UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 3

SECI1113 – COMPUTATIONAL MATHEMATICS - SECTION01

(GROUP – T9)

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Question 1

$T(R^2) = R^2$

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$$

let $x = \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$, $y = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$

$$T(x+y) = T\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$$
$$= \begin{bmatrix} 2(a_1+b_1) - (a_2+b_2) \\ a_1 + b_1 + 3(a_2+b_2) \end{bmatrix}$$
$$= \begin{bmatrix} 2a_1 + 2b_1 - a_2 - b_2 \\ a_1 + b_1 + 3a_2 + 3b_2 \end{bmatrix}$$

$$T(x) + T(y) = T\begin{bmatrix} a_1 \\ a_2 \end{bmatrix} + T\begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$$
$$= \begin{bmatrix} 2a_1 - a_2 \\ a_1 + 3a_2 \end{bmatrix} + \begin{bmatrix} 2b_1 - b_2 \\ b_1 + 3b_2 \end{bmatrix}$$
$$= \begin{bmatrix} 2a_1 - a_2 + 2b_1 - b_2 \\ a_1 + 3a_2 + b_1 + 3b_2 \end{bmatrix}$$
$$= T\begin{bmatrix} a_1 + b_1 \\ a_2 + b_2 \end{bmatrix}$$
$$= T(x+y)$$
$$\therefore T(x) + T(y) = T(x+y)$$

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Question 1

let c be any real number.

$$cx = c\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$
$$= \begin{bmatrix} ca_1 \\ ca_2 \end{bmatrix}$$

$$T(cx) = T\begin{bmatrix} ca_1 \\ ca_2 \end{bmatrix}$$
$$= \begin{bmatrix} 2ca_1 - ca_2 \\ ca_1 + 3ca_2 \end{bmatrix}$$
$$= \begin{bmatrix} c(2a_1 - a_2) \\ c(a_1 + 3a_2) \end{bmatrix}$$
$$= c\begin{bmatrix} 2a_1 - a_2 \\ a_1 + 3a_2 \end{bmatrix}$$
$$= cT\begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$
$$= cT(x)$$

$$\therefore T(cx) = cT(x)$$

So, $T: R^2 \to R^2$ is a linear transformation.

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Question 2

$T(R^3) \to R^3$

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ 2xy \\ x+z+1 \end{bmatrix}$$

let $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}, \ y = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}, \ z = \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$

$$T(cx+y+z) \stackrel{?}{=} cT(x) + T(y) + T(z)$$

$$T(cx+y+z) \stackrel{?}{=} cT\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + T\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + T\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$T(cx+y+z) = T\begin{bmatrix} cx_1+y_1+z_1 \\ cx_2+y_2+z_2 \\ cx_3+y_3+z_3 \end{bmatrix}$$

$$= \begin{bmatrix} c(x_1+y_1+z_1)+(x_2+y_2+z_2)-(x_3+y_3+z_3) \\ 2c(x_1+y_1+z_1)(x_2+y_2+z_2) \\ c(x_1+y_1+z_1)+(x_3+y_3+z_3)+1 \end{bmatrix}$$

$$= \begin{bmatrix} c(x_1+y_1+z_1)+x_2+y_2+z_2-x_3-y_3-z_3 \\ 2c(x_1+y_1+z_1)(x_2+y_2+z_2) \\ c(x_1+y_1+z_1)+x_3+y_3+z_3+1 \end{bmatrix}$$

$$= \begin{bmatrix} c(x_1+x_2-x_3)+(y_1+y_2-y_3)+(z_1+z_2-z_3) \\ 2c(x_1x_2+x_1y_2+x_1z_2+y_1x_2+y_1y_2+y_1z_2+ \\ z_1x_2+z_1y_2+z_1z_2) \\ cx_1+x_3+cy_1+y_3+cz_1+z_3+1 \end{bmatrix}$$

$$T(cx+y+z) = \begin{bmatrix} c(x_1+x_2-x_3)+(y_1+y_2-y_3)+(z_1+z_2-z_3) \\ c\left[2x_1(x_2+y_2+z_2)\right]+2y_1(x_2+y_2+z_2)+2z_1(x_2+y_2+z_2) \\ c(x_1+x_3)+(y_1+y_3)+(z_1+z_3)+1 \end{bmatrix}$$

Question 2

$$\cancel{T(cX+y+z) = \begin{bmatrix} c(x_1+x_2-x_3) + (y_1+y_2-y_3) + (z_1+z_2+z_3) \\ 2c(x_1+y_1+z_1)(x_2+y_2+z_2) \\ c(x_1+y_1+z_1)+x_3+y_3+z_3+1 \end{bmatrix}}$$

$$cT(x)+T(y)+T(z) = cT\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + T\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} + T\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix}$$

$$= c\begin{bmatrix} x_1+x_2-x_3 \\ 2x_1x_2 \\ x_1+x_3+1 \end{bmatrix} + \begin{bmatrix} y_1+y_2-y_3 \\ 2y_1y_2 \\ y_1+y_3+1 \end{bmatrix} + \begin{bmatrix} z_1+z_2-z_3 \\ 2z_1z_2 \\ z_1+z_3+1 \end{bmatrix}$$

$$= \begin{bmatrix} c(x_1+x_2-x_3) + (y_1+y_2-y_3) + (z_1+z_2-z_3) \\ c(2x_1x_2) + (2y_1y_2) + (2z_1z_2) \\ c(x_1+x_3+1)+y_1+y_3+z_1+z_3+2 \end{bmatrix}$$

$$\therefore T(cX+y+z) \neq cT(x)+T(y)+T(z)$$

$$T\begin{bmatrix} cX \\ y \\ z \end{bmatrix} \neq cT(x)+T(y)+T(z)$$

Therefore, $T: R^3 \to R^3$ is not a linear transformation.

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3. a) $T(u) = Au$, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$, $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$T(e_1) = Ae_1$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(1) + 2(0) + 3(0) \\ 2(1) + 1(0) + 3(0) \\ 1(1) + 3(0) + 2(0) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \text{*}$$

$T(e_2) = Ae_2$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1(0) + 2(1) + 3(0) \\ 2(0) + 1(1) + 3(0) \\ 1(0) + 3(1) + 2(0) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \text{*}$$

$T(e_3) = Ae_3$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1(0) + 2(0) + 3(1) \\ 2(0) + 1(0) + 3(1) \\ 1(0) + 3(0) + 2(1) \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix} \text{*}$$

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Question 3

b) $T(3e_1 - 4e_2 + 6e_3) = 3T(e_1) - 4T(e_2) + 6T(e_3)$

$$= 3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} + \begin{bmatrix} 18 \\ 18 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 20 \\ 3 \end{bmatrix} \; *$$

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Question 4

4. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ -2x + 4y \end{bmatrix}$

$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$= N(T)$

$T(v) = \begin{bmatrix} 1 - 2(3) \\ -2(1) + 4(3) \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 10 \end{bmatrix}$

$\therefore$ Since $T(v)$, $\begin{bmatrix} -5 \\ 10 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, v is not in $N(T)$.

Assignment 3

Question 5

$$T(v_1) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad T(v_2) = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \quad T(v_3) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

a) $c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

$$c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] \quad R_1 \to R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 2 & 5 & -1 & 1 \\ -1 & 0 & -1 & -2 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + M_{21} R_1 \\ M_{21} = -\frac{2}{1} = -2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ -1 & 0 & -1 & -2 \end{array}\right] \quad R_3 \to R_3 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2 \to \frac{1}{5} R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + M_{13} R_3 \\ M_{13} = -\frac{2}{1} = -2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2 \to R_2 + R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$= c_1 = 2 \quad c_2 = -\frac{3}{5} \quad c_3 = 0$

$w \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ is in range of T.

Assignment 3

Question 5

$$T(v_1) = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \quad T(v_2) = \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \quad T(v_3) = \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

a) $c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

$$c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$= \left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] \quad R_1 \to R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 2 & 5 & -1 & 1 \\ -1 & 0 & -1 & -2 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + M_{21} R_1 \\ M_{21} = \frac{-2}{1} = -2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ -1 & 0 & -1 & -2 \end{array}\right] \quad R_3 \to R_3 + R_1$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2 \to \frac{1}{5} R_2$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right] \quad \begin{array}{l} R_1 \to R_1 + M_{13} R_3 \\ M_{13} = -\frac{2}{1} = -2 \end{array}$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & -1 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right] \quad R_2 \to R_2 + R_3$$

$$= \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -\frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$= c_1 = 2 \quad c_2 = -\frac{3}{5} \quad c_3 = 0$

$w \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$ is in range of T.

Assignment 3

Question 5

b) Find basis for R(T).

$$= \begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} \quad R_1 \to R_3$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 2 & 5 & -1 \\ -1 & 0 & -1 \end{bmatrix} \quad \begin{array}{l} R_2 \to R_2 + M_{21}R_1 \\ M_{21} = -\frac{2}{1} = -2 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -5 \\ -1 & 0 & -1 \end{bmatrix} \quad R_3 \to R_3 + R_1$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \to \frac{1}{5} R_2$$

$$= \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{array}{l} R_1 \to R_1 + M_{13}R_3 \\ m_{13} = -\frac{2}{1} = -2 \end{array}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad R_2 \to R_2 + R_3$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R(T) = \text{span} \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

c) $\dim(N(T)) = 3$