ASSIGNMENT 4

SECI1113 – COMPUTATIONAL MATHEMATICS - SECTION01

(GROUP – T9)

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Question 1

a) $|\varepsilon| = 2$ mph

$\alpha = 60$ mph

$|\varepsilon| = |\alpha - \bar{\alpha}|$

$|2| = |60 - \bar{\alpha}|$

$\pm 2 = 60 - \bar{\alpha}$

$-58 / -62 = -\bar{\alpha}$

$\bar{\alpha} = 58/62$

For $\bar{\alpha} = 58$

$\varepsilon_r = \left|\frac{\alpha - \bar{\alpha}}{\alpha}\right|$

$= \left|\frac{60-58}{60}\right|$

$= \frac{2}{60}$

$= 0.0333$

For $\bar{\alpha} = 62$

$\varepsilon_r = \left|\frac{60-62}{60}\right|$

$= \frac{2}{60}$

$= 0.0333$

$\varepsilon_r = 0.0333 \times 100\%$

$= 3.33\%$

∴ The relative error is 3.33 %.

No. Date

Question 1

b)

$$f'(x) \approx \frac{f(x+h) - f(x)}{h} \quad ; h = 0.1$$

$|\epsilon|$ $f'(2) = ?$ $\quad f(x) = x$

$f(x) = x$

$f(2) = 2$

$f(h) = h$

$f(0.1) = 0.1$

$f(x+h) = f(x) + f(h)$

$= 2 + 0.1$

$= 2.1$

$$f'(2) = \frac{f(x+h) - f(x)}{h}$$

$$= \cancel{f(2) + f(h) - f(2)}$$

$$= \frac{f(2) + f(0.1) - f(2)}{0.1}$$

$$= \frac{2.1 - 2}{0.1}$$

$$= 1$$

$$|\epsilon| = |f(2) - f'(2)|$$

$$= |2 - 1|$$

$$|\epsilon| = 1$$

$\therefore$ Absolute error for $f'(2)$ for $f(x) = x$ is 1.

No. Date

Question 1

c.) $6.232 + 7.0321 + 5.43 - 3.385$

Exact calculation gives result as follow :

$6.232 + 7.0321 + 5.43 - 3.385 = 15.3091$

Modulus error is given by,

$|\varepsilon_3| \leqslant 0.0005 + 0.00005 + 0.005 + 0.0005 = 0.00605$

Therefore, the result is between

$15.3091 + 0.00605$ and $15.3091 - 0.00605$

15.31515 and 15.30305

Both of these values are same if rounded to 1 ~~dem~~ decimal point.

Therefore, the best ~~appro~~ ~~appo~~ approximate value is 15.3 (1 demical point).

Question 2

$x^6 - x - 1 = 0$

$\varepsilon = 0.001$

Accuracy 3 demical points, $\varepsilon = 0.001 \times \frac{1}{2} = 0.0005$

So, do calculations in 4 ~~dem~~ decimal points.

Let $f(x) = x^6 - x - 1$

$(a,b) = [1,2]$

$f(1) = (1)^6 - 1 - 1 = -1 \ (<0)$

$f(2) = (2)^6 - 2 - 1 = 61 \ (>0)$

$c = \frac{a+b}{2}$

$c_1 = \frac{1+2}{2} = 1.5$

$\cancel{f(x_0)}$ $f(c_1) = f(1.5) = 1.5^6 - 1.5 - 1 = 8.8906 \ (>0)$

$c_2 = \frac{1+1.5}{2} = 1.25$

$f(c_2) = f(1.25) = 1.25^6 - 1.25 - 1 = 1.5647 \ (>0)$

$c_3 = \frac{1+1.25}{2} = 1.125$

$f(c_3) = f(1.125) = 1.125^6 - 1.125 - 1 = -0.0977 \ (<0)$

$c_4 = \frac{1.125+1.25}{2} = 1.1875$

$f(c_4) = f(1.1875) = 1.1875^6 - 1.1875 - 1 = 0.6167 \ (>0)$

$c_5 = \frac{1.125+1.1875}{2} = 1.1563$

$f(c_5) = f(1.1563) = 1.1563^6 - 1.1563 - 1 = 0.2338 \ (>0)$

$\cancel{c_6}$ $c_6 = \frac{1.125+1.1563}{2} = 1.1407$

$f(c_6) = f(1.1407) = 1.1407^6 - 1.1407 - 1 = 0.0624 \ (>0)$

$c_7 = \frac{1.125+1.1407}{2} = 1.1329$

$f(c_7) = f(1.1329) = 1.1329^6 - 1.1329 - 1 = \cancel{0} -0.0187 \ (<0)$

$$c_8 = \frac{1.1329 + 1.1407}{2} = 1.1368$$

$$f(c_8) = f(1.1368) = 1.1368^6 - 1.1368 - 1 = 0.0215 \quad (>0)$$

$$c_9 = \frac{1.1329 + 1.1368}{2} = 1.1349$$

$$f(c_9) = f(1.1349) = 1.1349^6 - 1.1349 - 1 = 0.0018 \quad (>0)$$

$$c_{10} = \frac{1.1329 + 1.1349}{2} = 1.1339$$

$$f(c_{10}) = f(1.1339) = 1.1339^6 - 1.1339 - 1 = -0.0085 \quad (<0)$$

$$\cancel{c_{11} = \frac{1.1339 + 1.1349}{2} = 1.1344}$$

$$\cancel{f(c_{11}) = f(1.1344) = 1.1344^6 - 1.1344 - 1 \approx -0.0136} \quad (<0)$$

$$\cancel{c_{12} = 1.1334 + 1.1349}$$

$$\cancel{c_{12}} = c_{11} = \frac{1.1339 + 1.1349}{2} = 1.1344$$

$$f(c_{11}) = f(1.1344) = 1.1344^6 - 1.1344 - 1 = -0.0033 \quad (<0)$$

$$c_{12} = \frac{1.1344 + 1.1349}{2} = 1.1347$$

$$f(c_{12}) = f(1.1347) = 1.1347^6 - 1.1347 - 1 = -0.0002 \quad (<0)$$

~~In the~~ The $f(c_{12}) = 0.0002 < 0.0005$.

The root of $f(x)$ in the interval $[1,2]$ is $1.1347 \approx 1.135$

(approximate to 3 decimal places).

No. Date

i	a	b	f(a)	f(b)	c	f(c)
1	1	2	-1	61	1.5	8.8906
2	~~b~~ 1	1.5	-1	8.8906	1.25	1.5647 ~~1.15~~
3	1	1.25	-1	1.5647 ~~1.15~~	1.125	-0.0977
4	1.125	1.25	-0.0977	1.5647	1.1875	0.6167
5	1.125	1.1875	-0.0977	0.6167	1.1563	0.2338
6	1.125	1.1563	-0.0977	0.2338	1.1407	0.0624
7	1.125	1.1407	-0.0977	0.0624	1.1329	-0.0187
8	1.1329	1.1407	-0.0187	0.0624	1.1368	0.0215
9	1.1329	1.1368	-0.0187	0.0215	1.1349	~~0.02~~ 0.0018
10	1.1329	1.1349	-0.0187	0.0018	1.1339	-0.0085
11	1.1339	1.1349	-0.0085	0.0018	1.1344	-0.0033
12	1.1344	1.1349	-0.0033	0.0018	1.1347	-0.0002

No.

Date

Question 3

Secant Method

$f(x) = \cos(x) + 2\sin(x) + x^2$, $\varepsilon = 0.0005$, $x_0 = 0$, $x_1 = -0.1$

$f(x_0) = f(0) = \cos(0) + 2\sin(0) + (0)^2$
$= 1$

$f(x_1) = f(-0.1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$
$= 0.8053$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} = \frac{(0)(0.8053) - (-0.1)(1)}{(0.8053) - (1)}$$
$$= -0.5136$$

$f(x_2) = f(-0.5136) = \cos(-0.5136) + 2\sin(-0.5136) + (-0.5136)^2$
$= 0.1521$

$$x_3 = \frac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \frac{(-0.1)(0.1521) - (-0.5136)(0.8053)}{(0.1521) - (0.8053)}$$
$$= -0.6099$$

$f(x_3) = f(-0.6099) = \cos(-0.6099) + 2\sin(-0.6099) + (-0.6099)^2$
$= 0.0461$

No.

Date

$$x_4 = \frac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \frac{(-0.5136)(0.0461) - (-0.6099)(0.1521)}{(0.0461) - (0.1521)}$$

$$= -0.6518$$

$$f(x_4) = f(-0.6518) = \cos(-0.6518) + 2\sin(-0.6518) + (-0.6518)^2$$

$$= 0.0066$$

$$x_5 = \frac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)} = \frac{(-0.6099)(0.0066) - (-0.6518)(0.0461)}{(0.0066) - (0.0461)}$$

$$= -0.6588$$

$$f(x_5) = f(-0.6588) = \cos(-0.6588) + 2\sin(-0.6588) + (-0.6588)^2$$

$$= 0.0004$$

i	x_{i-1}	x_i	x_{i+1}	$f(x_{i+1})$
1	0	-0.1	-0.5136	0.1521
2	-0.1	-0.5136	-0.6099	0.0461
3	-0.5136	-0.6099	-0.6518	0.0066
4	-0.6099	-0.6518	-0.6588	0.0004

$|f(x_5)| = 0.0004 < 0.0005$, $x = x_5 = -0.6588 = -0.66$

$\therefore x = -0.66$

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Date

Question 3

Newton's Method

$f(x) = \cos(x) + 2\sin(x) + x^2$, $\epsilon = 0.0005$, $x_0 = -0.1$

$f'(x) = -\sin(x) + 2\cos(x) + 2x$

$f(x_0) = f(-0.1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$
$= 0.8053$

$f'(x_0) = f'(-0.1) = -\sin(-0.1) + 2\cos(-0.1) + 2(-0.1)$
$= 1.8898$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = (-0.1) - \frac{(0.8053)}{(1.8898)}$$
$$= -0.5261$$

$f(x_1) = f(-0.5261) = \cos(-0.5261) + 2\sin(-0.5261) + (-0.5261)^2$
$= 0.1372$

$f'(x_1) = f'(-0.5261) = -\sin(-0.5261) + 2\cos(-0.5261) + 2(-0.5261)$
$= 1.1795$

Date

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = (-0.3261) - \frac{(0.1372)}{(1.1795)}$$

$$= -0.6424$$

$$f(x_2) = f(-0.6424) = \cos(-0.6424) + 2\sin(-0.6424) + (-0.6424)^2$$

$$= 0.0151$$

$$f'(x_2) = f'(-0.6424) = -\sin(-0.6424) + 2\cos(-0.6424) + 2(-0.6424)$$

$$= 0.9156$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)} = (-0.6424) - \frac{(0.0151)}{(0.9156)}$$

$$= -0.6589$$

$$f(x_3) = f(-0.6589) = \cos(-0.6589) + 2\sin(-0.6589) + (-0.6589)^2$$

$$= 0.0003$$

No.

Date

i	x_i	$f(x_i)$	$f'(x_i)$
0	-0.1	0.8053	1.8898
1	-0.5261	0.1372	1.1795
2	-0.6424	0.0151	0.9156
3	-0.6589	0.0003	

$|f(x_3)| = 0.0003 < 0.0005$, $x = x_3 = -0.6589 = -0.66$

$\therefore x = -0.66$

The number of iterations for Newton's method is less than the number of iterations for Secant method. Therefore, we can conclude that Newton's method finds the root faster than Secant method.

Question 4

i) $\frac{1}{x} = 1.37$

$\frac{1}{x} - 1.37 = 0$

$f(x) = \frac{1}{x} - 1.37$

$= \frac{1}{x} - \alpha$

$f'(x) = -\frac{1}{x^2}$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$= x_n - \frac{\left(\frac{1}{x_n} - \alpha\right)}{\left(-\frac{1}{x_n^2}\right)}$$

$$= x_n - \boxed{\left(\frac{1 - \alpha x_n}{\cancel{x_n}}\right) \times \left(-x_n^{\cancel{2}}\right)}$$

$$= x_n - \boxed{(1 - \alpha x_n)(-x_n)}$$

$$= x_n - \left(-x_n + \alpha x_n^2\right)$$

$$= x_n + x_n - \alpha x_n^2$$

$$= 2x_n - \alpha x_n^2$$

$$= x_n(2 - \alpha x_n)$$ ※

No.

Date

Question 4

ii) $f(x) = \frac{1}{x} - 1.37$, $\varepsilon = 0.000000005$, $x_0 = 0.75$

$$f(x_0) = f(0.75) = \frac{1}{0.75} - 1.37$$

$$= -0.036666667$$

$$x_1 = (x_0)(2 - \alpha x_0) = (0.75)(2 - 1.37(0.75))$$

$$= 0.729375$$

$$f(x_1) = f(0.729375) = \frac{1}{0.729375} - 1.37$$

$$= 0.001036847$$

$$x_2 = (x_1)(2 - \alpha x_1) = (0.729375)(2 - 1.37(0.729375))$$

$$= 0.729926590$$

$$f(x_2) = f(0.729926590) = \frac{1}{0.729926590} - 1.37$$

$$= 0.000000783$$

No.

Date

$$x_3 = (x_2)(2-\alpha x_2) = (0.729926590)(2-1.37(0.729926590))$$
$$= 0.729927007$$

$$f(x_3) = \frac{1}{0.729927007} - 1.37$$
$$= 0.000000001$$

i	x_i	$f(x_i)$
0	0.75	-0.036666667
1	0.729375	0.001036847
2	0.729926590	0.000000783
3	0.729927007	0.000000001

$|f(x_3)| = 0.000000001 < 0.000000005$, $x = x_3 = 0.729927007$
$= 0.72992701$

$\therefore x = 0.72992701$

Question 5

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\det(S - \lambda I) = \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$$= \left| \begin{matrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{matrix} \right| \begin{matrix} 1-\lambda & -1 \\ -1 & 2-\lambda \\ 0 & -1 \end{matrix}$$

$$= [(1-\lambda)(2-\lambda)(1-\lambda)] + [(-1)(-1)(0)] + [(0)(-1)(-1)] - [(-1)(-1)(1-\lambda)] - [(1-\lambda)(-1)(-1)] - [(0)(2-\lambda)(0)]$$

$$= (-\lambda^3 + 4\lambda^2 - 5\lambda + 2) + 0 - (1-\lambda) - (1-\lambda) - 0$$

$$= -\lambda^3 + 4\lambda^2 - 5\lambda + 2 - 1 + \lambda - 1 + \lambda$$

$$= -\lambda^3 + 4\lambda^2 - 3\lambda$$

- Eigen values : $\lambda_1 = 3$, $\lambda_2 = 1$, $\lambda_3 = 0$

Question 5

$\lambda = 1$

$(S - \lambda I)x = 0$

$$\begin{bmatrix} 1-1 & -1 & 0 \\ -1 & 2-1 & -1 \\ 0 & -1 & 1-1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} -1 & 1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] -R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] -R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_1 \to R_1 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_3 \to R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = -t$

$y = 0$

$z = t$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -t \\ 0 \\ t \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t$$

- Eigenvector $(\lambda = 1) = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

QUESTION 5

$\lambda = 3$

$(S - \lambda I)x = 0$

$$\begin{bmatrix} 1-3 & -1 & 0 \\ -1 & 2-3 & -1 \\ 0 & -1 & 1-3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -2 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_1 \to R_1 - R_2 \quad \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_2 \to R_2 - R_1$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_3 \to R_3 - R_2 \quad \left[\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \begin{array}{l} -R_1 \\ -R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$x = t$
$y = -2t$
$z = t$

$$\begin{bmatrix} t \\ -2t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix} t$$

* Eigenvector $(\lambda = 3) = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

Question 5

$\lambda = 0$

$(S - \lambda I)x = 0$

$$\begin{bmatrix} 1-0 & -1 & 0 \\ -1 & 2-0 & -1 \\ 0 & -1 & 1-0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] R_2 \to R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] R_3 \to R_3 + R_2$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \begin{aligned} x &= t \\ y &= t \\ z &= t \end{aligned}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} t \\ t \\ t \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} t$$

- Eigenvector $(\lambda = 0) = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

Question 6

$$B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$$

a)

$R_1 : |\lambda - (-18)| < |20| + |-6|$

$= |\lambda + 18| < 26$

$= -44 < \lambda < 8$

$= \lambda \in (-44, 8)$

$R_2 = |\lambda - 14| < |(-12)| + |(-3)|$

$= |\lambda - 14| < 15$

$= -1 < \lambda < 29$

$= \lambda \in (-1, 29)$

$R_3 = |\lambda - 13| < |30| + |(-30)|$

$= |\lambda - 13| < 60$

$= -47 < \lambda < 73$

$= \lambda \in (-47, 73)$

∴ Eigenvalues $B = \lambda \in (-47, 73)$

Subject : .... No : ....

Date :

Question 6

b)

$$v = Bv^{(1)}$$

$$= \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} \qquad m_1 = 13 \qquad v^{(1)} = \frac{v}{m_1}$$

$$= \frac{1}{13} \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix}$$

$$v^{(1)} = \begin{bmatrix} -\frac{6}{13} \\ -\frac{3}{13} \\ 1 \end{bmatrix} = \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1 \end{bmatrix}$$

$$|v^{(1)} - v^{(0)}| > 0.05$$

$$v = Bv^{(1)}$$

$$= \begin{bmatrix} -18 & 20 & -6 \\ -20 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} -2.3090 \\ -0.6932 \\ 6.0790 \end{bmatrix} \qquad m_2 = 6.0790$$

$$v^{(2)} = \frac{v}{m_2}$$

$$= \frac{1}{6.079} \begin{bmatrix} -2.3090 \\ -0.6932 \\ 6.079 \end{bmatrix}$$

$$v^{(2)} = \begin{bmatrix} -0.3798 \\ -0.1140 \\ 1 \end{bmatrix}$$

$$|v^{(2)} - v^{(1)}| > 0.05$$

Subject : .... No : .... Date :

Question 6

b)

k	$(v^{(k)})^T$			$(Bv^{(k)})^T$			m_{k+1}
0	0	0	1	-6	-3	13	13
1	-0.4615	-0.2308	1	-2.3090	2.9988	6.0790	6.0790
2	-0.3798	-0.1140	1	-1.4436	-0.0384	5.0260	5.0260
3	-0.2872	-0.0076	1	-0.9824	0.8400	4.6120	4.6120
4	-0.2130	0.0737	1	-0.8920	0.5878	4.3990	4.3990
5	-0.1573	0.1336	1	-0.4966	0.7580	4.2730	4.2730
6	-0.1162	0.1774	1				

$$|v^{(6)} - v^{(5)}| < 0.05$$

$$\lambda_1 \approx m_6 = 4.2730$$

$$v_1 = v^{(6)} = \begin{bmatrix} -0.1162 \\ 0.1774 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 44 & -3 \\ 30 & -20 & 13 \end{bmatrix} \begin{bmatrix} -0.1162 \\ 0.1774 \\ 1 \end{bmatrix} = 4 \begin{bmatrix} -0.1162 \\ 0.1774 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -0.3604 \\ 0.8780 \\ 4.1920 \end{bmatrix} = \begin{bmatrix} -0.4648 \\ 0.7096 \\ 4 \end{bmatrix}$$

Subject : .... No : .... Date :

Question 6

c) $A = B$ shifted

$A = B - \lambda_1 I = B - 4I$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}$$

$$v^{(0)} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \qquad \varepsilon = 0.05$$

$$v = Av^{(0)}$$

$$= \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$v = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \qquad m_1 = 9$$

$$v^{(1)} = \frac{v}{m_1}$$

$$= \frac{1}{9}\begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} = \begin{bmatrix} -0.6667 \\ -0.3333 \\ 1 \end{bmatrix}$$

$$v = Av^{(1)}$$

$$= \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} -0.6667 \\ -0.3333 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.0014 \\ 1.6674 \\ -1.0020 \end{bmatrix} \qquad m_2 = 2.0014$$

$$v^{(2)} = \frac{v}{m_2} = \frac{1}{2.0014}\begin{bmatrix} 2.0014 \\ 1.6674 \\ -1.0020 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.8331 \\ -0.5006 \end{bmatrix}$$

Subject : .... No : .... Date :

D)

k	$(v^{(k)})^T$			$(Av^{(k)})^T$			m_{k+1}
0	0	0	1	-6	-3	9	9
1	-0.6667	-0.3333	1	2.0014	1.6674	-1.0020	2.0014
2	1	0.8331	-0.5006	-2.3349	-2.1672	0.5016	-2.3344
3	1	0.9284	-0.2149	-2.1426	-2.0713	0.2189	-2.1426
4	1	0.9667	-0.0998	-2.0672	-2.0336	0.1008	-2.0672
5	1	0.9837	-0.0988	-2.0332	-2.0166	0.0498	-2.0332
6	1	0.9918	-0.0245				

$|v^{(6)} - v^{(5)}| < \varepsilon$ $\therefore \lambda$ shifted $= m_6 = -2.0332$

$= x - 2$

$$v = \begin{bmatrix} 1 \\ 0.9918 \\ -0.0245 \end{bmatrix}$$

$\lambda_3 = \lambda$ shifted $+ \lambda_1$

$= -2 + 4$

$\lambda_3 = 2$

$Ay = \lambda y$

$$\begin{bmatrix} 1.9830 \\ 1.9587 \\ -0.0725 \end{bmatrix} = \begin{bmatrix} 2 \\ 1.9836 \\ -0.049 \end{bmatrix}$$

No : ....

Subject : .... Date :

d) $\sum_{i=1}^{3} \lambda i = \sum_{i=1}^{3} a_{ii}$

$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33}$

$4 + \lambda_2 + 2 = -18 + 14 + 13$

$\lambda_2 + 6 = 9$

$\lambda_2 = 9 - 6$

$\lambda_2 = 3$