SECI2143 PROBABILITY & STATISTICAL DATA ANALYSIS

SEMESTER II 2020/2021

ASSIGNMENT 3

No.	TEAM OMEGA	
1	AFIF HAZMIE ARSYAD BIN AGUS	A20EC0176
2	AHMAD AIMAN HAFIZI BIN MUHAMMAD	A20EC0177
3	LUQMAN ARIFF BIN NOOR AZHAR	A20EC0202
4	MUHAMMAD IMRAN HAKIMI BIN MOHD SHUKRI	A20EC0213

Lecturer:
DR. CHAN WENG HOWE

Submission Data:
3 JULY 2021

No.: .... Date: ....

Assignment 3

a)

Q1

$n = 25$

$\bar{x} = 19.5$

$\sigma = 9.88$

$CI = 90\%$

$z_{\alpha/2} = 1.645$

$$\bar{x} \pm z\left(\frac{\sigma}{\sqrt{n}}\right) = 19.5 \pm (1.645)\left(\frac{9.88}{\sqrt{25}}\right)$$

$$= 19.5 \pm 3.25$$

$$= (16.25, 22.75) \text{ seconds}$$

b)

$n = 360$

$CI = 99\%$

$z_{\alpha/2} = 2.575$

$\hat{p} = \frac{160}{360}$

$\hat{p} = 0.444$

$$\text{Standard Error} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

$$= \sqrt{\frac{(0.444)(1-0.444)}{360}}$$

$$SE = 0.026$$

$$\text{Confidence Interval} = \hat{p} \pm z(SE)$$

$$= 0.444 \pm 2.575(0.026)$$

$$= 0.444 \pm 0.067$$

$$= (0.377, 0.511)$$ food stores offer promotions

(Q1) a) ~~$H_0: p = 0.10$~~ ~~$H_1: p \neq 0.10$~~ $H_0: p = 0.10$ $H_1: p < 0.10$

$n = 350$

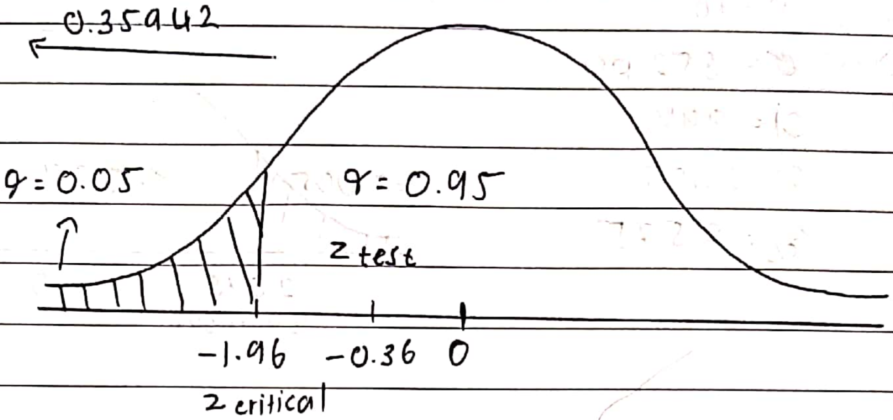
$\alpha = 0.05$

$CI = 0.95$

$Z_{\alpha/2} = 1.96$

Left-tailed test

$\hat{p} = \dfrac{33}{350}$



$\hat{p} = 0.0943$

Test statistics

$$Z_{test} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$$= \frac{0.0943 - 0.10}{\sqrt{\frac{(0.0943)(1 - 0.0943)}{350}}}$$

$Z_{test} = -0.36$

Since $Z_{test} > Z_{critical}$,
fail to reject H_0

$P(Z < -0.36) = 0.35942$

P-value $= 0.35942$

since P-value $> \alpha$
fail to reject H_0

There ~~is~~ are no sufficient evidance that company ABC's incorrect test result is less than 10%

Q2) b) $H_0: \mu = 58000\,psi$ $\quad H_1: \mu \neq 58000\,psi$

$\bar{x} = 58{,}400\,psi$

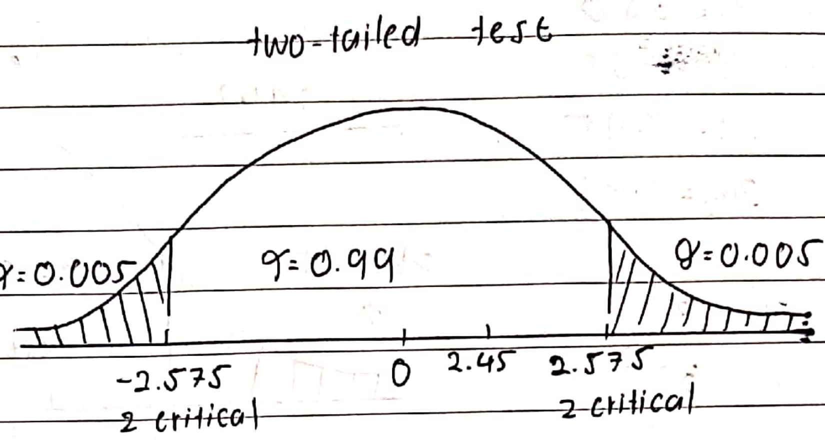
$n = 16$

$\sigma = 652\,psi$

$CI = 99\%$

$\alpha = 0.005$

$z_{\alpha/2} = 2.575$



Test statistics

$$z_{test} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$= \frac{58{,}400 - 58{,}000}{\left(\frac{652}{\sqrt{16}}\right)}$$

$z_{test} = 2.45$

$P(z < -2.45) = 0.0714$

$P(z < 2.45) = 1 - 0.99286$

$= 0.0714$

$P\text{-value} = 2(0.0714)$

$= 0.1428$

since $z_{test}\,(2.45) < z_{critical}\,(2.575)$

fail to reject H_0

Since P-value > 0.005

fail to reject H_0

There are sufficient ~~reason~~ evidence to support the claims that the average compressive strength of steel is 58,000 psi.

3

(Q3)

a) $H_0: \mu = 10 \quad H_1: \mu \neq 10$

$n = 10$

$\alpha = 0.01$

CI : 99%

$$\bar{x} = \frac{10.3 + 9.9 + 10.2 + 10.1 + 9.7 + 9.9 + 9.8 + 10.3 + 10.0 + 10.4}{10}$$

$\bar{x} = 10.06$

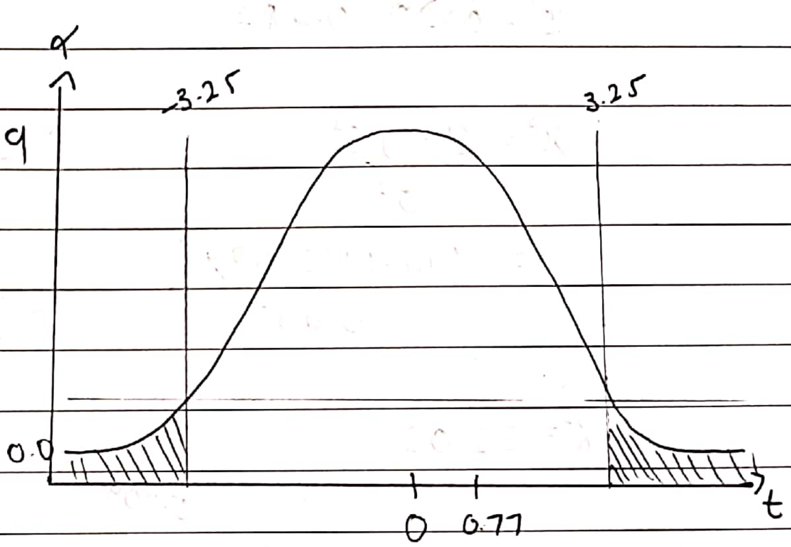
~~$s = 3^2 + (-1)^2 + 2^2 + 0.1$~~

$$s^2 = \frac{(0.3)^2 + (-0.1)^2 + 0.2^2 + 0.1^2 + (-0.3)^2 + (-0.1)^2 + (-0.2)^2 + 0.3^2 + 0^2 + 0.4^2}{10-1}$$

$s = \sqrt{0.06}$ $\quad df = n-1 = 10-1 = 9$

$s = 0.245$

$t_{\alpha/2, n-1} = t_{0.005, 9} = 3.25$



Test statistics

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{10.06 - 10.00}{0.245/\sqrt{10}}$$

$= 0.77$

~~P-value = P(z<3~~

$P(z < -0.77) = 0.22065$

$P(z > 0.77) = 1 - 0.77935$

$= 0.22065$

P-value $= 2(0.22065)$

$= 0.4413$

since $t < t_{\alpha/2, n-1}$

fail to reject H_0

since P-value $> \alpha$

fail to reject H_0

It is statistically significant to support the claims of ~~the averag~~ Brand X of having an average of 10 l of car lubricant is true

4

Scanned with CamScanner

(Q3) b) H_0: ~~AR~~ $\sigma = 0.44kg$ H_1: $\sigma \neq 0.44kg$ (two-tailed test)

$n = 14$

$CI = 95\%$

$\alpha = 0.05$

$$\bar{x} = \frac{3.70 + 4.31 + 3.73 + 4.33 + 3.33 + 2.58 + 4.47 + 3.55 + 4.66 + 3.68 + 3.02 + 4.09 + 2.36 + 3.35}{14}$$

$$s = \frac{\Sigma (x-\bar{x})^2}{n-1}$$

$\bar{x} = 3.65$

$$s^2 = \frac{(0.05)^2 + (0.66)^2 + (0.08)^2 + 0.68^2 + (-0.32)^2 + (-1.07)^2 + (0.82)^2 + (-0.1)^2 + 1.01^2 + 0.03^2 + (-0.63)^2 + 0.44^2 + (-1.29)^2 + (-0.3)^2}{14-1}$$

$s^2 = \cancel{0.20}\ 0.4877$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$= \frac{(14-1)(0.477)}{0.44^2}$$

$\chi^2 = 32.03$

$\chi^2_{1-\alpha/2,\, n-1} = \chi^2_{1-0.025,\, 14-1}$

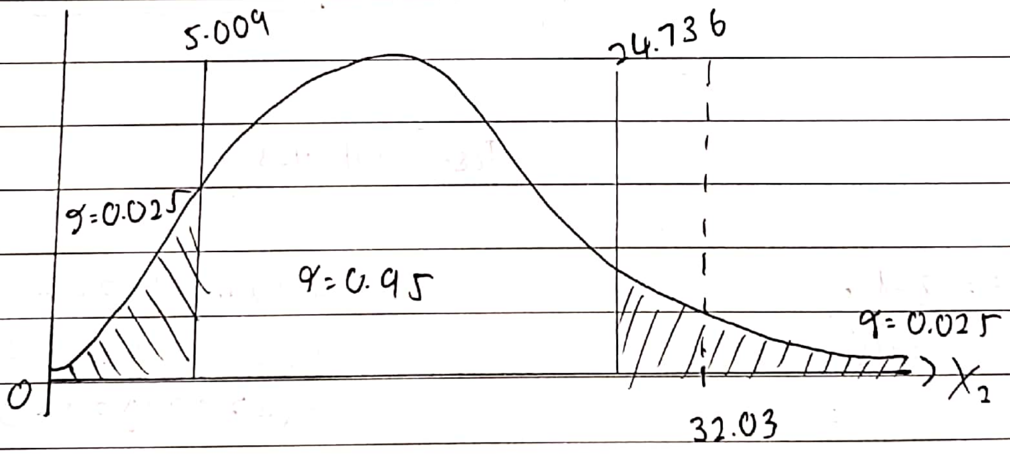
$= \chi^2_{0.975,\, 13}$

$= 5.009$

$\chi^2_{\alpha/2,\, n-1} = \chi^2_{0.025,\, 14-1}$

$= \chi^2_{0.025,\, 13}$

$= 24.736$



~~Since $\chi^2 > \chi^2_{\alpha/2, n-1}$~~

Since $\chi^2 > \chi^2_{0.025,13}$

reject Ho

It is true that vitamin supplement do change the standard deviation of babies's birth weight, statistically.

5

Scanned with CamScanner

Q4 $H_0 : \sigma^2 = 0.18 \text{ inch}^2$ $H_1 : \sigma^2 > 0.18 \text{ inch}^2$ (right-tailed test)

$n = 101$

$\sigma^2 = 0.18$

$s^2 = 0.165$

$\alpha = 0.05$

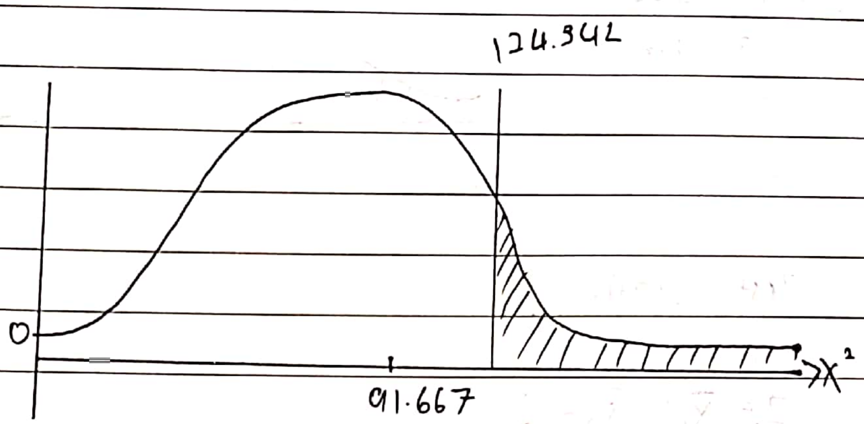
$$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$$

$$\chi^2_{\alpha, n-1} = \chi^2_{0.05,100} = 124.342$$

$$= \cancel{\frac{((97+1)(0.18))}{(1}}$$

$$= \frac{(101-1)(0.165)}{0.18}$$

$$\chi^2 = 91.6767$$



Since $\chi^2 < \chi^2_{0.05,100}$

fail to reject H_0

We have significant evidence to accuse that the new joint inspector is not making satisfactory measurements.

5

Date: ....

(Q5) $H_0 = \sigma_1^2 - \sigma_2^2 = 0$ $\quad$ $H_0: \mu_1 - \mu_2 = 0$

$H_1 = \sigma_1^2 - \sigma_2^2 \neq 0$ $\quad$ $H_1: \mu_1 - \mu_2 \neq 0$ (two-tailed test)

$\mu_1 = 76/4$ $\quad$ $\bar{x}_1 = 76.4$

$\mu_2 = 71.2$ $\quad$ $\bar{x}_2 = 71.2$

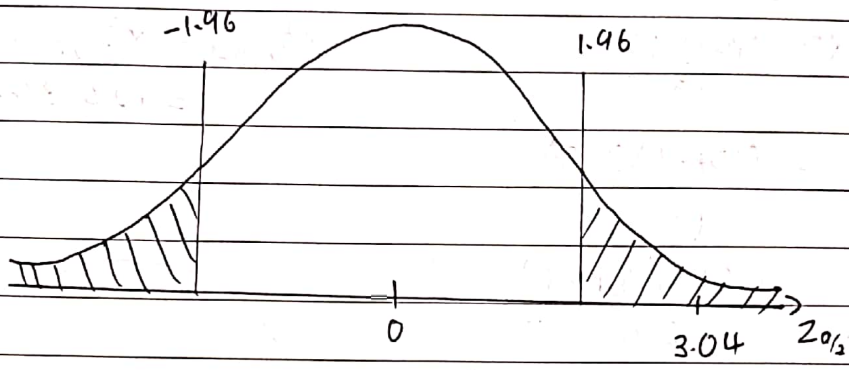
$\sigma_1^2 = 25.3$

$\sigma_2^2 = 22.2$

$\alpha = 0.05$

$z_{\alpha/2} = z_{0.025}$

$= 1.96$



Test statistic

$$z_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$= \frac{76.4 - 71.2 - 0}{\sqrt{\frac{25.3}{15} + \frac{22.2}{18}}}$$

$z_0 = 3.04$

Since $z_0 > z_{0.025}$

reject H_0

There is sufficient evidence to conclude that there are difference in the mean spending between the two population

6

6) $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$ $\alpha = 0,05$ $t_{0.05, 35} = 1.645$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(16-1)(3,6)^2 + (21-1)(2,5)^2}{16 + 21 - 2} = 9,13$$

$s_p = 3,02$

$$t_0 = \frac{93,6 - 96,5}{(3,02)\sqrt{\frac{1}{16} + \frac{1}{21}}}$$

$= -4,89$

$\therefore$ since $t_0 = -4,89 < 1,645$, fail to reject H_0. Insufficient evidence to show that standard deviation from batch 1 greater than batch 2

7) $\bar{D} = \dfrac{(203-225) + (390-410) + (389-402) + (279-285) + (333-355) + (213-240) + (410-444) + (364-370) + (470-501) + (464-490)}{10}$

$H_0: \mu_D = 0$

$H_1: \mu_D > 0$

$= -20,7$

$\bar{D}^2 = 484 + 400 + 169 + 36 + 484 + 729 + 1156 + 36 + 961 + 676 = 5131$

$$S_D = \sqrt{\frac{5131 - \frac{(-207)^2}{10}}{9}}$$

$= 9,70$

$$t = \frac{-20,7 - 0}{\frac{9,7}{\sqrt{10}}}$$

$= -6,75$

$t_{0.05, 9} = 1,833$

$\therefore$ since $-6,75 < 1,833$, reject H_0. Insufficient evidence to show training increases the number of words spelled correctly

NO.:

Date:

Questions 8 (10mark)

x	y	xy	x^2	y^2
0.27	2	0.54	0.0729	4
1.41	3	4.23	1.9881	9
2.19	3	6.57	4.7961	9
2.83	6	16.98	8.0089	36
2.19	4	8.76	4.7961	16
1.81	2	3.62	3.2761	4
0.85	1	0.85	0.7225	1
3.05	5	15.25	9.3025	25
Σ 14.6	26	56.8	32.9632	104

i) $r = \dfrac{n\Sigma xy - \Sigma x \Sigma y}{\sqrt{[n\Sigma x^2 - (\Sigma x)^2][n\Sigma y^2 - (\Sigma y)^2]}}$

$= \dfrac{8(56.8) - 14.6(26)}{\sqrt{[8(32.9632) - (14.6)^2][8(104) - (26)^2]}}$

$= \dfrac{454.4 - 379.6}{\sqrt{(50.5456)(156)}} = \dfrac{74.8}{88.7981}$

$r = 0.8423$.

ii) $H_0 : p = 0$

$H_1 : p \neq 0$

$\alpha = 0.05/2$

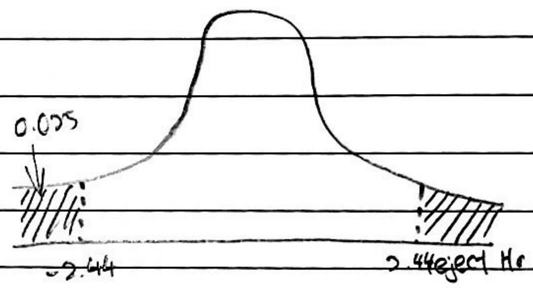
$= 0.025 = 2.4469$

$df = 8 - 2$

$t = \dfrac{r}{\sqrt{\dfrac{1-r^2}{n-2}}}$

$= \dfrac{0.8423}{\sqrt{\dfrac{1 - .8423^2}{6}}}$

$= 3.8278$.



∴ since 3.8278 > 2.4469, reject Ho. there is no sufficient evidence of a linear relationship between weight of plastic usage and size of household.

iii) $\alpha = 0.01/2$

$= 0.005 = 3.707$

∴ since 3.8278 > 3.707, reject Ho. The result in (ii) does not change as not enough evidence.

Question 9.

(i) $\sum xy = 25431$

$\sum x^2 = 133336$

$\sum x = 1076$

$\sum y = 216$

$$b_1 = \frac{25431 - \frac{(1076)(216)}{9}}{133336 - \frac{1076^2}{9}}$$

$= -0.0837$

$b_0 = 24 - (-0.0837)(119.56)$

$= 34.0068$

$\hat{y} = 34.0068 - 0.0837x$ #

(ii) $\hat{y} = 34.0068 - 0.0837(125)$

$= 23.54$

≈ 24

Question 10.

$\alpha = 0.01$

$H_0 = \mu_1 = \mu_2 = \mu_3$

H_1 = one mean is different (at least)

Memory booster :

$n = 5$

$$\bar{x} = \frac{70 + 77 + 83 + 90 + 97}{5}$$

$$= \frac{417}{5}$$

$$= 83.4$$

$$s = \sqrt{(70 - 83.4)^2 + (77 - 83.4)^2 + (83 - 83.4)^2 + (90 - 83.4)^2 + (97 - 83.4)^2}$$

$= 10.6$

Placebo :

$n = 5$

$$\bar{x} = \frac{37 + 43 + 50 + 57 + 63}{5} = 50$$

$$s = \sqrt{\frac{(37-50)^2 + (43-50)^2 + (50-50)^2 + (57-50)^2 + (63-50)^2}{5-1}}$$

$= 10.44$

Without treatment :

$n = 5$

$$\bar{x} = \frac{3 + 10 + 17 + 23 + 30}{5} = 16.6$$

$$s = \sqrt{\frac{(3-16.6)^2 + (10-16.6)^2 + (17-16.6)^2 + (23-16.6)^2 + (30-16.6)^2}{5-1}}$$

$= 10.6$

mean between samples:

$$\bar{\bar{x}} = \frac{83.4 + 50 + 16.6}{3} = 50$$

std deviation between samples:

$$S_{\bar{x}} = \sqrt{\frac{(83.4-50)^2 + (50-50)^2 + (16.6-50)^2}{3-1}}$$

$$= 33.4$$

Variance between samples:

$$nS_{\bar{x}}^2 = 5(33.4)^2 = 5577.8$$

variance within samples:

$$S_p^2 = \frac{(10.6)^2 + (10.44)^2 + (10.6)^2}{3}$$

$$= 111.24$$

Test statistic, F:

$$F = \frac{5577.8}{111.24} = 50.14$$

numerator = 3-1 = 2, denominator = 3(5-1) = 12

F critical value = 6.93, α = 0.01

Thus, we reject the null hypothesis, H_0 because F test statistic is greater than F critical value. There is enough evidence to state that at least one of the means is different.