ASSIGNMENT 1

SECI1113 – COMPUTATIONAL MATHEMATICS - SECTION01

(GROUP – T9)

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QUESTION 1

(1,-1) and (2,1)

$2x - y = 3$ $\qquad$ $x + 3y = 5$

$2(1) - (-1) = 3$ $\qquad$ $1 + 3(-1) = 5$

$3 = 3$ $\qquad$ $-2 \neq 5$

(1,-1) is not solution ~~this~~ to this system.

$2x - y = 3$ $\qquad$ $x + 3y = 5$

$2(2) - (1) = 3$ $\qquad$ $(2) + 3(1) = 5$

$3 = 3$ $\qquad$ $5 = 5$

(2,1) is the solution to this system.

QUESTION 2

a) $x - y = 2$ $\qquad$ $2x - 2y = 4$

$x = 2 + y$ $\qquad$ $2(2+y) - 2y = 4$

$4 + 2y - 2y = 4$

$2y - 2y = 4 - 4$

$0 = 0$

b) $x - y = 1$ $\qquad$ $x - y = 3$

$x = 1 + y$ $\qquad$ $(1+y) - y = 3$

$1 + y - y = 3$

$y - y = 3 - 1$

$0 = 2$

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QUESTION 3

a) $\begin{pmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{pmatrix}$

$$\det = 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$$

$$= 1((1)(2) - (1)(1)) - 0((5)(2) - (0)(1)) + 3((5)(1) - (0)(1))$$
$$= 1(1) - 0(10) + 3(5)$$
$$= 16 \#$$

(b) $\begin{pmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{pmatrix}$

$$\det = 1\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 0\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 1((0)(-1) - (1)(1)) - (-1)((-1)(-1) - (0)(1)) + 0((-1)(1) - (0)(0))$$
$$= 1(-1) + 1(1) + 0(-1)$$
$$= 0$$

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Question 4.

a) Let $\begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$ be A.

Inverse of A, $A^{-1} = \frac{1}{3(1) - 0} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$$A^{-1} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$$

b) Let $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ be B

$$\text{Det}(B) = 1\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$$

$$= 1(1+0) - 0(0) + 0(0)$$

$$= 1$$

$\det B_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 + 2 = 3$

$\det B_{12} = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = 0$

$\det B_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$\det B_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\det B_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\det B_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\det B_{31} = \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} = 0$

$\det B_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = -2$

$\det B_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

Cofactor matrix B $= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

Adjoint B $= \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Inverse of B, $B^{-1} = \frac{1}{|B|}$ (Adjoint B)

$$= \frac{1}{1} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

Question 5

a)

$$\begin{aligned} x_1 + 2x_2 - 3x_3 &= 9 \\ 2x_1 - x_2 + x_3 &= 0 \\ 4x_1 - x_2 + x_3 &= 4 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right]$$

$$m_{21} = -\frac{a_{21}}{a_{11}} = \frac{-2}{1} = -2 \qquad m_{32} = -\frac{a_{32}}{a_{22}} = -\frac{-9}{-5} = -\frac{9}{5}$$

$$m_{31} = -\frac{a_{31}}{a_{11}} = \frac{-4}{1} = -4$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 + m_{21}R_1 \\ R_3 \rightarrow R_3 + m_{31}R_1 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & -9 & 13 & -32 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & -9 & 13 & -32 \end{array}\right] \quad R_3 \rightarrow R_3 + m_{32}R_2 \quad \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} \end{array}\right]$$

Row echelon form

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} \end{array}\right] \rightarrow \begin{array}{l} x_1 + 2x_2 - 3x_3 = 9 \\ -5x_2 + 7x_3 = -18 \\ \frac{2}{5}x_3 = \frac{2}{5} \end{array}$$

Back-substitution

$\frac{2}{5}x_3 = \frac{2}{5}$

$x_3 = \frac{2}{5} \times \frac{5}{2}$

$x_3 = 1$

$-5x_2 + 7x_3 = -18$

$-5x_2 + 7(1) = -18$

$-5x_2 = -18 - 7$

$-5x_2 = -25$

$x_2 = 5$

$x_1 + 2x_2 - 3x_3 = 9$

$x_1 + 2(5) - 3(1) = 9$

$x_1 + 10 - 3 = 9$

$x_1 + 7 = 9$

$x_1 = 2$

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Question #5

b) $x_1 + x_2 + 2x_3 = 8$
$-x_1 - 2x_2 + 3x_3 = 1$
$3x_1 - 7x_2 + 4x_3 = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$m_{21} = -\frac{a_{21}}{a_{11}} = -\frac{-1}{1} = 1$ $\quad m_{32} = -\frac{a_{32}}{a_{22}} = -\frac{-10}{1} = -10$

$m_{31} = -\frac{a_{31}}{a_{11}} = -\frac{3}{1} = -3$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \quad \begin{array}{l} R_2 \to R_2 + m_{21}R_1 \\ R_3 \to R_3 + m_{31}R_1 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right] \quad R_3 \to R_3 + m_{32}R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right]$$

Row echelon form

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right] \to \begin{array}{l} x_1 + x_2 + 2x_3 = 8 \\ -x_2 + 5x_3 = 9 \\ -52x_3 = -104 \end{array} \to -x_3 = -2$$

c.

Back-substitution

~~$-52x_3 = -104$~~
$x_3 = 2$

$-x_2 + 5x_3 = 9$
$-x_2 + 5(2) = 9$
$-x_2 = 9 - 10$
$-x_2 = -1$
$x_2 = 1$

$x_1 + x_2 + 2x_3 = 8$
$x_1 + 1 + 2(2) = 8$
$x_1 + 1 + 4 = 8$
$x_1 = 3$

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Question 6

a)

$$\begin{aligned} x_1 - 3x_2 - 2x_3 &= 0 \\ -x_1 + 2x_2 + x_3 &= 0 \\ 2x_1 + 4x_2 + 6x_3 &= 0 \end{aligned} \rightarrow \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right]$$

$m_{21} = -\frac{a_{21}}{a_{11}} = -\frac{-1}{1} = 1$ $\quad m_{12} = -\frac{a_{12}}{a_{22}} = -\frac{-3}{1} = 3$ $\quad m_{23} = -\frac{a_{23}}{a_{13}} = -1$

$m_{31} = -\frac{a_{31}}{a_{11}} = -\frac{2}{1} = -2$ $\quad m_{13} = -\frac{a_{13}}{a_{23}} = -1$ $\quad m_{32} = -\frac{a_{32}}{a_{22}} = -1$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \begin{array}{l} R_2 \rightarrow R_2 + m_{21}R_1 \\ R_3 \rightarrow R_3 + m_{31}R_1 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right] \begin{array}{l} R_2 \rightarrow (-1)R_2 \\ R_3 \rightarrow (\frac{1}{10})R_3 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \rightarrow R_1 + m_{12}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \rightarrow R_1 + m_{13}R_1R_2 \\ R_2 \rightarrow R_2 + m_{23}R_1R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{array}\right] \begin{array}{l} R_3 \rightarrow R_3 + m_{32}R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0$, $x_2 = 0$ and $x_3 = 0$

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Question 6

b.)

$2r + s = 3$

$4r + s = 7$

$2r + 5s = -1$

$\rightarrow \begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{bmatrix}$

$\begin{bmatrix} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{bmatrix}$ $R_1 \rightarrow \left(\frac{1}{2}\right) R_1$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{bmatrix}$

$m_{21} = -\frac{a_{21}}{a_{11}} = -4$

$\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{bmatrix}$ $R_2 \rightarrow R_2 + m_{21} R_1$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 1 \\ 2 & 5 & -1 \end{bmatrix}$

$\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 1 \\ 2 & 5 & -1 \end{bmatrix}$ $R_2 \rightarrow (-1) R_2$ $\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -1 \\ 2 & 5 & -1 \end{bmatrix}$

$m_{12} = -\frac{a_{12}}{a_{22}} = -\frac{\frac{1}{2}}{1} = -\frac{1}{2}$

$\begin{bmatrix} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -1 \\ 2 & 5 & -1 \end{bmatrix}$ $R_1 \rightarrow R_1 + m_{12} R_2$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 2 & 5 & -1 \end{bmatrix}$

$m_{31} = -\frac{a_{31}}{a_{11}} = -\frac{2}{1} = -2$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 2 & 5 & -1 \end{bmatrix}$ $R_3 \rightarrow R_3 + m_{31} R_1$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 5 & -5 \end{bmatrix}$

$m_{32} = -\frac{a_{32}}{a_{22}} = -5$

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 5 & -5 \end{bmatrix}$ $R_3 \rightarrow R_3 + m_{32} R_2$ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$

$r = 2$

$s = -1$

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Question 7

a) Doolittle

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$A \quad X = B$$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$L \quad U = A$$

$U_{11} = 2 \qquad U_{12} = 1 \qquad U_{13} = 4$

$L_{21}U_{11} = 8$

$L_{21}(2) = 8$

$L_{21} = 4$

$U_{22} + L_{21}U_{12} = -3$

$U_{22} + (4)(1) = -3$

$U_{22} + 4 = -3$

$U_{22} = -7$

$U_{23} + L_{21}U_{13} = 2$

$U_{23} + (4)(4) = 2$

$U_{23} = -14$

$U_{11}L_{31} = 4$

$L_{31}(2) = 4$

$L_{31} = 2$

$L_{32}U_{22} + L_{31}U_{12} = 11$

$(-7)L_{32} + (2)(1) = 11$

$L_{32}(-7) + 2 = 11$

$L_{32} = \frac{-9}{7}$

$U_{33} + U_{23}L_{32} + U_{13}L_{31} = -1$

$U_{33} + (-14)(\frac{-9}{7}) + (4)(2) = -1$

$U_{33} + 18 + 8 = -1$

$U_{33} = -27$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$L \quad Y = B$

$a = 12$

$4a + b = 20$

$4(12) + b = 20$

$b = -28$

$2a - \frac{9}{7}b + c = 33$

$2(12) - \frac{9}{7}(-28) + c = 33$

$24 + 36 + c = 33$

$c = -27$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$U \quad X = Y$

$-27z = -27$

$z = 1$ ✱

$-7y - 14z = -28$

$-7y - 14(1) = -28$

$-7y - 14 = -28.$

$y = 2$ ✱

$2x + y + 4z = 12$

$2x + 2 + 4(1) = 12$

$2x + 2 + 4 = 12$

$x = 3$ ✱

$2x + y + 4z = 12$

$2(3) + 2 + 4(1) = 12$

$6 + 2 + 4 = 12$

$12 = 12$ ✱

$8x - 3y + 2z = 20$

$8(3) - 3(2) + 2(1) = 20$

$24 - 6 + 2 = 20$

$20 = 20$ ✱

$4x + 11y - z = 33$

$4(3) + 11(2) - 1 = 33$

$12 + 22 - 1 = 33$

$33 = 33$ ✱

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b) Crout

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$A \quad X = B$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$L \quad U = A$

$L_{11} = 2 \qquad L_{11}U_{12} = 1 \qquad L_{11}U_{13} = 4$

$U_{12}(2) = 1 \qquad (2)L_{13} = 4$

$U_{12} = \frac{1}{2} \qquad L_{13} = 2$

$L_{21} = 8 \qquad L_{22} + L_{21}U_{12} = -3 \qquad L_{22}U_{23} + L_{21}U_{13} = 2$

$L_{22} + (8)(\frac{1}{2}) = -3 \qquad (-7)U_{23} + (8)(2) = 2$

$L_{22} + 4 = -3 \qquad -7U_{23} + 16 = 2$

$L_{22} = -7 \qquad U_{23} = 2$

$L_{31} = 4 \qquad L_{32} + L_{31}U_{12} = 11 \qquad L_{33} + L_{32}U_{23} + L_{31}U_{13} = -1$

$L_{32} + (4)(\frac{1}{2}) = 11 \qquad L_{33} + (9)(2) + (4)(2) = -1$

$L_{32} + 2 = 11 \qquad L_{33} + 18 + 8 = -1$

$L_{32} = 9 \qquad L_{33} = -27$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$L \quad Y = B$

$2a = 12 \qquad 8a - 7b = 20 \qquad 4a + 9b - 27c = 33$

$a = 6 \qquad 8(6) - 7b = 20 \qquad 4(6) + 9(4) - 27c = 33$

$48 - 7b = 20 \qquad 24 + 36 - 27c = 33$

$b = 4 \qquad -27c = -27$

$c = 1$

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$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$z = 1$ ✱

$y + 2z = 4$

$y + 2(1) = 4$

$y + 2 = 4$

$y = 2$ ✱

$x + \frac{1}{2}y + 2z = 6$

$x + \frac{1}{2}(2) + 2(1) = 6$

$x + 1 + 2 = 6$

$x = 3$ ✱

$2x + y + 4z = 12$

$2(3) + 2 + 4(1) = 12$

$6 + 2 + 4 = 12$

$12 = 12$ ✱

$8x - 3y + 2z = 20$

$8(3) - 3(2) + 2(1) = 20$

$24 - 6 + 2 = 20$

$20 = 20$ ✱

$4x + 11y - z = 33$

$4(3) + 11(3) - (1) = 33$

$12 + 22 - 1 = 33$

$33 = 33$ ✱

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Question 8

$$\|u-v\| = \sqrt{(\sqrt{2}-0)^2 + (1-2)^2 + (-1-2)^2}$$

$$= \sqrt{2+1+9}$$

$$= \sqrt{12}$$

$$= 2\sqrt{3}$$

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Question 9

$U = (2, 1, 2)$ and $V = (3, 3, 1)$

a) $U \cdot V = (2)(3) + (1)(3) + (2)(1)$

$= 6 + 3 + 2$

$= 11$

b) $U \times V = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix} = (i)\begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} - (j)\begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} + (k)\begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix}$

$= (i)(1-6) - (j)(2-6) + (k)(6-3)$

$= -5i + 4j + 3k$

$= (-5 \quad 4 \quad 3)$

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Question 10

a) $C_1U_1 + C_2U_2 + C_3U_3 = V$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right]$$

$m_{21} = \frac{-a_{21}}{a_{11}} = -1$

$m_{31} = \frac{-a_{31}}{a_{11}} = -1$

$1 + (-1)(1) = 0$

$2 + (-1)(1) = 1$

$-1 + (-1)(2) = -3$

$-2 + (-1)(1) = -3$

$R_2 \to R_2 + m_{21}R_1$

$R_3 \to R_3 + m_{31}R_1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$1 + (-0)(1) = 0$

$3 + (-1)(1) = 2$

$1 + (-1)(2) = -1$

$5 + (-1)(1) = 4$

$m_{12} = \frac{-a_{12}}{a_{22}} = -1$

$m_{32} = \frac{-a_{32}}{a_{22}} = -2$

$R_3 \to R_3 + m_{32}R_2$

$R_1 \to R_1 + m_{12}R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$0 + (-2)(0) = 0$

$2 + (-2)(1) = 0$

$-1 + (-2)(-3) = 5$

$4 + (-2)(-3) = 10$

$R_3 \to \frac{1}{5}R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$m_{13} = \frac{-a_{13}}{a_{33}} = -5$

$m_{23} = \frac{-a_{23}}{a_{33}} = 3$

$1 + (-1)(0) = 1$

$1 + (-1)(1) = 0$

$2 + (-1)(-3) = 5$

$1 + (-1)(-3) = 4$

$R_1 \to R_1 + m_{13}R_3$

$R_2 \to R_2 + m_{23}R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$C_1 = -6$

$C_2 = 3$

$C_3 = 2$

$1 + (-5)(0) = 1$

$0 + (-5)(0) = 0$

$5 + (-5)(1) = 0$

$4 + (-5)(2) = -6$

$0 + (7)(0) = 0$

$1 + (7)(0) = 1$

$-3 + (3)(1) = 0$

$-3 + (3)(2) = 3$

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$$C_1U_1 + C_2U_2 + C_3U_3 = V$$

$$(-6)\begin{bmatrix}1\\1\\1\end{bmatrix} + (3)\begin{bmatrix}1\\2\\3\end{bmatrix} + (2)\begin{bmatrix}2\\-1\\1\end{bmatrix} = \begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$$\begin{bmatrix}-6\\-6\\-6\end{bmatrix} + \begin{bmatrix}3\\6\\9\end{bmatrix} + \begin{bmatrix}4\\-2\\2\end{bmatrix} = \begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$$\begin{bmatrix}1\\-2\\5\end{bmatrix} = \begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$\therefore$ V is a linear combination of U_1, U_2 and U_3.

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b) $C_1U + C_2V + C_3W = 0$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 5 & 1 & 2 & 0 \end{array}\right]$$

$m_{21} = \frac{-a_{21}}{a_{11}} = -2$

$m_{31} = \frac{-a_{31}}{a_{11}} = -5$

$2 + (-2)(1) = 0$

$5 + (-2)(2) = 1$

$5 + (-2)(1) = 3$

$0 + (-2)(0) = 0$

$5 + (-5)(1) = 0$

$R_2 \to R_2 + m_{21}R_1$, $R_3 \to R_3 + m_{31}R_1$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -9 & -3 & 0 \end{array}\right]$$

$m_{12} = \frac{-a_{12}}{a_{22}} = -2$

$m_{32} = \frac{-a_{32}}{a_{22}} = 9$

$1 + (-5)(2) = -9$

$2 + (-5)(1) = -3$

$0 + (-5)(0) = 0$

$R_1 \to R_1 + m_{12}R_2$, $R_3 \to R_3 + m_{32}R_2$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 24 & 0 \end{array}\right]$$

$1 + (-2)(0) = 1$

$2 + (-2)(1) = 0$

$1 + (-2)(3) = -5$

$0 + (-2)(0) = 0$

$R_3 \to \frac{1}{24}R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$m_{13} = \frac{-a_{13}}{a_{33}} = -5$

$m_{23} = \frac{-a_{23}}{a_{33}} = -3$

$0 + (9)(0) = 0$

$-9 + (9)(1) = 0$

$-3 + (9)(3) = 24$

$0 + (9)(0) = 0$

$R_1 \to R_1 + m_{13}R_3$, $R_2 \to R_2 + m_{23}R_3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$1 + (-5)(0) = 1$

$0 + (-5)(0) = 0$

$5 + (-5)(1) = 0$

$0 + (-5)(0) = 0$

$0 + (-3)(0) = 0$

$1 + (-3)(0) = 1$

$3 + (-3)(1) = 0$

$0 + (-3)(0) = 0$

∴ Since C_1, C_2, and C_3 are trivial, they are linearly independent.