ASSIGNMENT 2
CHAPTER 3 – GENERAL VECTOR SPACES
SECI/SCSI 1113 COMPUTATIONAL MATHEMATICS
20202021-2

1. Let $V = \left\{ \begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} \middle| t \in \mathbb{R} \right\}$. Determine whether *V* is a vector space by verifying Axiom 3, Axiom 7 and Axiom 8 if all the other axioms are satisfied. Defined vectors addition and multiplication as: (12 marks)

$$\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} = \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$c\begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} = \begin{bmatrix} 1+ct \\ 2-ct \\ 3+2ct \end{bmatrix}$$

2. Let

$$S = \left\{ \left\{ \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} \right\} \middle| a, b, c \in \mathbb{R} \right\}$$

   a. Show that *S* is a subspace of M_{2x2}. (5 marks)
   b. Is $\begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ in *S*? (3 marks)

3. Let $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} \right\}$.

   a. Is span(*S*)=R^3? (5 marks)
   b. Determine whether *S* is the basis of R^3. (5 marks)

4. Given B_1 and B_2 as the bases of R^2.

$$B_1 = \left\{ \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \end{bmatrix} \right\} \; and \; B_2 = \left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ -1 \end{bmatrix} \right\}$$

   Find

   a. The coordinates of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ relative to B_2. (2 marks)
   b. The transition matrix from B_2 to B_1. (6 marks)
   c. $[v]_{B_1}$ if $[v]_{B_2} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$. (2 marks)