quad$ $1 + 3(-1) = 1 - 3$

$= 3 = 3$ $\quad\quad$ $= -2 \neq 5$

$\therefore$ $(1,-1)$ is not the solution to the given system of equations.

For point $(2,1)$,

$2(2) - (1) = 4 - 1$ $\quad\quad$ $2 + 3(1) = 2 + 3$

$= 3 = 3$ $\quad\quad$ $= 5 = 5$

$\therefore$ $(2,1)$ is the solution to the given system of equations.

Question 2 :

a) $x - y = 2$ $\quad\quad$ $2x - 2y = 4$

$x = 2 + y$ $\quad\quad$ $2(2+y) - 2y = 4$

$4 + 2y - 2y = 4$

$0 = 4 - 4$

$0 = 0$

$\therefore$ Infinite solution, consistent

b) $x - y = 1$ $\quad\quad$ $x - y = 3$

$x = 1 + y$ $\quad\quad$ $1 + y - y = 3$

$0 = 2$

$\therefore$ No solution, Inconsistent

Question 3 :

a) $\begin{vmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}$

$C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix}$ $\quad\quad$ $C_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix}$ $\quad\quad$ $C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

$= (1)\left((1)(2) - (1)(1)\right)$ $\quad\quad$ $= (-1)\left((5)(2) - (1)(0)\right)$ $\quad\quad$ $= (1)\left((5)(1) - (1)(0)\right)$

$= (2 - 1)$ $\quad\quad$ $= (-1)(10 - 0)$ $\quad\quad$ $= (5 - 0)$

$= 1$ $\quad\quad$ $= -10$ $\quad\quad$ $= 5$

det $= (1)(1) + (0)(-10) + 3(5)$

$= 1 + 0 + 15$

$= 16$

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Question 3 :

b) $\begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}$

$C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix}$
$= (1)\big((0)(-1) - (1)(1)\big)$
$= (0 - 1)$
$= -1$

$C_{12} = (-1)^{1+2} \begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix}$
$= (-1)\big((-1)(-1) - (1)(0)\big)$
$= (-1)(1 - 0)$
$= -1$

$C_{13} = (-1)^{1+3} \begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$
$= (1)\big((-1)(1) - (0)(0)\big)$
$= (-1 - 0)$
$= -1$

$\det = 1(-1) + (-1)(-1) + (0)(-1)$
$= -1 + 1 + 0$
$= 0$

Question 4 :

a) let $A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{(3)(1) - (0)(0)} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \dfrac{1}{3} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix}$

b) let

$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$\det A = (1)\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - (0)\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + (0)\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$
$= (1)\,((1)(1) - (-2)(0)) - 0 + 0$
$= (1)(1 - 0)$
$= 1$

$\det A_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = \big((1)(1) - (-2)(0)\big) = 1$

$\det A_{12} = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = \big((0)(1) - (-2)(0)\big) = 0$

$\det A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = \big((0)(0) - (1)(0)\big) = 0$

$\det A_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = \big((0)(1) - (0)(0)\big) = 0$

$\det A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = ((1)(1) - (0)(0) = 1$

$\det A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = \big((1)(0) - (0)(0)\big) = 0$

$\det A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} = \big((0)(-2) - (0)(1)\big) = 0$

$\det A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = \big((1)(-2) - (0)(0)\big) = -2$

$\det A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = \big((1)(1) - (0)(0)\big) = 1$

cofactor matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$

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Question 4:

b)

Cofactor matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ Ad adjoint $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

$$A^{-1} = \frac{1}{1}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 5:

a)
$$x_1 + 2x_2 - 3x_3 = 9$$
$$2x_1 - x_2 + x_3 = 0$$
$$4x_1 - x_2 + x_3 = 4$$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right)$$

$R_3 = R_3 - 2R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 0 & 1 & -1 & 4 \end{array}\right)$$

$R_2 = R_2 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 1 & -1 & 4 \end{array}\right)$$

$R_3 = 5R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & 2 & 2 \end{array}\right)$$

$$x_1 + 2x_2 - 3x_3 = 9$$
$$-5x_2 + 7x_3 = -18$$
$$2x_3 = 2 \qquad x_3 = 1$$

$$-5x_2 + 7(1) = -18$$
$$-5x_2 = -18 - 7$$
$$x_2 = -25 \div (-5)$$
$$x_2 = 5 \qquad x_2 = 5$$

$$x_1 = 9 - 2(5) + 3(1)$$
$$= 9 - 10 + 3$$
$$= 2 \qquad x_1 = 2$$

$\therefore x_1 = 2, \quad x_2 = 5, \quad x_3 = 1$

Question 5 :

b) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

$R_2 = R_2 + R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right)$$

$R_3 = R_3 - 3R_1$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right)$$

$R_3 = R_3 - 10R_2$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right)$$

$x_1 + x_2 + 2x_3 = 8$

$-x_2 + 5x_3 = 9$

$-52x_3 = -104$

$x_3 = -104 \div -52$

$= 2$ $\qquad x_3 = 2$

$-x_2 = 9 - 5(2)$

$x_2 = -(9-10)$

$= -(-1)$

$= 1$ $\qquad x_2 = 1$

$x_1 = 8 - (1) - 2(2)$

$= 8 - 1 - 4$

$= 3$ $\qquad x_1 = 3$

$\therefore x_1 = 3 , \quad x_2 = 1 , \quad x_3 = 2$

Question 6 :

a) $x_1 - 3x_2 - 2x_3 = 0$

$-x_1 + 2x_2 + x_3 = 0$

$2x_1 + 4x_2 + 6x_3 = 0$

$$\left(\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right)$$

$R_2 = R_2 + R_1 \qquad R_3 = R_3 - 2R_1$

$$\left(\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right)$$

$R_3 = \frac{1}{10} R_3 + R_2$

$$\left(\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$R_2 = -R_2$

$$\left(\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$R_1 = R_1 + 3R_2$

$$\left(\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

let $x_3 = t$ $\qquad x_3 = t$

$x_2 + x_3 = 0$

$x_2 = -t$ $\qquad x_2 = -t$

$x_1 + x_3 = 0$

$x_1 = -x_3$

$x_1 = -t$ $\qquad x_1 = -t$

$\therefore x_1 = -t , \quad x_2 = -t , \quad x_3 = t$

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Question 6 :

b)

$$2r + s = 3$$
$$4r + s = 7$$
$$2r + 5s = -1$$

$$\left(\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right)$$

$\therefore\ r = 2$

$s = -1$

$R_3 = R_3 - R_1 \quad R_2 = R_2 - 2R_1$

$$\left(\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{array}\right)$$

$R_1 = R_1 + R_2 \quad R_3 = R_3 + 4R_2$

$$\left(\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right)$$

3

2

1

$R_2 = -R_2 \quad R_1 = R_1 \div 2$

$$\left(\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right)$$

Question 7 :

a) Doolittle

$$2x + y + 4z = 12$$
$$8x - 3y + 2z = 20$$
$$4x + 11y - z = 33$$

$$\left(\begin{array}{ccc|c} 2 & 1 & 4 & 12 \\ 8 & -3 & 2 & 20 \\ 4 & 11 & -1 & 33 \end{array}\right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$S_1 = 12$

$4(12) + S_2 = 20$

$S_2 = -28$

$2S_1 - \frac{9}{7}S_2 + S_3 = 33$

$S_3 = 33 - 2(12) + \frac{9}{7}(-28)$

$= -27$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$-27z = -27$

$z = 1$ $\qquad z = 1$

$-7y - 14(1) = -28$

$-7y = -14$

$y = 2$ $\qquad y = 2$

$2x + y + 4z = 12$

$2x + (2) + 4(1) = 12$

$2x = 12 - 6$

$2x = 6$

$x = 3$ $\qquad x = 3$

$\therefore\ x = 3,\ y = 2,\ z = 1$

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Question 7 :

b) Crout :

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix} \qquad \begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$S_1 = 12 \div 2 = 6$.

$8S_1 - 7S_2 = 20$

$S_2 = (20 - 8(6)) \div -7$

$= 4$

$4(6) + 9(4) - 27S_3 = 33$

$-27S_3 = -27$

$S_3 = 1$

$z = 1$ $\qquad z = 1$

$y + 2(1) = 4$

$y = 4 - 2 = 2$ $\qquad y = 2$

$x + \frac{1}{2}(2) + 2(1) = 6$

$x = 6 - 1 - 2$

$x = 3$ $\qquad x = 3$

$\therefore x = 3, y = 2, z = 1$

Question 8 :

$u = (\sqrt{2}, 1, -1)^T \qquad v = (0, 2, 2)^T$

$d(u,v) = \sqrt{(\sqrt{2}-0)^2 + (1-2)^2 + (-1-2)^2}$

$= \sqrt{2 + 1 + 9}$

$= \sqrt{12}$

Question 9 :

$u = (2, 1, 2) \qquad v = (3, 3, 1)$

a) $u \cdot v = (2)(3) + (1)(3) + (2)(1)$

$= 6 + 3 + 2$

$= 11$

b) $u \times v = ((1)(1) - (2)(3), -((2)(1) - (2)(3)), (2)(3) - (1)(3))$

$= (1 - 6, -(2 - 6), 6 - 3)$

$= (-5, 4, 3)$

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Question 10 :

a)

$$C_1\begin{pmatrix}1\\1\\1\end{pmatrix} + C_2\begin{pmatrix}1\\2\\3\end{pmatrix} + C_3\begin{pmatrix}2\\-1\\1\end{pmatrix} = \begin{pmatrix}1\\-2\\5\end{pmatrix}$$

$$\begin{bmatrix}1 & 1 & 2\\1 & 2 & -1\\1 & 3 & 1\end{bmatrix}\begin{bmatrix}C_1\\C_2\\C_3\end{bmatrix} = \begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 1\\1 & 2 & -1 & -2\\1 & 3 & 1 & 5\end{array}\right]$$

$R_2 = R_2 - R_1 \quad R_3 = R_3 - R_1$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 1\\0 & 1 & -3 & -3\\0 & 2 & -1 & 4\end{array}\right]$$

$R_3 = R_3 - 2R_2$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 1\\0 & 1 & -3 & -3\\0 & 0 & 5 & 10\end{array}\right]$$

$R_3 = \frac{1}{5}R_3$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 1\\0 & 1 & -3 & -3\\0 & 0 & 1 & 2\end{array}\right]$$

$R_2 = R_2 + 3R_3$

$$\left[\begin{array}{ccc|c}1 & 1 & 2 & 1\\0 & 1 & 0 & 3\\0 & 0 & 1 & 2\end{array}\right]$$

$$-6\begin{bmatrix}1\\1\\1\end{bmatrix} + 3\begin{bmatrix}1\\2\\3\end{bmatrix} + 2\begin{bmatrix}2\\-1\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$R_1 = R_1 - R_2 - 2R_3$

$$\left[\begin{array}{ccc|c}1 & 0 & 0 & -6\\0 & 1 & 0 & 3\\0 & 0 & 1 & 2\end{array}\right]$$

$\therefore$ since $C_1 = -6$, $C_2 = 3$, $C_3 = 2$,

therefore, v is a linear combination of u_1, u_2 and u_3

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b) $\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 5 & 1 & 2 & 0 \end{array}\right]$$

$R_3 = R_3 - 2R_2 - R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 0 & -11 & -9 & 0 \end{array}\right]$$

$R_2 = R_2 - 2R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -11 & -9 & 0 \end{array}\right]$$

$R_3 = R_3 + 11R_2$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 24 & 0 \end{array}\right]$$

$R_3 = \frac{1}{24} R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_2 = R_2 - 3R_3$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$R_1 = R_1 - 2R_2 - R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$0\begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + 0\begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + 0\begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

since $c_1 = c_2 = c_3 = 0$,

Thus $\{u, v, w\}$ are linearly independent