UNIVERSITI TEKNOLOGI MALAYSIA

ASSIGNMENT 2

SECI1113 – COMPUTATIONAL MATHEMATICS - SECTION01

(GROUP – T9)

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Question 1

$$V = \left\{ \begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} \middle| \, t \in \mathbb{R} \right\}$$

Axiom 3 : $(u+v)+w = u+(v+w)$

let $u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$ $v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$ $w = \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$

$(u+v)+w$

$$= \left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} \right) + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2+t_3) \\ 2-(t_1+t_2+t_3) \\ 3+(2t_1+2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+(t_2+t_3) \\ 2-(t_2+t_3) \\ 3+(2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \left(\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix} \right)$$

$= u+(v+w)$ (Addition is associative)

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Question 1

Axiom 7 : $k(u+v) = ku + kv$

let c be k

$$c(u+v) = cu + cv$$

let $u = \begin{bmatrix} 1+t_1 \\ \cancel{2+t}\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$ $\quad V = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$

$$c(u+v) = c\left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} \right)$$

$$= c \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+c(t_1+t_2) \\ 2-c(t_1+t_2) \\ 3+c(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct_1+ct_2) \\ 2-(ct_1+ct_2) \\ 3+(2ct_1+2ct_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{bmatrix} + \begin{bmatrix} 1+ct_2 \\ 2-ct_2 \\ 3+2ct_2 \end{bmatrix}$$

$$\neq c \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + c \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$$

$$\neq cu + cv$$

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Question 1

Axiom 8 : $(c+d)v = cv + dv$

let $v = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$

$$(c+d)v = (c+d)\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(c+d)\,t_1 \\ 2-(c+d)\,t_1 \\ 3+(c+d)\,2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct_1+dt_1) \\ 2-(ct_1+dt_1) \\ 3+(2ct_1+2dt_1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{bmatrix} + \begin{bmatrix} 1+dt_1 \\ 2-dt_1 \\ 3+2dt_1 \end{bmatrix}$$

$$= c\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + d\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+t_1 \end{bmatrix}$$

$$= cv + dv \quad \#$$

$\therefore$ V is a vector space since Axiom 3, Axiom 7 and Axiom 8 are satisfied.

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Question 2

2 $S = \left\{ \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} \middle| a,b,c \in R \right\}$

a) For closed under addition :

Let $U : \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$ Let $V : \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$

$$U + V = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix} + \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1-b_1)+(a_2-b_2) & (a_1)+(a_2) \\ (b_1+c_1)+(b_2+c_2) & (a_1-c_1)+(a_2-c_2) \end{bmatrix}$$

$$= \begin{bmatrix} a_1+a_2-b_1-b_2 & a_1+a_2 \\ b_1+b_2+c_1+c_2 & a_1+a_2-c_1-c_2 \end{bmatrix}$$

= S is closed under addition.

For closed under scalar multiplication :

$$KU = k \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$$

$$= \begin{bmatrix} ka_1-kb_1 & ka_1 \\ kb_1+kc_1 & ka_1-kc_1 \end{bmatrix}$$

= S is closed under scalar multiplication

= S is a subspace of $M_{2\times 2}$

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Question 2

b) $\begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix}$, $\begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$

$a-b=5$ $\qquad a=3 \qquad$ $b+c=-2$

$3-b=5$ $\qquad\qquad\qquad$ $-2+c=-2$

$b=3-5$ $\qquad\qquad\qquad$ $c=0$

$b=-2$

$=$ So, $\begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S.

Assignment 2

Question 3

3. a) $S = \left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}, \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} \right\}$

$$= c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} + c_5 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x \\ 2 & 3 & 2 & 6 & 4 & y \\ 3 & -1 & -1 & 1 & 5 & z \end{array}\right] \begin{array}{l} R_2 \to R_2 - 2R \\ R_3 \to R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x \\ 0 & 5 & 0 & 6 & 10 & y - 2x \\ 0 & 0 & -4 & -\frac{7}{5} & 10 & z - \frac{2}{5}y + \frac{4}{5}x \end{array}\right]$$

- Span $(S) = R^3$ exists.

Assignment 2

Question 3

D) $\begin{bmatrix} 1 & -1 & 1 & 0 & -3 \\ 2 & 3 & 2 & 6 & 4 \\ 3 & -1 & -1 & 1 & 5 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} : \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 2 & 3 & 2 & 6 & 4 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right]$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 2 & 3 & 2 & 6 & 4 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right] \begin{array}{l} R_2 : R_2 - 2R_1 \\ R_3 : R_3 - 3R_1 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 0 & 2 & -4 & 1 & 14 & 0 \end{array}\right] R_2 : \frac{1}{5} R_2$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & 0 \\ 0 & 2 & -4 & 1 & 14 & 0 \end{array}\right] \begin{array}{l} R_1 : R_1 + R_2 \\ R_3 : R_3 - 2R_2 \end{array}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 6/5 & -1 & 0 \\ 0 & 1 & 0 & 6/5 & 2 & 0 \\ 0 & 0 & -4 & -7/5 & 10 & 0 \end{array}\right] R_3 : -\frac{1}{4} R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 1 & 6/5 & 3/2 & 0 \\ 0 & 1 & 0 & 6/5 & 2 & 0 \\ 0 & 0 & 0 & 7/20 & -5/2 & 0 \end{array}\right] R_1 : R_1 - R_3$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & 0 & 17/20 & 3/2 & 0 \\ 0 & 1 & 0 & 6/5 & 2 & 0 \\ 0 & 0 & 1 & 7/20 & -5/2 & 0 \end{array}\right]$$

$(c_4 = s,\ c_5 = t)$

$c_1 + \frac{17}{20} s + \frac{3}{2} t = 0$

$c_1 = -\frac{17}{20} s - \frac{3}{2} t$

$c_2 + \frac{6}{5} s + 2t = 0$

$c_2 = -\frac{6}{5} s - 2t$

$c_3 + \frac{7}{20} s - \frac{5}{2} t = 0$

$c_3 = -\frac{7}{20} s + \frac{5}{2} t$

- S is not linearly independent. So S is not basis of R^3.

Question 4

a.)

let $A_1\begin{bmatrix}1\\1\end{bmatrix} + A_2\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$$A = \left[\begin{array}{cc|c}1 & 3 & 1\\ 1 & -1 & 0\end{array}\right] \xrightarrow{R_2 \to R_1 - R_2} \left[\begin{array}{cc|c}1 & 3 & 1\\ 0 & 4 & 1\end{array}\right]$$

$4A_2 = 1$

$A_2 = \frac{1}{4}$

$\cancel{A_1 + 3A_2 = 1}$

$\cancel{\frac{1}{4} + 3A_2 = 1}$

$\cancel{3A_2}$

$A_1 + 3A_2 = 1$

$A_1 + 3\left(\frac{1}{4}\right) = 1$

$A_1 \cancel{x} + \frac{3}{4} = 1$

$A_1 = \frac{1}{4}$

$$\frac{1}{4}\begin{bmatrix}1\\1\end{bmatrix} + \frac{1}{4}\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$\therefore$ Coordinates of $\begin{bmatrix}1\\0\end{bmatrix}$ relative to $B_2 = \begin{bmatrix}\frac{1}{4}\\ \frac{1}{4}\end{bmatrix}$ ※

b.) let $\cancel{A}\ a_1\begin{bmatrix}1\\-1\end{bmatrix} + a_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$, $\cancel{b}\ b_1\begin{bmatrix}1\\-1\end{bmatrix} + b_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}3\\-1\end{bmatrix}$

$a_1 = 1$

$-a_1 + 2a_2 = 1$

$-1 + 2a_2 = 1$

$2a_2 = 2$

$a_2 = 1$

$b_1 = 3$

$\cancel{b}\ -b_1 + 2b_2 = -1$

$-3 + 2b_2 = -1$

$\cancel{+}\ 2b_2 = 2$

$b_2 = 1$

$[V_1]_{B_1} = \begin{bmatrix}1\\1\end{bmatrix}$, $[V_2]_{B_1} = \begin{bmatrix}3\\1\end{bmatrix}$

$$[I]^{B_1}_{B_2} = \begin{bmatrix}1 & 3\\ 1 & 1\end{bmatrix}$$

c.) $[V]_{B_1} = [I]^{B_1}_{B_2}[V]_{B_2}$

$[V]_{B_1} = \begin{bmatrix}1 & 3\\ 1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$

$[V]_{B_1} = \begin{bmatrix}11\\5\end{bmatrix}$ ※