



UTM
UNIVERSITI TEKNOLOGI MALAYSIA

**SECI2143 PROBABILITY & STATISTICAL
DATA ANALYSIS**

SEMESTER II 2020/2021

ASSIGNMENT 3

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Assignment 3

a)

$$\begin{aligned} \text{Q1 } n &= 25 & \bar{x} \pm z \left(\frac{\sigma}{\sqrt{n}} \right) &= 19.5 \pm (1.645) \left(\frac{9.88}{\sqrt{25}} \right) \\ \bar{x} &= 19.5 & & \\ \sigma &= 9.88 & & \\ CI &= 90\% & &= 19.5 \pm 3.25 \\ z_{\alpha/2} &= 1.645 & &= (16.25, 22.75) \text{ seconds} \end{aligned}$$

b)

$$\begin{aligned} n &= 360 & \text{Standard Error} &= \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} \\ CI &= 99\% & & \\ z_{\alpha/2} &= 2.575 & &= \sqrt{\frac{(0.444)(1-0.444)}{360}} \\ \hat{p} &= \frac{160}{360} & & \\ \hat{p} &= 0.444 & SE &= 0.026 \end{aligned}$$

$$\text{Confidence Interval} = \hat{p} \pm z(SE)$$

$$= 0.444 \pm 2.575(0.026)$$

$$= 0.444 \pm 0.067$$

$$= (0.377, 0.511) \text{ food}$$

stores

offer

promotions

(Q2) a) $H_0: p = 0.10$ $H_1: p < 0.10$ $H_0: p = 0.10$ $H_1: p < 0.10$

$n = 350$

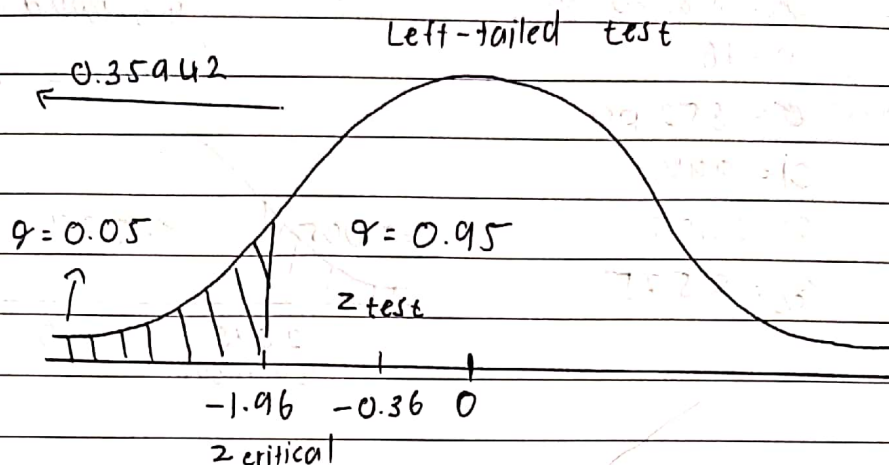
$\alpha = 0.05$

$CI = 0.95$

$2\alpha/2 = 1.96$

$\hat{p} = \frac{33}{350}$

$\hat{p} = 0.0943$



Test statistics

$$z_{\text{test}} = \frac{\hat{p} - p}{\sqrt{\frac{pq}{n}}}$$

$= \frac{0.0943 - 0.10}{\sqrt{\frac{(0.0943)(1 - 0.0943)}{350}}}$

$= -0.36$

$P(Z < -0.36) = 0.35942$

P-value = 0.35942

since P-value $> \alpha$

fail to reject H_0

$z_{\text{test}} = -0.36$

since $z_{\text{test}} > z_{\text{critical}}$,
fail to reject H_0

are no

There is sufficient evidence that company
ABC's incorrect test result is less than 10%

22

$$b) H_0: \mu = 58000 \text{ psi} \quad H_1: \mu \neq 58000 \text{ psi}$$

$$\bar{x} = 58,400 \text{ psi}$$

$$n = 16$$

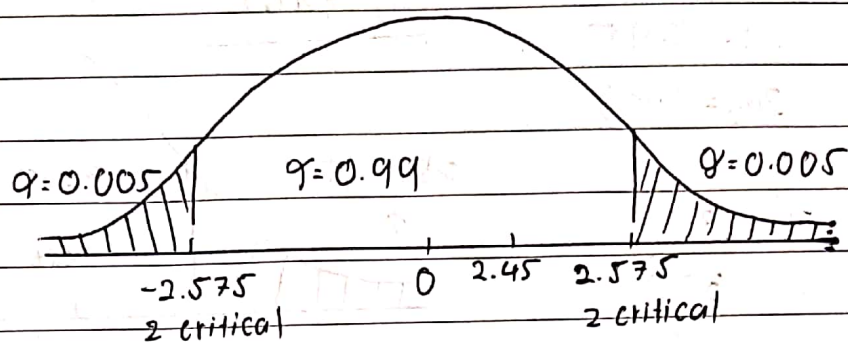
$$\sigma = 652 \text{ psi}$$

$$CI = 99\%$$

$$\alpha = 0.005$$

$$z_{\alpha/2} = 2.575$$

two-tailed test



Test statistics

$$z_{\text{test}} = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$$

$$P(Z < -2.45) = 0.0714$$

$$P(Z < 2.45) = 1 - 0.99286$$

$$= 0.0714$$

$$= \frac{58,400 - 58,000}{\left(\frac{652}{\sqrt{16}} \right)}$$

$$P\text{-value} = 2(0.0714)$$

$$= 0.1428$$

$$z_{\text{test}} = 2.45$$

since $z_{\text{test}} (2.45) < z_{\text{critical}} (2.575)$

fail to reject H_0

since $P\text{-value} > 0.005$

fail to reject H_0

evidence

There are sufficient ~~reason~~ evidence to support the claims that the average compressive strength of steel is 58,000 psi.

3

Q3

$$a) H_0: \mu = 10 \quad H_1: \mu \neq 10$$

$$n = 10$$

$$\alpha = 0.01$$

$$CI = 99\%$$

$$\bar{x} = \frac{10.3 + 9.9 + 10.2 + 10.1 + 9.7 + 9.9 + 9.8 + 10.3 + 10.0 + 10.4}{10}$$

$$\bar{x} = 10.06$$

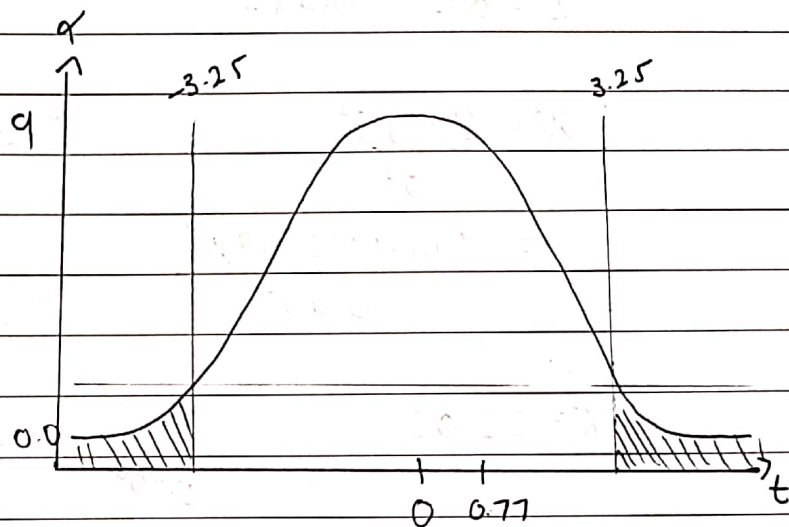
$$s^2 = 3^2 + (-1)^2 + 2^2 + 0.1^2$$

$$s^2 = \frac{(0.3)^2 + (-0.1)^2 + 0.2^2 + 0.1^2 + (-0.3)^2 + (-0.1)^2 + (-0.2)^2 + 0.3^2 + 0^2 + 0.4^2}{10-1}$$

$$s = \sqrt{0.06} \quad df = n-1 = 10-1 = 9$$

$$s = 0.245$$

$$t_{\alpha/2, n-1} = t_{0.005, 9} = 3.25$$



Test statistics

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

$$= \frac{10.06 - 10.00}{0.245 / \sqrt{10}}$$

$$= 0.77$$

since $t < t_{\alpha/2, n-1}$

fail to reject H_0

$$P\text{-value} = P(2 < 3)$$

$$P(2 < -0.77) = 0.22065$$

$$P(2 > 0.77) = 1 - 0.77935 = 0.22065$$

$$P\text{-value} = 2(0.22065) = 0.4413$$

since $P\text{-value} > \alpha$

fail to reject H_0

It is statistically significant to support the claims of the average Brand X of having an average of 101 of car lubricant is true

(4)

Q3) b) $H_0: \sigma = 0.44 \text{ kg}$ $H_1: \sigma \neq 0.44 \text{ kg}$ (two-tailed test)

$n = 14$

$CI = 95\%$

$\alpha = 0.05$

$\bar{x} = 3.70 + 4.31 + 3.73 + 4.33 + 3.33 + 2.58 + 4.47 +$

$3.55 + 4.66 + 3.68 + 3.02 + 4.09 + 2.36 + 3.35$

14

$s = \frac{\sum (x_i - \bar{x})^2}{n-1}$

$\bar{x} = 3.65$

$s^2 = (0.05)^2 + (0.66)^2 + (0.08)^2 + 0.68^2 + (-0.32)^2 + (-1.07)^2 + (0.82)^2 +$

$(-0.1)^2 + 1.01^2 + 0.03^2 + (-0.63)^2 + 0.44^2 + (-1.29)^2 + (-0.3)^2$

$14-1$

$s^2 = 0.4877$

$\chi^2 = \frac{(n-1)s^2}{\sigma^2}$

$= \frac{(14-1)(0.477)}{0.44^2}$

0.44^2

$\chi^2 = 32.03$

$\chi^2_{1-\alpha/2, n-1} = \chi^2_{1-0.025, 14-1}$

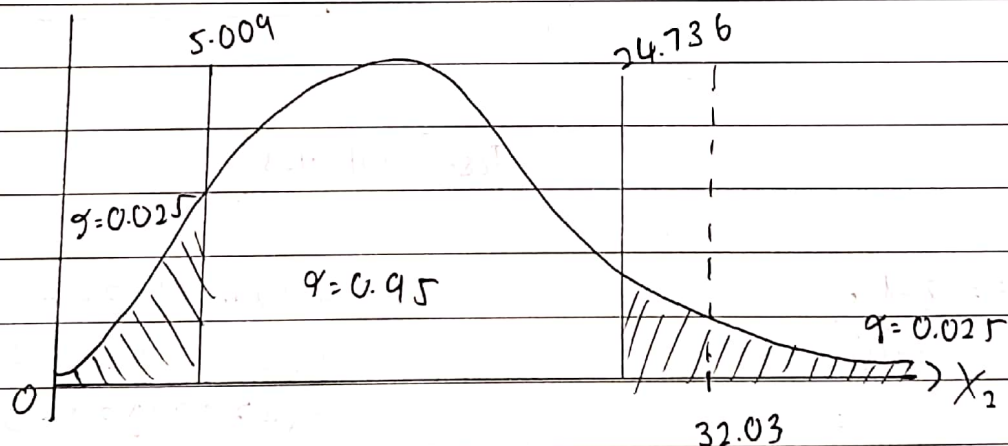
$= \chi^2_{0.975, 13}$

$= 5.009$

$\chi^2_{\alpha/2, n-1} = \chi^2_{0.025, 14-1}$

$= \chi^2_{0.025, 13}$

$= 24.736$



Since $\chi^2 > \chi^2_{\alpha/2, n-1}$

Since $\chi^2 > \chi^2_{0.025, 13}$

reject H_0

It is true that vitamin supplement do change the standard deviation of babies's birth weight, statistically.

5

Q4) $H_0: \sigma^2 = 0.18 \text{ inch}^2$ $H_1: \sigma^2 > 0.18 \text{ inch}^2$ (right-tailed test)

$n = 101$

$\sigma^2 = 0.18$

$s^2 = 0.165$

$\alpha = 0.05$

$$X^2 = \frac{(n-1)s^2}{\sigma^2}$$

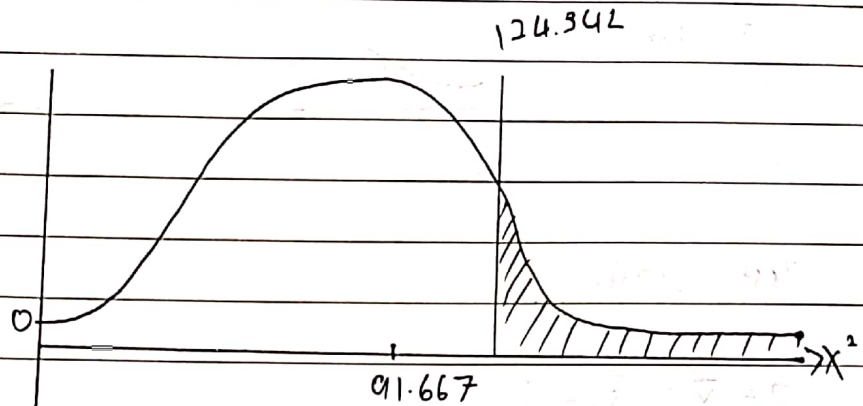
$$= \frac{(101-1)(0.165)}{0.18}$$

$$= \frac{(100)(0.165)}{0.18}$$

$X^2 = 91.667$

$$X^2_{\alpha, n-1} = X^2_{0.05, 100}$$

$$= 124.342$$



Since $X^2 < X^2_{0.05, 100}$

fail to reject H_0

We have significant evidence to accuse ^{that} the new joint inspector is not making satisfactory measurements.

(Q5)

$$H_0: \sigma_1^2 - \sigma_2^2 = 0$$

$$H_0: \mu_1 - \mu_2 = 0$$

$$H_1: \sigma_1^2 - \sigma_2^2 \neq 0$$

$$H_1: \mu_1 - \mu_2 \neq 0 \text{ (two-tailed test)}$$

$$\mu_1 = 76.4$$

$$\bar{x}_1 = 76.4$$

$$\mu_2 = 71.2$$

$$\bar{x}_2 = 71.2$$

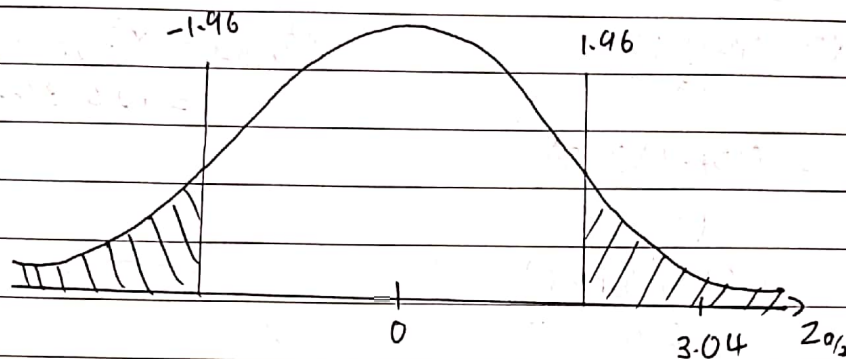
$$\sigma_1^2 = 25.3$$

$$\sigma_2^2 = 22.2$$

$$\alpha = 0.05$$

$$Z_{\alpha/2} = Z_{0.025}$$

$$= 1.96$$



Test Statistic

:

$$Z_0 = \frac{\bar{x}_1 - \bar{x}_2 - \Delta_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$

$$\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$= 76.4 - 71.2 - 0$$

$$\sqrt{\frac{25.3}{15} + \frac{22.2}{18}}$$

$$Z_0 = 3.04$$

Since $Z_0 > Z_{0.025}$

Reject H_0

There is sufficient evidence to conclude that there are difference in the mean spending between the two population

(6)

6) $H_0: \mu_1 = \mu_2$ $H_1: \mu_1 > \mu_2$ $\alpha = 0.05$ $t_{0.05, 35} = 1.645$

$$s_p^2 = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$$

$$= \frac{(16 - 1)(3.6)^2 + (21 - 1)(2.5)^2}{16 + 21 - 2} = 9.13$$

$$s_p = 3.02$$

$$t_0 = \frac{93.6 - 98.5}{(3.02) \sqrt{\frac{1}{16} + \frac{1}{21}}}$$

$$= -4.89$$

$\therefore$ since $t_0 = -4.89 < 1.645$, fail to reject

H_0 . Insufficient evidence to show that standard deviation from batch 1 greater than batch 2

7) $\bar{D} = \frac{(203 - 225) + (370 - 440) + (389 - 402) + (279 - 285) + (355 - 355) + (213 - 240) + (410 - 440) + (364 - 370) + (470 - 501) + (464 - 490)}{10}$

$$= -20.7$$

$$\bar{D}^2 = 484 + 440 + 169 + 36 + 484 + 729 + 1156 + 36 + 961 + 676 = 5131$$

$$s_D = \sqrt{\frac{5131 - \frac{(-20.7)^2}{10}}{9}}$$

$$= 9.70$$

$$b = \frac{-20.7 - 0}{\frac{9.7}{\sqrt{10}}}$$

$$= -6.75$$

$$t_{0.05, 9} = 1.833$$

$\therefore$ since $-6.75 < 1.833$, reject H_0 . Insufficient evidence to show training increases the number of words spelled correctly

Questions 8 (10mark)

X	Y	xy	x ²	y ²	i) $r = \frac{n \sum xy - \sum x \sum y}{\sqrt{[n \sum x^2 - (\sum x)^2][n \sum y^2 - (\sum y)^2]}}$
0.27	2	0.54	0.0729	4	
1.41	3	4.23	1.9881	9	
2.19	3	6.57	4.7961	9	$= \frac{8(56.8) - 14.6(26)}{\sqrt{[8(32.9632) - (14.6)^2][8(104) - (26)^2]}}$
2.83	6	16.98	8.0089	36	
2.19	4	8.76	4.7961	16	
1.81	2	3.62	3.2761	4	$= \frac{454.4 - 379.6}{\sqrt{(50.5456)(156)}} = \frac{74.8}{88.7981}$
0.85	1	0.85	0.7225	1	
2.05	5	10.25	9.3025	25	
$\sum$ 14.6	26	56.8	32.9632	104	$r = 0.8423$

ii) $H_0: \rho = 0$

$H_1: \rho \neq 0$

$$t = \frac{r}{\sqrt{\frac{1-r^2}{n-2}}}$$

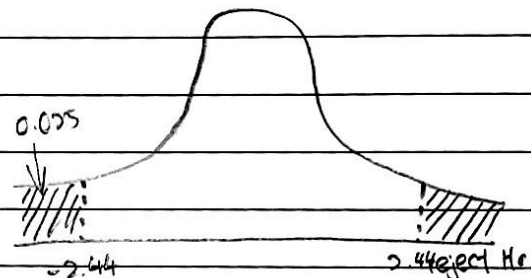
$\alpha = 0.05/2$

$= 0.025 = 2.4469$

$df = 8 - 2$

$$= \frac{0.8423}{\sqrt{\frac{1-0.8423^2}{6}}}$$

$= 3.8278$



$\therefore$ Since $3.8278 > 2.4469$, reject H_0 . There is no sufficient evidence of a linear relationship between weight of plastic usage and size of household.

iii) $\alpha = 0.01/2$

$= 0.005 = 3.707$

$\therefore$ Since $3.8278 > 3.707$, reject H_0 . The result in (ii) does not change as not enough evidence.