COURSE:

SECI2143-02 PROBABILITY & STATISTICAL DATA ANALYSIS

FACULTY:

FACULTY OF ENGINEERING

SCHOOL:

SCHOOL OF COMPUTING

TITLE:

ASSIGNMENT 2

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QUESTION 1

The probability that indicates the subject will be rated as very easy, $P(A) = 0.08$, easy, $P(B) = 0.29$, average $P(C) = 0.34$, difficult, $P(D) = 0.17$ or very difficult, $P(E) = 0.12$.

Very easy = $P(A) = 0.08$
Easy = $P(B) = 0.29$
Average = $P(C) = 0.34$
Difficult = $P(D) = 0.17$
Very Difficult = $P(E) = 0.12$

Find the probabilities that the subjects will be rated based on the following categories:

a) Average, easy or very easy

$$P(C \cup B \cup A) = P(C) + P(B) + P(A)$$
$$= 0.34 + 0.29 + 0.08$$
$$= 0.71$$

b) Average, difficult or very difficult

$$P(C \cup D \cup E) = P(C) + P(D) + P(E)$$
$$= 0.34 + 0.17 + 0.12$$
$$= 0.63$$

c) Neither very difficult nor very easy

$$P(D \cup A) = P(D) + P(A)$$
$$= 0.17 + 0.08$$
$$= 0.25$$

Neither very difficult nor very easy:

$$P(D \cup A)' = 1 - P(D \cup A)$$
$$= 1 - 0.25$$
$$= 0.75$$

d) Difficult or very easy

$$P(D \cup A) = P(D) + P(A)$$
$$= 0.17 + 0.08$$
$$= 0.25$$

or

$$1 - P(D \cup A)'$$
$$= 1 - 0.75$$
$$= 0.25$$

Question 2

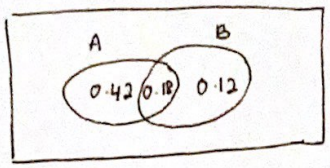
$P(A) = 0.60$

$P(B) = 0.30$

$P(A \cup B) = 0.80$

a) $P(A \cap B) = P(A) \cdot P(B)$
$= 0.60\ (0.30)$
$= 0.18$

b)



$P(A') = 1 - P(A)$
$= 1 - 0.60$
$= 0.40$

$P(B') = 1 - P(B)$
$= 1 - 0.30$
$= 0.70$

c) $P(A' \cap B) = P(A') \cdot P(B)$
$= 0.40\ (0.30)$
$= 0.12$

d) $P(A' \cap B') = P(A') \cdot P(B')$
$= 0.40\ (0.70)$
$= 0.28$

e) $P(A' \cup B') = P(A') + P(B') - P(A' \cap B')$
$= 0.40 + 0.70 - 0.28$
$= 0.82$

f) $P(A \cap B') = P(A) \cdot P(B')$
$= 0.60\ (0.70)$
$= 0.42$

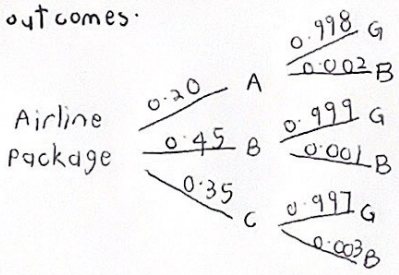
QUESTION 3

Airline services provides various packages for their customers booking during holiday season. There are 20%, 45% and 35% of the customers booked for package A, B and C respectively. Given the probability experienced with bad services are as follow:

Package A : 0.002
Package B : 0.001
Package C : 0.003

Based on the above situation, answer the following questions:

a) construct a tree diagram based on the situation given and determine their associated probabilities, including the intersection probabilities together with the outcomes.



Let G = Good service, B = Bad service

Package A :
Bad service = 0.002
Good service = 1 - 0.002
= 0.998

Package B :
Bad service = 0.001
Good service = 1 - 0.001
= 0.999

Package C :
Bad service = 0.003
Good service = 1 - 0.003
= 0.997

b) Based on the answer in (a), determine the probability that a customer experienced with a good service from the airlines.

Package A :
$P(A \cap G) = P(A) \times P(G)$
$= 0.20 \times 0.998$
$= 0.1996$

Package B
$P(B \cap G) = P(B) \times P(G)$
$= 0.45 \times 0.999$
$= 0.4496$

Package C
$P(C \cap G) = 0.35 \times 0.997$
$= 0.3490$

$P(G) = 0.1996 + 0.4496 + 0.3490$
$= 0.9982$

c) Given that the package is operating smoothly, what is the probability that it is in Package B?

$$P(B|G) = \frac{P(B \cap G)}{P(G)}$$

$$= \frac{P(B) \times P(G)}{P(G)}$$

$$= \frac{0.45 \times 0.999}{0.9982}$$

$$= 0.4504$$

Question 4.

banana sold : 70% (0.7)
Kurma sold : 30% (0.3)

x : no. of dairy product purchased among 20 customer.

a) Binomial Random Variable.
- independent trials
- Fixed number is next 20 customer purchased.

b) $P(X=8) = b(8; 20, \frac{7}{10})$

$= \binom{20}{8}\left(\frac{7}{10}\right)^{8}\left(\frac{3}{10}\right)^{12}$

$= 3.8593 \times 10^{-3}$

c) $P(7 \leq x \leq 10) = P(x=7) + P(x=8) + P(x=9) + P(x=10)$

$= b(7; 20, \frac{7}{10}) + b(8; 20, \frac{7}{10}) + b(9; 20, \frac{7}{10}) + b(10; 20, \frac{7}{10})$

$= \left[\binom{20}{7}\left(\frac{7}{10}\right)^{7}\left(\frac{3}{10}\right)^{13}\right] + \left[\binom{20}{8}\left(\frac{7}{10}\right)^{8}\left(\frac{3}{10}\right)^{12}\right] +$

$\left[\binom{20}{9}\left(\frac{7}{10}\right)^{9}\left(\frac{3}{10}\right)^{11}\right] + \left[\binom{20}{10}\left(\frac{7}{10}\right)^{10}\left(\frac{3}{10}\right)^{10}\right]$

$= (1.0178 \times 10^{-3}) + (3.8593 \times 10^{-3}) + (0.0120) + (0.0308)$

$= 0.0477$

d) $P(3 \leq x \leq 5) = P(x=3) + P(x=4) + P(x=5)$

$= b(3; 20, \frac{3}{10}) + b(4; 20, \frac{3}{10}) + b(5; 20, \frac{3}{10})$

$= \left[\binom{20}{3}\left(\frac{3}{10}\right)^{3}\left(\frac{7}{10}\right)^{17}\right] + \left[\binom{20}{4}\left(\frac{3}{10}\right)^{4}\left(\frac{7}{10}\right)^{16}\right] + \left[\binom{20}{5} + \left(\frac{3}{10}\right)^{5}\left(\frac{7}{10}\right)^{15}\right]$

$= 0.0716 + 0.1304 + 0.1789$

$= 0.3809$

QUESTION 5

A = student pilot pass the test B = student pilot fail the test

$P(A) = 0.7$

$P(B) = 1 - 0.7$

$P(B) = 0.3$

Let X be the number of trial to pass the test

Since the trial is independent, X has a negative binomial distribution with $k=1$, $p = 0.7$

a) ~~on~~ pass on the third try;

$x = 3$, $k=1$, $p = 0.7$, $q = 0.3$

i) $P(X = x) = \binom{x-1}{k-1} p^k q^{x-k}$

$= \binom{3-1}{1-1} (0.7)^1 (0.3)^{3-1}$

$= \binom{2}{0} (0.7)^1 (0.3)^2$

$P(X = 3) = 0.063$

∴ The probability the student pass on third try is 0.063.

ii) Before the fourth try;

$x = 4$, $k = 1$, $p = 0.7$, $q = 0.3$

$P(X < 4) = P(1 \leq X \leq 3)$

$= \sum_{x=1}^{3} b(x; 1, 0.7)$

$= \binom{1-1}{1-1} (0.7)^1 (0.3)^{1-1} + \binom{2-1}{1-1} (0.7)^1 (0.3)^{2-1} + \binom{3-1}{1-1} (0.7)^1 (0.3)^{3-1}$

$= \binom{0}{0} (0.7)(1) + \binom{1}{0} (0.7)(0.3) + \binom{2}{0} (0.7)(0.3)^2$

$= 0.7 + 0.21 + 0.063$

$= 0.973$

∴ Probability of student passing before fourth try is 0.973.

iii. At the fifth after seventh try,

$x = 7,\ k = 5,\ p = 0.7,\ q = 0.3$

$P(X = x) = \binom{x-1}{k-1} p^k q^{x-k}$

$= \binom{7-1}{5-1} (0.7)^5 (0.3)^{7-5}$

$= \binom{6}{4} (0.7)^5 (0.3)^2$

$P(X = 7) = 0.227$

∴ The probability a student will pass at fifth after seventh trials is 0.227. #

Probability student will pass within 5th to 7th try

$x = 5, 7 \quad k = 1 \quad p = 0.7,\ q = 0.3$

$P(5 \leq X \leq 7)$

$= \sum_{x=5}^{7} b(x; 1, 0.7)$

$= \binom{5-1}{1-1} (0.7)^1 (0.3)^{5-1} +$

$\binom{6-1}{1-1} (0.7)^1 (0.3)^{6-1} +$

$\binom{7-1}{1-1} (0.7)^1 (0.3)^{7-1}$

$= \binom{4}{0} (0.7)^1 (0.3)^4 + \binom{5}{0} (0.7)^1 (0.3)^5 + \binom{6}{0} (0.7)^1 (0.3)^6$

$= 5.67 \times 10^{-3} + 1.701 \times 10^{-3} + 5.103 \times 10^{-4}$

$= 0.007881$

∴ Probability student will pass within 5th to 7th is 0.007881 #

Note:
Since the question can be interpret in 2 ways, if we assume a student will pass at fifth at trying for seventh time, the probability is 0.227.

b) Mean and Variance

$r = 5,\ p = 0.7\ \ q = 0.3$

Mean, $\mu = E(x)$

$= \frac{r}{p}$

$= \frac{5}{0.7}$

$\mu = 7.143$

Variance, $\sigma^2 = V(x)$

$= \frac{r(q)}{p^2}$

$= \frac{(5)(0.3)}{(0.7)^2}$

$= 3.061$

QUESTION 6

At QT sugar Factory the amounts which go into bag of sugar are supposed to be normally distributed with mean 36 kg and standard deviation 0.1 kg. once every 30 minutes a bag is selected from the production line and its contents are noted precisely. If the amount of the bag goes below 35.8 kg or above 36.2kg, then the bag will be declared out of control.

a) $\mu = 36kg, \sigma = 0.1kg$

$z = \frac{X - \mu}{\sigma}$ $\longrightarrow$ $z_2 = \frac{36.2 - 36}{0.1}$

$= 2$

$z_1 = \frac{35.8 - 36}{0.1}$

$= -2$

$(-2 < z < 2)$

$P(X < 35.8) + P(X > 36.2)$

$= P(z < -2) + P(>> 2)$

$= 0.0228 + (1 - 0.9772)$

$p = 0.0456$

b) $n = 16, p = 0.0456$

$P(X = 0) = \binom{16}{0} 0.0456^0 (1 - 0.0456)^{16}$

$= 0.4739$

c) $n = 16, p = 0.0456$

$P(X = 1) = \binom{16}{1} 0.0456^1 (1 - 0.0456)^{15}$

$= 0.3623$

d) $P(X < 35.8) + P(X > 36.2)$

$= \left(\frac{35.8 - 37}{0.4}\right) + \left(\frac{36.2 - 37}{0.4}\right)$

$= P(z < -3) + P(z > -2)$

$= 0.0013 + (1 - 0.0028)$

$= 0.9785$

QUESTION 7

Mean (μ) = 28 Standard Deviation (σ) = 8

a) Proportion of the duration between 20 and 40 days old

$$z = \frac{x - \mu}{\sigma} \qquad z = \frac{x - \mu}{\sigma}$$

$$z = \frac{20 - 28}{8} \qquad = \frac{40 - 28}{8}$$

$$z = -1 \quad , \qquad z = 1.5$$

$$P(20 < x < 40) = P(-1.0 < z < 1.5)$$

$$= P(z < 1.5) - P(z < -1.0)$$

$$= 0.93319 - 0.15866$$

$$P(20 < x < 40) = 0.77453$$

b) Propation of the duration less than 30 days old.

$$z = \frac{x - \mu}{\sigma} \qquad P(x < 30.0) = P(z < 0.25)$$

$$= \frac{30 - 28}{8} \qquad = 0.59871 \ \text{※}$$

$$z = 0.25$$

c) Number of Days in which 75% of all claims are above.

Since,

$P(X < x) = 0.75$

Then,

$P(z < z) = 0.75 \quad , z = 0.67$

$$z = \frac{x - \mu}{\sigma}$$

$$z\sigma = x - \mu$$

$$x = z\sigma + \mu$$

$$= (0.67)(8) + 28$$

$$x = 33.36$$