UNIVERSITI TEKNOLOGI MALAYSIA

SCHOOL OF COMPUTING
Faculty of Engineering

COMPUTATIONAL MATHEMATICS

SEMESTER II 2020/2021

ASSIGNMENT 1

T7

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No.: Assignment 1 (due 19/4) Date:

1) System of linear equation:

$2x-y=3$

$x+3y=5$

For point $(1,-1)$,

$2(1)-(-1)=3$

$1+3(-1)=-2 \quad (-2 \neq 5)$

$\therefore$ Since the point $(1,-1)$ does not satisfy the equation $x+3y=5$, $(1,-1)$ is not the solution of the system of equation.

For point $(2,1)$,

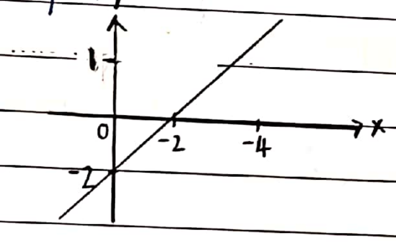
$2(2)-1=3$

$2+3(1)=5$

$\therefore$ Since the point $(2,1)$ satisfy both the equation, $(2,1)$ is a solution of the system.

2a) $x-y=2$

$2x-2y=4$



$x=2+y$

$2(2+y)-2y=4$

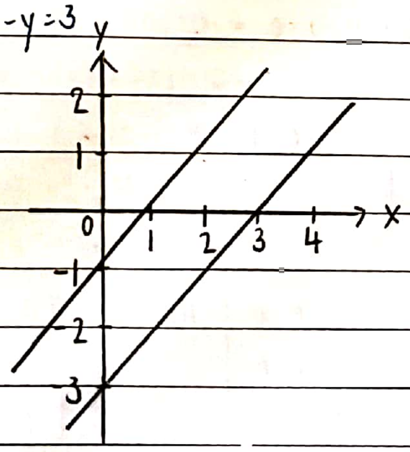
$4+2y-2y=4$

$0=0$ (infinite solution)

$\therefore$ The system is redundant because the second equation is equivalent to the first one. The solution is infinite and consistent.

b) $x-y=1$

$x-y=3$



$x=1+y$

$1+y-y=3$

$0=2$ (no solution)

$\therefore$ There are two different answer for the same equation. Referring to the graph, the system consists of two parallel lines that never cross. The system is inconsistent and there is no solutions for this system.

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3a) $A = \begin{vmatrix} 1 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 1 & 2 \end{vmatrix}$

$\det A = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= (-1)^{1+1} a_{11} \det A_{11} + (-1)^{1+2} a_{12} \det A_{12} + (-1)^{1+3} a_{13} \det A_{13}$

$= 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1(2-1) - 0 + 3(5-0)$

$= 16$

b) $A = \begin{vmatrix} 1 & -1 & 0 \\ -1 & 0 & 1 \\ 0 & 1 & -1 \end{vmatrix}$

$\det A = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= (-1)^{1+1} a_{11} \det A_{11} + (-1)^{1+2} a_{12} \det A_{12} + (-1)^{1+3} a_{13} \det A_{13}$

$= 1\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 0\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(0-1) - (-1)(1) + 0$

$= 0$

4a) $A = \begin{bmatrix} 3 & 0 \\ 0 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{3(1)-0} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix}$

b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$ $\quad \det A = 1\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - 0\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + 0\begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

$= 1$

$\det A_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$ $\qquad \det A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} = 0$

$\det A_{12} = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = 0$ $\qquad \det A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = -2 - 0 = -2$

$\det A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$ $\qquad \det A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$\det A_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\det A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

$\det A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

Cofactor matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$ $\quad$ Adjoint $= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

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$$A^{-1} = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

5a) $x_1 + 2x_2 - 3x_3 = 9$

$2x_1 - x_2 + x_3 = 0$

$4x_1 - x_2 + x_3 = 4$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 4 & -1 & 1 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 4R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & -9 & 13 & -32 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 9/5 R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & 2/5 & 2/5 \end{array}\right]$$

$\frac{2}{5} x_3 = \frac{2}{5}$ $\quad -5x_2 + 7x_3 = -18$ $\quad x_1 + 2x_2 - 3x_3 = 9$

$x_3 = 1$ $\quad -5x_2 + 7(1) = -18$ $\quad x_1 + 2(5) - 3(1) = 9$

$x_2 = 5$ $\quad x_1 = 2$

b) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 3R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 10R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -104 \end{array}\right]$$

$-52x_3 = -104$ $\qquad$ $x_2 - 5x_3 = -9$ $\qquad$ $x_1 + x_2 + 2x_3 = 8$

$x_3 = 2$ $\qquad$ $x_2 - 5(10) = -9$ $\qquad$ $x_1 + 1 + 2(2) = 8$

$x_2 = 1$ $\qquad$ $x_1 = 3$

6a) $x_1 - 3x_2 - 2x_3 = 0$

$-x_1 + 2x_2 + x_3 = 0$

$2x_1 + 4x_2 + 6x_3 = 0$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_1} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 10R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - 3R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore$ There is no solution for the system.

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b) $2r + s = 3$

$4r + s = 7$

$2r + 5s -1$

$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 2 & 5 & -1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 4R_2} \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_2} \left[\begin{array}{cc|c} 2 & 0 & 4 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to 1/2 R_1} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore\ r = 2,\ s = -1$

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7a) $2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$u_{11} = \frac{2}{1} = 2$ $\quad u_{12} = \frac{1}{1} = 1$ $\quad u_{13} = \frac{4}{1} = 4$ $\quad u_{23} = \frac{2 - 4(4)}{1} = -14$

$l_{21} = \frac{8}{2} = 4$ $\quad l_{31} = \frac{4}{2} = 2$ $\quad u_{22} = \frac{-3 - 4(1)}{1} = -7$ $\quad l_{32} = \frac{11 - (2)(1)}{-7} = -\frac{9}{7}$ $\quad u_{33} = \frac{-1 - 2(4) - (-14)(-\frac{9}{7})}{1} = -27$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$Y_1 = \frac{12}{1} = 12$ $\quad Y_2 = \frac{20 - (4)(12)}{1} = -28$ $\quad Y_3 = \frac{33 - 2(12) - (-\frac{9}{7})(-28)}{1} = -15$

$$Y = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$z = \frac{-27}{-27} = 1$ $\quad y = \frac{-28 - (-14)(1)}{-7} = 2$ $\quad x = \frac{12 - 1(2) - 4(1)}{2} = 3$

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7b) $$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix}\begin{bmatrix} 1 & u_{12} & u_{13} \\ 0 & 1 & u_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$l_{11} = 2$ $\quad l_{21} = 8$ $\quad l_{31} = 4$ $\quad u_{12} = \frac{1}{2} = \frac{1}{2}$ $\quad u_{13} = \frac{4}{2} = 2$

$l_{22} = -3 - 8(\frac{1}{2}) = -7$ $\quad u_{23} = \frac{2-8(2)}{-7} = 2$ $\quad l_{32} = 11 - 4(\frac{1}{2}) = 9$ $\quad l_{33} = -1 - 4(2) - 9(2) = -27$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$y_1 = \frac{12}{2} = 6$ $\quad y_2 = \frac{20 - 8(6)}{-7} = 4$ $\quad y_3 = \frac{33 - 4(6) - (9)(4)}{-27} = 1$

$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$z = \frac{1}{1} = 1$ $\quad y = \frac{4 - 2(1)}{1} = 2$ $\quad x = \frac{6 - \frac{1}{2}(2) - 2(1)}{1} = 3$

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8) $d(u,v) = \sqrt{(\sqrt{2}-0)^2 + (1-2)^2 + (-1-2)^2}$

$= 2\sqrt{3}$

9a) $u \cdot v = 2(3) + 1(3) + 2(1)$

$= 11$

b) $u \times v = (1(1) - 2(3),\ 2(3) - 2(1),\ 2(3) - 1(0))$

$= (-5\ \ 4\ \ 6)$

10a) $$c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & -1 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right] \xrightarrow{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 1 & 3 & 1 & 5 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 2R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 0 & 4 \end{array}\right]$$

$\therefore$ Since it has no solution in the linear system, v cannot be expressed as a linear combination of u_1, u_2 and u_3.

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b) $c_1 \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 5 \\ 1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 5 & 5 \\ 5 & 1 & 2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 5 & 1 & 2 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 5 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - 5R_1} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -9 & -3 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + 9R_2} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 24 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to \frac{1}{24} R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 3R_3} \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - 2R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 - R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ Solving the linear equations, we found that $c_1 = 0$, $c_2 = 0$, $c_3 = 0$.
This indicates that the system has only the trivial solution and we can conclude that the vectors u, v and w are linearly independent.