UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

COMPUTATIONAL MATHEMATICS

SEMESTER II 2020/2021

ASSIGNMENT 3

T7

NAME OF STUDENTS	MATRIC NO.
Muhammed Munir Bin Mohd Azmi	A20EC8031
Chong Kah Wei	A20EC0027
Iman Ehsan Bin Hassan	A20EC0048

LECTURER : DR RAZANA BINTI ALWEE

No.: Date:

1. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x-y \\ x+3y \end{bmatrix}$

$T(x,y) = (2x-y, x+3y)$

Let $x = (x_1, y_1)$ and $y = (x_2, y_2)$ be vectors in $\mathbb{R}^2$ and let c be any real number.

$x+y = (x_1, y_1) + (x_2, y_2)$

$\quad = (x_1+x_2, y_1+y_2)$

$T(x+y) = T(x_1+x_2, y_1+y_2)$

$\quad = (2(x_1+x_2)-(y_1+y_2), x_1+x_2+3(y_1+y_2))$

$\quad = (2x_1+2x_2-y_1-y_2, x_1+x_2+3y_1+3y_2)$

$\quad = (2x_1-y_1+2x_2-y_2, x_1+3y_1+x_2+3y_2)$

$\quad = (2x_1-y_1, x_1+3y_1), (2x_2-y_2, x_2+3y_2)$

$\quad = T(x)+T(y)$

$cx = c(x_1, y_1) = (cx_1, cy_1)$

$T(cx) = T(cx_1, cy_1)$

$\quad = (2cx_1-cy_1, cx_1+3cy_1)$

$\quad = c(2x_1-y_1, x_1+3y_1)$

$\quad = cT(x)$

Since $T(x+y) = T(x)+T(y)$ and $T(cx) = cT(x)$, T is a linear transformation.

No.: Date:

2. $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ 2xy \\ x+z+1 \end{bmatrix}$

$T(x, y, z) = (x+y-z,\ 2xy,\ x+z+1)$

Let $x = (x_1, y_1, z_1)$, $y = (x_2, y_2, z_2)$ and $z = (x_3, y_3, z_3)$ be vectors in $\mathbb{R}^3$ and let c be any real number.

$x+y+z = (x_1, y_1, z_1) + (x_2, y_2, z_2) + (x_3, y_3, z_3)$

$= (x_1+x_2+x_3,\ y_1+y_2+y_3,\ z_1+z_2+z_3)$

$T(x+y+z) = (x_1+x_2+x_3+y_1+y_2+y_3-(z_1+z_2+z_3),\ 2(x_1+x_2+x_3)(y_1+y_2+y_3),$
$x_1+x_2+x_3+z_1+z_2+z_3+1)$

$= (x_1+x_2+x_3+y_1+y_2+y_3-z_1-z_2+z_3,\ 2x_1y_1+2x_1y_2+2x_1y_3+2x_2y_1+2x_2y_2$
$+2x_2y_3+2x_3y_1+2x_3y_2+2x_3y_3,\ x_1+x_2+x_3+z_1+z_2+z_3+1)$

$T(x)+T(y)+T(z) = (x_1+y_1-z_1,\ 2x_1y_1,\ x_1+z_1+1) + (x_2+y_2-z_2,\ 2x_2y_2,\ x_2+z_2+1)$
$+ (x_3+y_3-z_3,\ 2x_3y_3,\ x_3+z_3+1)$

$= (x_1+y_1-z_1+x_2+y_2-z_2+x_3+y_3-z_3,\ 2x_1y_1+2x_2y_2+2x_3y_3,\ x_1+z_1+1$
$+x_2+z_2+1+x_3+z_3+1)$

$T(x+y+z) \neq T(x)+T(y)+T(z)$

$cx = c(x_1, y_1, z_1) = (cx_1, cy_1, cz_1)$

$T(cx) = T(cx_1, cy_1, cz_1)$

$= (cx_1+cy_1-cz_1,\ 2cx_1(cy_1),\ cx_1+cz_1+1)$

$= (c(x_1+y_1-z_1),\ c(2cx_1y_1),\ c(x_1+z_1+1/c))$

$= c(x_1+y_1-z_1,\ 2cx_1y_1,\ x_1+z_1+1/c)$

$cT(x) = c(x_1+y_1-z_1,\ 2x_1y_1,\ x_1+z_1+1)$

$T(cx) \neq cT(x)$

Since $T(x+y+z) \neq T(x)+T(y)+T(z)$ and $T(cx) \neq cT(x)$, T is not a linear transformation.

3. $T(u) = Au$, $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$

a) Find $T(e_1)$, $T(e_2)$ and $T(e_3)$

$$T(u) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} u$$

$$T(e_1) = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \qquad T(e_2) = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \qquad T(e_3) = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

b) Find $T(3e_1 - 4e_2 + 6e_3)$

$$T(3e_1 - 4e_2 + 6e_3) = 3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 \\ 6 \\ 3 \end{bmatrix} - \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix} + \begin{bmatrix} 18 \\ 18 \\ 12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 20 \\ 3 \end{bmatrix}$$

No.:

Date:

4. $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x-2y \\ -2x+4y \end{bmatrix}$

$T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} 1-2(3) \\ -2(1)+4(3) \end{bmatrix}$

$= \begin{bmatrix} -5 \\ 10 \end{bmatrix}$

$\neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

Since $T\left(\begin{bmatrix} 1 \\ 3 \end{bmatrix}\right) \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ is not in $N(T)$

5a) $c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$

$$c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 + 2R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 0 & 5 & -3 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \to \frac{1}{5} R_2]{R_2 \to R_2 + 3R_3} \left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 0 & 1 & 0 & -3/5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \to -R_1]{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & -3/5 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Solution is $c_1 = 2$, $c_2 = -3/5$, $c_3 = 0$, $w \in R(T)$.

b) $$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 + 2R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \to \frac{1}{5} R_2]{R_2 \to R_2 + 3R_3} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \to -R_1]{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

Leading 1s are in columns 1, 2 and 3, basis of $R(T) = \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$

No.: Date:

c) $c_1T(v_1) + c_2T(v_2) + c_3T(v_3) = 0$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 + 2R_1} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & 5 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_2 \to \frac{1}{5}R_2]{R_2 \to R_2 + 3R_3} \left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \to -R_1]{R_1 \to R_1 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 = 0,\ c_2 = 0,\ c_3 = 0$

$$N(T) = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

Since the null space consists only of the zero vector, it has no basis.

Thus,

$\dim(N(T)) = 0$

Scanned with CamScanner

No.:

Date:

6a $T\left(\begin{bmatrix}-1\\1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\3\\2\end{bmatrix}$ $T\left(\begin{bmatrix}-1\\-1\\1\end{bmatrix}\right)=\begin{bmatrix}0\\-1\\0\end{bmatrix}$ $T\left(\begin{bmatrix}0\\1\\1\end{bmatrix}\right)=\begin{bmatrix}1\\2\\2\end{bmatrix}$

$a_1\begin{bmatrix}0\\0\\1\end{bmatrix}+a_2\begin{bmatrix}1\\0\\-1\end{bmatrix}+a_3\begin{bmatrix}-1\\-1\\0\end{bmatrix}=\begin{bmatrix}0\\3\\2\end{bmatrix}$

$b_1\begin{bmatrix}0\\0\\1\end{bmatrix}+b_2\begin{bmatrix}1\\0\\-1\end{bmatrix}+b_3\begin{bmatrix}-1\\-1\\0\end{bmatrix}=\begin{bmatrix}0\\-1\\0\end{bmatrix}$

$c_1\begin{bmatrix}0\\0\\1\end{bmatrix}+c_2\begin{bmatrix}1\\0\\-1\end{bmatrix}+c_3\begin{bmatrix}-1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\2\\2\end{bmatrix}$

$$\left[\begin{array}{ccc|c}0&1&-1&0\\0&0&-1&3\\1&-1&0&2\end{array}\right]\xrightarrow{R_1\leftrightarrow R_3}\left[\begin{array}{ccc|c}1&-1&0&2\\0&0&-1&3\\0&1&-1&0\end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{ccc|c}1&-1&0&2\\0&1&-1&0\\0&0&-1&3\end{array}\right]$$

$-a_3=3$ $\qquad a_2-a_3=0$ $\qquad a_1-a_2=2$

$a_3=-3$ $\qquad a_2-(-3)=0$ $\qquad a_1-(-3)=2$

$a_2=-3$ $\qquad a_1=-1$

$$\left[\begin{array}{ccc|c}0&1&-1&0\\0&0&-1&-1\\1&-1&0&0\end{array}\right]\xrightarrow{R_1\leftrightarrow R_3}\left[\begin{array}{ccc|c}1&-1&0&0\\0&0&-1&-1\\0&1&-1&0\end{array}\right]$$

$$\xrightarrow{R_2\leftrightarrow R_3}\left[\begin{array}{ccc|c}1&-1&0&0\\0&1&-1&0\\0&0&-1&-1\end{array}\right]$$

$-b_3=-1$ $\qquad b_2-b_3=0$ $\qquad b_1-b_2=0$

$b_3=1$ $\qquad b_2-1=0$ $\qquad b_1-1=0$

$b_2=1$ $\qquad b_1=1$

No.: Date:

$$\left[\begin{array}{ccc|c} 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \\ 1 & -1 & 0 & 2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right]$$

$$\xrightarrow{R_2 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 2 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & -1 & 2 \end{array}\right]$$

$-C_3 = 2$ $\quad C_2 - C_3 = 1$ $\quad C_1 - C_2 = 2$

$C_3 = -2$ $\quad C_2 - (-2) = 1$ $\quad C_1 - (-1) = 2$

$C_2 = -1$ $\quad C_1 = 1$

Thus,

$$[T]_B^{B'} = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 1 & -1 \\ -3 & 1 & -2 \end{bmatrix}$$

b) $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+z \\ 2y-x \\ y+z \end{bmatrix}$

$$T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2+3 \\ 2(1)-(-2) \\ 1+3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$$a_1\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + a_2\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + a_3\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & -1 & 0 & -2 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & 1 & 3 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_3} \left[\begin{array}{ccc|c} -1 & -1 & 0 & -2 \\ 0 & -2 & 0 & -2 \\ 1 & 1 & 1 & 3 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_1} \left[\begin{array}{ccc|c} -1 & -1 & 0 & -2 \\ 0 & -2 & 0 & -2 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$a_3 = 1$ $\quad -2a_2 = -2$ $\quad -a_1 - a_2 = -2$

$a_2 = 1$ $\quad -a_1 - (1) = -2$

$a_1 = 1$

No.: Date:

$$[v]_B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[T(v)]_{B'} = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 1 & -1 \\ -3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ -3 \\ -4 \end{bmatrix}$$

$$T(v) = 1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - 4 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$