Assignment 4
ERROR AND NONLINEAR SOLUTIONS

Question 1

a) If a speedometer's absolute error is 2 mph and it measures a speed of 60 mph, what is the relative error of this measurement expressed as a percentage?

b) Find the absolute error for $f'(2)$ for $f(x) = x$, given that

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \text{ and } h = 0.1.$$

c) Find the most accurate value for the following operation where each number has been rounded to a certain decimal point:

6.232 + 7.0321 + 5.43 – 3.385

Question 2

Find a root of the equation

$$x^6 - x - 1 = 0 \qquad \text{accurate to within } \varepsilon = 0.001.$$

In [1,2] using bisection method.

Question 3

For a non-linear function, $f(x) = \cos(x) + 2\sin(x) + x^2$, find the root using the following method:

a. Secant method, $x_0 = 0$; $x_1 = -0.1$
b. Newton method, $x_0 = -0.1$

Then, give some comments by comparing your results between these two methods. Do calculation in 4 decimal points and approximate the root in 2 decimal points.

Question 4

In his final maths exam, Ahmad needs to solve the equation

$$1/x = 1.37$$

However, his calculator has a problem where it only can add, subtract and multiply. Help Ahmad to solve this equation using the Newton method and the defective calculator.

i) Prove that Newton formula can be writen as

$X_{n+1} = x_n(2 - \alpha x_n)$, if $\alpha = 1.37$

ii) Then used formula in (i) to find x correct to 8 decimal places if $x_0 = 0.75$

Question 5

Use characteristic polynomial to find the eigen value and eigen vector for A

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

Question 6

1. Matrix ***B*** is given as follows,

$$B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$$

a) Use the Gerschgorin's Circle Theorem to determine a region containing all the eigenvalues of ***B***.
b) Find the dominant eigenvalue (λ_1) and the corresponding eigenvector of matrix ***B*** using Power method. Use $\boldsymbol{v}^{(0)} = [0, 0, 1]^T$.
c) Find the smallest eigenvalue (λ_3) and the corresponding eigenvector of matrix ***B*** using Shifted Power method. Use $\boldsymbol{v}^{(0)} = [0, 0, 1]^T$.
d) Find the intermediate eigenvalue (λ_2) of matrix ***B.***

Note: Do calculation in 4 decimal points with $\varepsilon = 0.05$. Round up the dominant eigenvalue (λ_1) in (b) and the smallest eigenvalue (λ_3) in (c) to the nearest integer.