UTM
UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 5

SECI1113 – COMPUTATIONAL MATHEMATICS - SECTION01

(GROUP – T9)

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No. Date

Question 1

Forward – difference Table

k	Year (X_k)	Birth rate (Y_k)	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7	0.8	-6.2	3.7	5.2
1	1985	31.5	-5.4	-2.5	8.9	
2	1995	26.1	-7.9	6.4		
3	2005	18.2	-1.5			
4	2015	16.7				

In Year 1980, $x_k = 1980$, between 1975 and 1985, $x_0 = 1975$, $y_0 = 30.7$.

$h = 1985 - 1975 = 10$

$r = \frac{1980 - 1975}{10} = 0.5$

Polynomial

$$P_4(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$P_4(1980) = 30.7 + (0.5)(0.8) + \left[\frac{0.5(0.5-1)}{2} \times -6.2\right] + \left[\frac{0.5(0.5-1)(0.5-2)}{6} \times 3.7\right] + \left[\frac{0.5(0.5-1)(0.5-2)(0.5-3)}{24} \times 5.2\right]$$

$P_4(1980) = 30.7 + 0.4 + 0.775 + 0.2312 - 0.2031$

$P_4(1980) = 31.9031$

$\therefore y(1980) \approx P_4(1980) = 31.9031$ (4 dp)

Birth rate in 1980 is 31.9031.

No. Date

Question 1

Newton Backward-difference Table

k	Year (x)	Birth Rate (Y)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
0	1975	30.7				
1	1985	31.5	0.8			
2	1995	26.1	-5.4	-6.2		
3	2005	18.2	-7.9	-2.5	3.7	
4	2015	16.7	-1.5	6.4	8.9	5.2

In Year 2010, $X = 2010$, between 2005 and 2015, $x_4 = 2015$, $y_4 = 16.7$

$h = 1985 - 1975 = 10$

$r = \frac{2010-2015}{10} = -0.5$

Polynomial

$$P_4(x) = y_4 + r\nabla y_4 + \frac{r(r+1)}{2!}\nabla^2 y_4 + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_4 + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_4$$

$$P_4(2010) = 16.7 + (-0.5)(-1.5) + \left[\frac{-0.5(-0.5+1)}{2} \times 6.4\right] + \left[\frac{-0.5(-0.5+1)(-0.5+2)}{6} \times 8.9\right] + \left[\frac{-0.5(-0.5+1)(-0.5+2)(-0.5+3)}{24} \times 5.2\right]$$

$P_4(2010) = 16.7 + 0.75 - 0.8 - 0.5562 - 0.2031$

$P_4(2010) = 15.8906$

$\therefore y(2010) \approx P_4(2010) = 15.8906$ (4 dp)

Birth rate in Year 2010 is 15.8906.

Question 2

a) $p(x) = a_0 + a_1 x$

System of linear equations

$$\begin{bmatrix} S_0 & S_1 \\ S_1 & S_2 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} V_0 \\ V_1 \end{bmatrix}$$

where

$$S_j = \sum_{k=0}^{4} x_k^j \quad , j = 0, 1, 2$$

$$V_l = \sum_{k=0}^{4} x_k^l f_k \quad , l = 0, 1$$

Calculation table

	x_k^0	x_k^1	x_k^2	f_k	$x_k^0 f_k$	$x_k^1 f_k$
	1	1970	3880900	6	6	11820
	1	1975	3900625	8.6	8.6	16985
	1	1980	3920400	13.6	13.6	26928
	1	1985	3940225	18.7	18.7	37119.5
	1	1990	3960100	31.6	31.6	62884
	1	1995	3980025	53.2	53.2	106134
	1	2000	4000000	78.8	78.8	157600
	1	2005	4020025	100	100	200500
	1	2010	4040100	107.1	107.1	215271
$\sum_{k=0}^{4}$	9	17910	35642400	-	417.6	835241.5

In matrix form:

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

No. Date

Question 2

a) $\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 17910 & 35642400 & 835241.5 \end{array}\right]$

$R_2 = R_2 - (1990)R_1$

$\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 0 & 1500 & 4217.5 \end{array}\right]$

$1500a_1 = 4217.5$

$a_1 = 2.8117$

$9a_0 + 17910a_1 = 417.6$

$9a_0 + 17910(2.8117) = 417.6$

$9a_0 = 417.6 - 50357.547$

$9a_0 = -49939.947$

$a_0 = -5548.883$

$a_0 = -5548.883$

$a_1 = 2.8117$

∴ ~~The P(x)~~ Polynomial expression, $p(x) = 2.8117x - 5548.883$ ✱

b) In Year 1979, $x = 1979$

$p(1979) = 2.8117(1979) - 5548.883$

$= 15.4713$

$f(1979) \approx p(1979) = 15.4713$

∴ Total index in 1979 is 15.4713. ✱

No.

Date

Question 3

$$y(3.0) = P_4(3.0) = L_0(3.0)y_0 + L_1(3.0)y_1 + L_2(3.0)y_2 + L_3(3.0)y_3 + L_4(3.0)y_4$$

$$L_0(3.0) = \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)}$$
$$= \frac{(-3.5)}{(43.75)} = -0.08$$

$$L_1(3.0) = \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)}$$
$$= \frac{(-10.5)}{(-15)} = 0.70$$

$$L_2(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)}$$
$$= \frac{(5.25)}{(9.375)} = 0.56$$

$$L_3(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)}$$
$$= \frac{(2.625)}{(-13.125)} = -0.20$$

$$L_4(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)}$$
$$= \frac{1.5}{75} = 0.02$$

$$P_4(3.0) = (-0.08)(0.455) + (0.70)(1.615) + (0.56)(3.760) + (-0.20)(5.333) + (0.02)(7.857)$$
$$= 2.28464 \approx 2.2846 \text{ ✱}$$

No.

Date

Question 4

a) $f(x) = 2x^3 + x^2 - 4$, $h = 0.5$

For Central three-point :

$$f'(2.5) = \frac{f(2.5+0.5) - f(2.5-0.5)}{2(0.5)}$$

$$= \frac{f(3) - f(16)}{1}$$

$$= 59 - 16 = 43$$

$f(3) = 2(3)^3 + (3)^2 - 4$

$= 59$

$f(2) = 2(2)^3 + (2)^2 - 4$

$= 16$

$f(2.5) = 2(2.5)^3 + (2.5)^2 - 4$

$= 33.5$

$$f''(2.5) = \frac{f(2.5-0.5) - 2f(2.5) + f(2.5+0.5)}{(0.5)^2}$$

$$= \frac{f(2) - 2f(2.5) + f(3)}{(0.25)}$$

$$= \frac{16 - 2(33.5) + 59}{0.25}$$

$$= \frac{8}{0.25} = 32$$

No.

Date

For Central five-point:

$f(1.5) = 5$

$f(2) = 16$

$f(3) = 59$

$f(3.5) = 94$

$f(2.5) = 33.5$

$$f'(2.5) = \frac{1}{12(0.5)}\left[f(2.5-2(0.5) - 8f(2.5-0.5) + 8f(2.5+0.5) - f(2.5+2(0.5)\right]$$

$$= \frac{1}{6}\left[f(1.5) - 8f(2) + 8f(3) - f(3.5)\right]$$

$$= \frac{1}{6}\left[5 - 8(16) + 8(59) - 94\right]$$

$$= \frac{1}{6}(255) = 42.5$$

$$f''(2.5) = \frac{1}{12(0.5)^2}\left[-f(2.5-2(0.5) + 16f(2.5-0.5) - 30f(2.5) + 16f(2.5+0.5) - f(2.5+2(0.5)\right]$$

$$= \frac{1}{3}\left[-f(1.5) + 16f(2) - 30f(2.5) + 16f(3) - f(3.5)\right]$$

$$= \frac{1}{3}\left[-(5) + 16(16) - 30(33.5) + 16(59) - 94\right]$$

$$= \frac{1}{3}(96) = 32$$

No.

Date

b) $f(x) = 2x^3 + x^2 - 4$

$f'(x) = 6x^2 + 2x$

$f''(x) = 12x + 2$

$f'(2.5) = 6(2.5)^2 + 2(2.5)$

$= 42.5$

$f''(2.5) = 12(2.5) + 2$

$= 32$

c) For Central three-point :

First derivative :

Estimated $f'(2.5)$ value = 43

Exact $f'(2.5)$ value = 42.5

error = True value - approximated value

$= 42.5 - 43$

$= -0.5$

Second derivative :

Estimated $f''(2.5)$ value = 32

Exact $f''(2.5)$ value = 32

error = $32 - 32$

$= 0$

No. _____

Date _____

For central five-point :

First derivative :

Estimated $f'(2.5)$ value = 42.5

Exact $f'(2.5)$ value = 42.5

error = 42.5 - 42.5

= 0

Second derivative :

Estimated $f''(2.5)$ value = 32

Exact $f''(2.5)$ value = 32

Error = 32 - 32

= 0

No : .....

Subject : ..... Date : . .

QUESTION 5

Approximate $\int_0^3 \frac{2}{1+5x^2}$

(a) $N = 6$

$h = \frac{3-0}{6}$

$= 0.5$

i	x_i	$f(x_i) = \frac{2}{1+5x^2}$	
0	0.0	2.0	
1	0.5		0.8889
2	1.0		0.3333
3	1.5		0.1633
4	2.0		0.0952
5	2.5		0.0620
6	3.0	0.0435	
Total		2.0435	1.5427

$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{0.5}{2}\left[2.0435 + 2(1.5427)\right]$

$= 1.282225$

Subject : .....

No : .....

Date : .....

b) $h = 0.5$

i	x_i	$f(x_i) = \frac{2}{1.5x^2}$		
0	0.0	2.0		
1	0.5		0.8889	
2	1.0			0.3333
3	1.5		0.1623	
4	2.0			0.0952
5	2.5		0.0620	
6	3.0	0.0435		
Total		2.0435	1.1142	0.4285

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{0.5}{3}\left[2.0435 + 4(1.1142) + 2(0.4285)\right]$$

$$= 1.2262$$

No : .....

Subject : ..... Date : . .

QUESTION 5

c) $h = 0.25$

$b = 3$

i	x_i	$f(x_i) = \frac{2}{1+5x^3}$		
0	0.0	2.0		
1	0.25		1.5238	
2	0.50		0.8889	
3	0.75			0.5246
4	1.0		0.3333	
5	1.25		0.2270	
6	1.50			0.1633
7	1.75		0.1226	
8	2.0		0.0952	
9	2.25			0.0760
10	2.5		0.0620	
11	2.75		0.0515	
12	3.0	0.0435		
Total		2.0435	3.3043	0.7639

$$\int_0^3 \frac{2}{1.5x^3}\,dx = \frac{3(0.25)}{8}\left[2.0435 + 3(3.3043) + 2(0.7639)\right]$$

$$= 1.2641$$

No : .....

Subject : ..... Date :

QUESTION 5

d) $h_1 = 3$ $\quad h_2 = \frac{3}{2} = 1.5$ $\quad h_3 = \frac{1.5}{2} = 0.75$

$x_1 = 0 + 1.5 = 1.5$

$x_1 = 0 + 0.75 = 0.75$

$x_2 = 0.75 + 0.75 = 1.5$

$x_3 = 1.5 + 0.75 = 2.25$

$$R_{1,1} = \frac{3}{2}\left[\frac{2}{1+5(0)^2} + \frac{2}{1+5(3)^2}\right]$$

$$= 3.0652$$

$$R_{2,1} = \frac{1}{2}\left[3.0652 + 3f(1.5)\right]$$

$$= \frac{1}{2}\left[3.0652 + 3(0.1633)\right]$$

$$= 1.7776$$

$$R_{3,2} = \frac{4(1.7776) - 3.0652}{3}$$

$$= 1.3484$$

Subject : .....

No : .....

Date : .....

QUESTION 5

a) $R_{3,1} = \frac{1}{2}\left[R_{2,1} + 1.5\,(R_1 + f_3)\right]$

$= \frac{1}{2}\left[R_{2,1} + 1.5\,(f(0.75) + (2.25)\right]$

$= \frac{1}{2}\left[1.7776 + 1.5\,(0.5246 + 0.0760)\right]$

$= 1.3393$

$R_{3,2} = \frac{4(1.3393) - 1.7776}{3}$

$= 1.3392$

$R_{3,3} = \frac{16(1.1932) - 1.3484}{15}$

$= 1.1829$

i	hi	Ri,1	Ri,2	Ri,3
1	3	3.0652		
2	1.5	1.7776	1.3424	
3	0.75	1.3393	1.1932	1.1829