UTM

UNIVERSITI TEKNOLOGI MALAYSIA

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SECI2143-01

PROBABILITY & STATISTICAL DATA ANALYSIS

ASSIGNMENT 2

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QUESTION 1

a) $P(C) + P(B) + P(A) = 0.34 + 0.29 + 0.08$

$= 0.71$

b) $P(C) + P(D) + P(E) = 0.34 + 0.17 + 0.12$

$= 0.63$

c) $P(E') + P(A') = P(B) + P(C) + P(D)$

$= 0.29 + 0.34 + 0.17$

$= 0.80$

d) $P(D) + P(A) = 0.17 + 0.08$

$= 0.25$

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QUESTION 2

5

a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

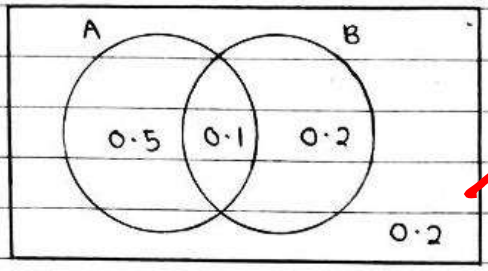
$0.8 = 0.6 + 0.3 - P(A \cap B)$

$P(A \cap B) = 0.6 + 0.3 - 0.8$

$= 0.1$

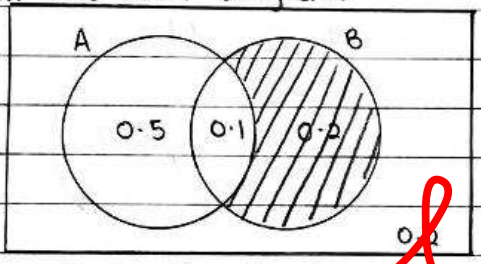
2

b)



3

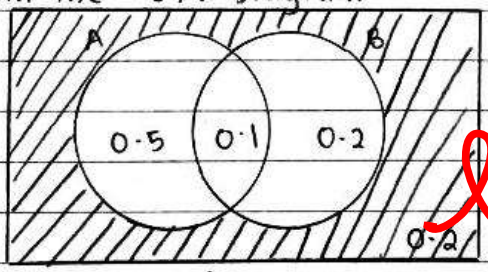
c) From the venn Diagram:



$P(A' \cap B) = 0.2$

2

d) From the venn Diagram:

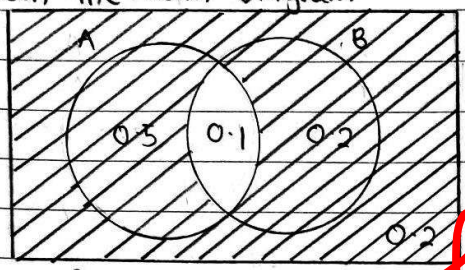


$P(A' \cap B') = 0.2$

2

QUESTION 2

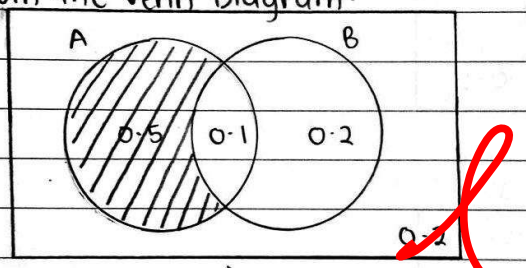
e) From the Venn Diagram



$P(A' \cup B') = 0.5 + 0.2 + 0.2$

$= 0.9$

f) From the venn Diagram:

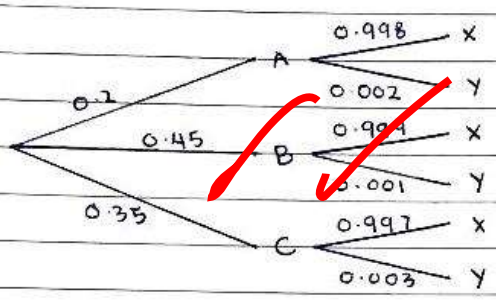


$P(A \cap B') = 0.5$

QUESTION 3

a) A = Package A, B = Package B, C = Package C

X = Good service, Y = Bad service



$P(A \cap X) = 0.2 \times 0.998 = 0.1996$

$P(A \cap Y) = 0.2 \times 0.002 = 0.0004$

$P(B \cap X) = 0.45 \times 0.999 = 0.44955$

$P(B \cap Y) = 0.45 \times 0.001 = 0.00045$

$P(C \cap X) = 0.35 \times 0.997 = 0.34895$

$P(C \cap Y) = 0.35 \times 0.003 = 0.00105$

b) $P(X) = P(A \cap X) + P(B \cap X) + P(C \cap X)$

$= 0.1996 + 0.44955 + 0.34895$

$= 0.9981$

c) $P(B|X) = \dfrac{P(B \cap X)}{P(X)}$

$= \dfrac{0.44955}{0.9981}$

$= 0.4504$

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QUESTION 4

a) Binomial random variable ✓1

b) $b(8; 20, 0{\cdot}7) = \binom{20}{8} (0{\cdot}7)^8 (0{\cdot}3)^{12}$ ✓2

$= 0{\cdot}0039$

c) $P(7 \leq x \leq 10) = b(7; 20, 0{\cdot}7) + b(8; 20, 0{\cdot}7) + b(9; 20, 0{\cdot}7) + b(10; 20, 0{\cdot}7)$

$= \binom{20}{7} (0{\cdot}7)^7 (0{\cdot}3)^{13} + \binom{20}{8} (0{\cdot}7)^8 (0{\cdot}3)^{12} + \binom{20}{9} (0{\cdot}7)^9 (0{\cdot}3)^{11}$

$+ \binom{20}{10} (0{\cdot}7)^{10} (0{\cdot}3)^{10}$

$= 0{\cdot}001018 + 0{\cdot}003859 + 0{\cdot}012207 + 0{\cdot}030817$ ✓8

$= 0{\cdot}0477$

d) $P(3 \leq x \leq 5) = b(3; 20, 0{\cdot}3) + b(4; 20, 0{\cdot}3) + b(5; 20, 0{\cdot}3)$

$= \binom{20}{3} (0{\cdot}3)^3 (0{\cdot}7)^{17} + \binom{20}{4} (0{\cdot}3)^4 (0{\cdot}7)^{16} + \binom{20}{5} (0{\cdot}3)^5 (0{\cdot}7)^{15}$

$= 0{\cdot}0716 + 0{\cdot}1304 + 0{\cdot}1789$ ✓9

$= 0{\cdot}3809$

QUESTION 5

a) $p = 0.7$

$q = 1 - 0.7 = 0.3$

$P(X = x) = b(x; k, p)$

$= \binom{x-1}{k-1} p^k q^{x-k}$

i) $P(X = 3) = b(3; 1, 0.7)$

$= \binom{3-1}{1-1} (0.7)^1 (0.3)^{3-1}$

$= \binom{2}{0} (0.7)^1 (0.3)^2$

$= 0.0630$

ii) $P(X < 4) = P(1 \leq x \leq 3)$

$= \sum b(x; 1, 0.7)$

$= b(1; 1, 0.7) + b(2; 1, 0.7) + b(3; 1, 0.7)$

$= \binom{1-1}{1-1} (0.7)^1 (0.3)^{1-1} + \binom{2-1}{1-1} (0.7)^1 (0.3)^{2-1} + \binom{3-1}{1-1} (0.7)^1 (0.3)^{3-1}$

$= \binom{0}{0} (0.7)^1 (0.3)^0 + \binom{1}{0} (0.7)^1 (0.3)^1 + \binom{2}{0} (0.7)^1 (0.3)^2$

$= 0.7000 + 0.2100 + 0.0630$

$= 0.9730$

iii) $P(X = 5) = b(7; 5, 0.7)$

$= \binom{7-1}{5-1} (0.7)^5 (0.3)^{7-5}$

$= \binom{6}{4} (0.7)^5 (0.3)^2$

$= 0.2269$

QUESTION 5

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b) $n = 5$

$p = 0.7$

$q = 1 - 0.7 = 0.3$

Mean $= np$

$= (5)(0.7)$ ✓

$= 3.5$

Variance $= npq$

$= (5)(0.7)(0.3)$

$= 1.05$ ✓

QUESTION 6

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a) $P(35.8 \leq x \leq 36.2)$

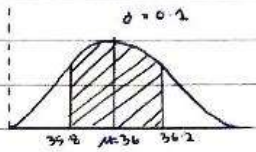
$= P\left(\frac{35.8-36}{0.1} \leq z \leq \frac{36.2-36}{0.1}\right)$ — 2

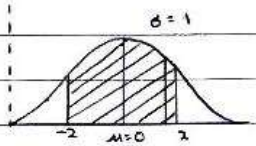
$= P(-2 \leq z \leq 2)$

$= P(z \geq 2) - P(z \geq -2)$ — 2

$= 0.9772 - 0.0228$

$= 0.9544$

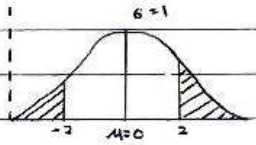




$P(\text{out of control})$

$= 1 - 0.9544$ — 1

$= 0.0456$



b) $x = 0,\ n = 16,\ p = 0.0456,\ q = 0.9544$ — 1/2

$b(x; n, p) = \binom{n}{x} p^x q^{n-x}$

$b(0; 16, 0.0456) = \binom{16}{0} (0.0456)^0 (0.9544)^{16}$ — 2

$= 0.4739$

c) $x = 1,\ n = 16,\ p = 0.0456,\ q = 0.9544$ — 1/2

$b(x; n, p) = \binom{n}{x} p^x q^{n-x}$

$b(1; 16, 0.0456) = \binom{16}{1} (0.0456)^1 (0.9544)^{15}$ — 2

$= 0.3623$

d) $P(35.8 \leq x \leq 36.2)$

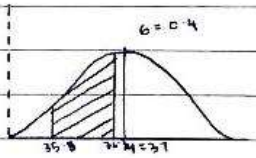
$= P\left(\frac{35.8-37}{0.4} \leq z \leq \frac{36.2-37}{0.4}\right)$ — 2

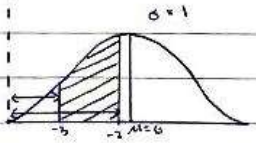
$= P(-3 \leq z \leq -2)$

$= P(z \geq -2) - P(z \geq -3)$ — 2

$= 0.0228 - 0.0013$

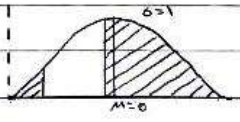
$= 0.0215$





$P(\text{out of control}) = 1 - 0.0215$

$= 0.9785$ — 1



QUESTION 7

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a) $P(20 \leq x \leq 40)$

$z = \frac{x-\mu}{\sigma} = \frac{20-28}{8} = -1$

$z = \frac{x-\mu}{\sigma} = \frac{40-28}{8} = 1.5$

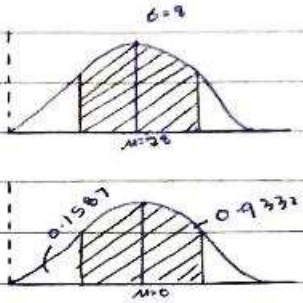
$P(z=-1) = 0.1587$

$P(z=1.5) = 0.9332$

$P(-1 \leq z \leq 1.5) = 0.9332 - 0.1587$

$= 0.7745 = 77.45\%$

6



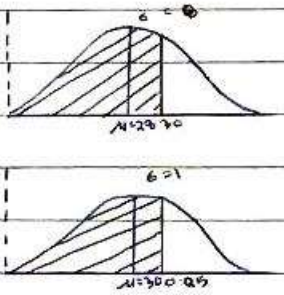
b) $P(X < 30)$

$z = \frac{x-\mu}{\sigma} = \frac{30-28}{8} = 0.25$

$P(z=0.25) = 0.5987$

$= 59.87\%$

4



c) $P(X > x) = 0.75$

$1 - P(X \leq x) = 0.75$

$P(X \leq x) = 0.25$

1

$P(z \leq -0.67) = 0.25$

$\frac{x-\mu}{\sigma} = -0.67$

$\frac{x-28}{8} = -0.67$

2

$x - 28 = -5.36$

$x = 22.64$

$x \approx 23$ days

2