ASSIGNMENT 3
CHAPTER 4 – LINEAR TRANSFORMATION
SECI/SCSI 1113 COMPUTATIONAL MATHEMATICS
20202021-2

1. Determine whether the function *T: R² →R²* by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$ is a linear transformation.

2. Determine whether the function *T: R³ →R³* by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + y - z \\ 2xy \\ x + z + 1 \end{bmatrix}$ is a linear transformation between vector space.

3. Define a linear operator *T: R³ →R³* by *T(u) = Au*, where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$
   a. Find $T(e_1)$, $T(e_2)$ and $T(e_3)$.
   b. Find $T(3e_1 - 4e_2 + 6e_3)$.

4. Define a linear operator *T: R² →R²* by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ -2x + 4y \end{bmatrix}$. Determine whether the vector $v = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is in *N(T)*.

5. Let *T: R³ →R³* be a linear operator and $B = \{v_1, v_2, v_3\}$ a basis for *R³*. Suppose

$$T(v_1) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, T(v_2) = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, T(v_3) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

   a. Determine whether $w = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ is in range of *T*.
   b. Find a basis for *R(T)*.
   c. Find *dim(N(T))*

6. *T: R² →R²* is a linear operator with *B* and *B'* ordered bases for *v*.

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x + z \\ 2y - x \\ y + z \end{bmatrix}$$

$$B = \left\{\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right\}, B' = \left\{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix}\right\}, v = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

   a. Find the basis representation for T relative to the ordered bases *B* and *B'*.
   b. Find *T(v)* using a direct computation and using the matrix representation.