UTM
UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

COMPUTATIONAL MATHEMATICS

SEMESTER II 2020/2021

ASSIGNMENT 2

T7

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1) addition: $\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} = \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$

multiplication: $c\begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} = \begin{bmatrix} 1+ct \\ 2-ct \\ 3+2ct \end{bmatrix}$

axiom 3:

$$\left[\begin{pmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{pmatrix} + \begin{pmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{pmatrix}\right] + \begin{pmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{pmatrix} = \begin{pmatrix} 1+(t_1+t_2)+1+t_3 \\ 2-(t_1+t_2)+2-t_3 \\ 3+(2t_1+2t_2)+3+2t_3 \end{pmatrix}$$

$$= \begin{pmatrix} 2+t_1+t_2+t_3 \\ -t_1-t_2-t_3 \\ 6+2t_1+2t_2+2t_3 \end{pmatrix}$$

$$\begin{pmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{pmatrix} + \left[\begin{pmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{pmatrix} + \begin{pmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{pmatrix}\right] = \begin{pmatrix} 1+t_1+1+(t_2+t_3) \\ 2-t_1+2-(t_2+t_3) \\ 3+2t_1+3+(2t_2+2t_3) \end{pmatrix}$$

$$= \begin{pmatrix} 2+t_1+t_2+t_3 \\ -t_1-t_2-t_3 \\ 6+2t_1+2t_2+2t_3 \end{pmatrix}$$

$\therefore$ The addition is associative. Thus, satisfied axiom 3.

axiom 7:

$$c\left[\begin{pmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{pmatrix} + \begin{pmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{pmatrix}\right] = c\begin{pmatrix} 1+t_1+1+t_2 \\ 2-t_1+2-t_2 \\ 3+2t_1+3+2t_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+ct_1+1+ct_2 \\ 2-ct_1+2-ct_2 \\ 3+2ct_1+3+2ct_2 \end{pmatrix}$$

$$= \begin{pmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{pmatrix} + \begin{pmatrix} 1+ct_2 \\ 2-ct_2 \\ 3+2ct_2 \end{pmatrix}$$

$$= c\begin{pmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{pmatrix} + c\begin{pmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{pmatrix}$$

$\therefore$ Thus, satisfied axiom 7.

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axiom 8:

$$(c+k)\begin{pmatrix}1+t\\2-t\\3+2t\end{pmatrix}=\begin{pmatrix}1+(c+k)t\\2-(c+k)t\\3+2(c+k)t\end{pmatrix} \qquad c\begin{pmatrix}1+t\\2-t\\3+2t\end{pmatrix}+k\begin{pmatrix}1+t\\2-t\\3+2t\end{pmatrix}=\begin{pmatrix}1+ct+1+kt\\2-ct+2-kt\\3+2ct+3+2kt\end{pmatrix}$$

$$=\begin{pmatrix}1+ct+kt\\2-ct-kt\\3+2ct+2kt\end{pmatrix} \qquad =\begin{pmatrix}2+ct+kt\\4-ct-kt\\6+2ct+2kt\end{pmatrix}$$

$\therefore$ $(c+k)V \neq (cV)+(kV)$. Thus, not satisfied axiom 7.

Therefore, V is not a vector space.

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2a) $S_1 = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$ $S_2 = \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$

$$S_1+S_2 = \begin{bmatrix} a_1-b_1+a_2-b_2 & a_1+a_2 \\ b_1+c_1+b_2+c_2 & a_1-c_1+a_2-c_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1+a_2)-(b_1+b_2) & a_1+a_2 \\ (b_1+b_2)+(c_1+c_2) & (a_1+a_2)-(c_1+c_2) \end{bmatrix}$$

$$= \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix}, \text{ where } a=a_1+a_2,\ b=b_1+b_2,\ c=c_1+c_2 \in \mathbb{R}$$

$\therefore$ The subset S is closed under addition.

$$kS_1 = \begin{bmatrix} ka_1-kb_1 & ka_1 \\ kb_1+kc_1 & ka_1-kc_1 \end{bmatrix}$$

$$= \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix}, \text{ where } a=ka_1,\ b=kb_1,\ c=kc_1 \in \mathbb{R}$$

$\therefore$ The subset S is closed under multiplication.

Thus, S is a subspace of $M_{2\times 2}$.

b) $S = \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix}$ $V = \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ $a-b=5$ $a=3$ $b+c=-2$ $a-c=3$

$a=3$ $a-b=5$ $b+c=-2$

$3-b=5$ $(-2)+c=-2$

$b=-2$ $c=0$

Substituting value a, b, c into vector S,

$$S = \begin{bmatrix} 3-(-2) & 3 \\ -2+0 & 3-0 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$$

$\therefore \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S.

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3a) Let $V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $V_2 = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix}$, $V_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $V_4 = \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix}$, $V_5 = \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$

$$C_1V_1 + C_2V_2 + C_3V_3 + C_4V_4 + C_5V_5 = C_1\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + C_2\begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + C_3\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + C_4\begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} + C_5\begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 2 & 3 & 2 & 6 & 4 & x_2 \\ 3 & -1 & -1 & 1 & 5 & x_3 \end{array}\right] \xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & -3 & x_1 \\ 2 & 3 & 2 & 6 & 4 & x_2 \\ 0 & 2 & -4 & 1 & 14 & x_3 - 3x_1 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & -3 & x_1 \\ 0 & 5 & 0 & 6 & 10 & x_2 - 2x_1 \\ 0 & 2 & 2 & 1 & 14 & x_3 - 3x_1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - \frac{2}{5}R_2} \left[\begin{array}{ccccc|c} 1 & -1 & -1 & 0 & -3 & x_1 \\ 0 & 5 & 0 & 6 & 10 & x_2 - 2x_1 \\ 0 & 0 & -4 & -\frac{7}{5} & 10 & x_3 - \frac{2}{5}x_2 - \frac{11}{5}x_1 \end{array}\right]$$

$\therefore$ Hence, span(s) = span$\left\{ \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \begin{pmatrix} -1 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \right\}$, span(s) = $\mathbb{R}^3$

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b) $\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 2 & 3 & 2 & 6 & 4 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 3 & -1 & -1 & 1 & 5 & 0 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 0 & 2 & -4 & 1 & 14 & 0 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 - \frac{2}{5} R_2} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 5 & 0 & 6 & 10 & 0 \\ 0 & 0 & -4 & -\frac{7}{5} & 10 & 0 \end{array}\right]$

$\therefore$ Since S is not linearly independent, S is not the basis of R^3.

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4a) $c_1\begin{bmatrix}1\\1\end{bmatrix}+c_2\begin{bmatrix}3\\-1\end{bmatrix}=\begin{bmatrix}1\\0\end{bmatrix}$

$\left[\begin{array}{cc|c}1 & 3 & 1\\1 & -1 & 0\end{array}\right]\xrightarrow{R_2\to R_2-R_1}\left[\begin{array}{cc|c}1 & 3 & 1\\0 & -4 & -1\end{array}\right]$

$-4c_2=-1$ $\qquad$ $c_1+3c_2=1$

$c_2=\frac{1}{4}$ $\qquad$ $c_1+3(\frac{1}{4})=1$

$c_1=\frac{1}{4}$

$[V]_{B_2}=\begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\end{bmatrix}$

$B_2\to B_1$

b) $c_1\begin{bmatrix}1\\-1\end{bmatrix}+c_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}1\\1\end{bmatrix}$ $\qquad$ $d_1\begin{bmatrix}1\\-1\end{bmatrix}+d_2\begin{bmatrix}0\\2\end{bmatrix}=\begin{bmatrix}3\\-1\end{bmatrix}$

$c_1+0=1$ $\qquad$ $-c_1+2c_2=1$ $\qquad$ $d_1+0=3$ $\qquad$ $-d_1+2d_2=-1$

$c_1=1$ $\qquad$ $-1+2c_2=1$ $\qquad$ $d_1=3$ $\qquad$ $-3+2d_2=-1$

$c_2=1$ $\qquad$ $d_2=1$

$[I]_{B_2}^{B_1}=\begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}$

c) $[V]_{B_1}=[I]_{B_2}^{B_1}[V]_{B_2}$

$=\begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$

$=\begin{bmatrix}11\\5\end{bmatrix}$