SEM 2 2020/2021 : SECTION 01

COMPUTATIONAL MATHEMATICS (SECI1113-01)

ASSIGNMENT 5

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Question 1

Question 1

birth rate in 1980 Let x_k = birth year

y_k = birth rate

k	x_k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7	0.8	-6.2	3.7	5.2
1	1985	31.5	-5.4	-2.5	8.9	
2	1995	26.1	-7.9	6.4		
3	2005	18.2	-1.5			
4	2015	16.7				

$x = 1980$

Reference value, $x_0 = 1975$

$h = 1985 - 1975 = 10$

$r = (x - x_0)/h = (1980 - 1975)/10 = 0.5$

$$P_4(1980) = 30.7 + (0.5)(0.8) + \frac{0.5(0.5-1)}{2!}(-6.7) + \frac{0.5(0.5-1)(0.5-2)}{3!}(3.7) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(5.2)$$

$$= 30.7 + 0.4 + 0.775 + 0.23125 + (-0.203125)$$

$$= 31.9031$$

$\therefore y(1980) \approx P_4(1980) = 31.9031$

No.: Date:

Question 1

birth rate in 2010

k	x_k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7				
1	1985	31.5	0.8			
2	1995	26.1	-5.4	-6.2		
3	2005	18.2	-7.9	-2.5	3.7	
4	2015	16.7	-1.5	6.4	8.9	5.2

$x = 2010$

Reference point, $x_4 = 2015$

$h = 2015 - 2005 = 10$

$r = (x - x_4)/h = (2010 - 2015)/10$

$= -0.5$

$$P_4(2010) = 16.7 + (-0.5)(-1.5) + \frac{(-0.5)(-0.5+1)}{2!}(6.4) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(8.9) + \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(5.2)$$

$$= 16.7 + 0.75 + (-0.8) + (-0.55625) + (-0.203125)$$

$$= 15.8906$$

$\therefore y(2010) \approx P_4(2010) = 15.8906$

Question 2

Question 2

k	x_k^0	x_k^1	x_k^2	f_k	$x_k^0 f_k$	$x_k^1 f_k$
0	1	1970	3880900	6.0	6.0	11820.0
1	1	1975	3900625	8.6	8.6	16985.0
2	1	1980	3920400	13.6	13.6	26928.0
3	1	1985	3940225	18.7	18.7	37119.5
4	1	1990	3960100	31.6	31.6	62884.0
5	1	1995	3980025	53.2	53.2	106134.0
6	1	2000	4000000	78.8	78.8	157600.0
7	1	2005	4020025	100.0	100.0	200500.0
8	1	2010	4040100	107.1	107.1	215271.0
$\sum_{i=0}^{8}$	9	17910	35642400	-	417.6	835241.5

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 9 & 17910 & 417.6 \\ 17910 & 35642400 & 835241.5 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{9} R_1} \left[\begin{array}{cc|c} 1 & 1990 & 46.4 \\ 17910 & 35642400 & 835241.5 \end{array}\right]$$

$$\downarrow R_2 \to R_2 - 17910 R_1$$

$$\left[\begin{array}{cc|c} 1 & 1990 & 46.4 \\ 0 & 1 & 2.8117 \end{array}\right] \xleftarrow{R_2 \to R_2/1500} \left[\begin{array}{cc|c} 1 & 1990 & 46.4 \\ 0 & 1500 & 4217.5 \end{array}\right]$$

$$\downarrow R_1 \to R_1 - 1990 R_2$$

$$\left[\begin{array}{cc|c} 1 & 0 & -5548.8167 \\ 0 & 1 & 2.8117 \end{array}\right]$$

$a_0 = -5548.8167$

$a_1 = 2.8117$

a) $p(x) = a_0 + a_1 x$

$p(x) = -5548.8167 + 2.8117x$

b) $f(1979) \approx p(1979) = -5548.8167 + 2.8117(1979)$

$= 15.5376$

Question 3

Question 3

$$L_0(3.0) = \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)}$$

$$= \frac{-3.5}{43.75}$$

$$= -0.08$$

$$L_1(3.0) = \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)}$$

$$= \frac{-10.5}{-15}$$

$$= 0.7$$

$$L_2(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)}$$

$$= \frac{5.25}{9.375}$$

$$= 0.56$$

$$L_3(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)}$$

$$= \frac{2.625}{13.125}$$

$$= -0.2$$

$$L_4(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)}$$

$$= \frac{1.5}{75}$$

$$= 0.02$$

$$\sum_{i=0}^{4} L_i(3.0) = L_0(3.0) + L_1(3.0) + L_2(3.0) + L_3(3.0) + L_4(3.0)$$

$$= -0.08 + 0.7 + 0.56 + (-0.2) + 0.02$$

$$= 1.00$$

$$P_4(3.0) = (-0.8)(0.455) + (0.7)(1.615) + (0.56)(3.75) + (-0.2)(5.333) + (0.02)(7.857)$$

$$= -0.0364 + 1.1305 + 2.1 + (-1.0666) + 0.15714$$

$$= 2.28464$$

$$\approx 2.2846$$

$$\therefore y(3.0) \approx P_4(3.0) = 2.2846$$

Question 4

Question 4

$f(x) = 2x^3 + x^2 - 4$, estimate $f'(2.5)$ and $f''(2.5)$

a) central three-point : $h = 0.5$

$$f'(x) \approx \frac{f(x+h) - f(x-h)}{2h}$$

$$f'(2.5) \approx \frac{f(2.5+0.5) - f(2.5-0.5)}{2(0.5)}$$

$$= \frac{f(3.0) - f(2.0)}{1.0}$$

$$= \frac{[2(3.0)^3 + (3.0)^2 - 4] - [2(2.0)^3 + (2.0)^2 - 4]}{1.0}$$

$$= 43$$

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x+h)}{h^2}$$

$$f''(2.5) \approx \frac{f(2.5-0.5) - 2f(2.5) + f(2.5+0.5)}{0.5^2}$$

$$= \frac{f(2.0) - 2f(2.5) + f(3.0)}{0.25}$$

$$= \frac{[2(2.0)^3 + (2.0)^2 - 4] - 2[2(2.5)^3 + (2.5)^2 - 4] + [2(3.0)^3 + (3.0)^2 - 4]}{0.25}$$

$$= \frac{16 - 2(33.5) + 50}{0.25}$$

$$= 32$$

central five-point : $h = 0.5$

$$f'(x) \approx \frac{1}{12h}[f(x-2h) - 8f(x-h) + 8f(x+h) - f(x+2h)]$$

$$f'(2.5) \approx \frac{1}{12(0.5)}[f(2.5-2(0.5)) - 8f(2.5-0.5) + 8f(2.5+0.5) - f(2.5+2(0.5))]$$

$$= \frac{1}{6}[f(1.5) - 8f(2.0) + 8f(3.0) - f(3.5)]$$

$$= \frac{1}{6}[((1.5)^3 + (1.5)^2 - 4) - 8((2.0)^3 + (2.0)^2 - 4) + 8((3.0)^3 + (3.0)^2 - 4 - ((3.5)^3 + (3.5)^2 - 4)]$$

$$= \frac{1}{6}[5 - 8(16) + 8(59) - 94]$$

$$= 42.5$$

$$f''(x) \approx \frac{1}{12h^2}[-f(x-2h) + 16(x-h) - 30f(x) + 16f(x+h) - f(x+2h)]$$

$$f''(2.5) \approx \frac{1}{12(0.5)^2}[-f(2.5-2(0.5)) + 16(2.5-0.5) - 30f(2.5) + 16f(2.5+0.5) - f(2.5+2(0.5))]$$

$$= \frac{1}{3}[-f(1.5) + 16f(2.0) - 30f(2.5) + 16f(3.0) - f(3.5)]$$

$$= \frac{1}{3}[-((1.5)^3 + (1.5)^2 - 4) + 16((2.0)^3 + (2.0)^2 - 4) - 30((2.5)^3 + (2.5)^2 - 4) + 16((3.0)^3 + (3.0)^2 - 4) - ((3.5)^3 + (3.5)^2 - 4)]$$

$$= \frac{1}{3}[-(5 + 16(16) - 30(33.5) + 16(59) - 94)]$$

$$= 32$$

b) $f(x) = 2x^3 + x^2 - 4$

$f'(x) = 3x^2 + 2x$

$f''(x) = 6x + 2$

$f'(x) = 3(2.5)^2 + 2(2.5)$

$= 42.5$

$f''(2.5) = 12(2.5) + 2$

$= 32$

e) central three-point:

$f'(2.5) \approx 43$

$f'(2.5) = 42.5$

$\varepsilon = 42.5 - 43$

$= -0.5$ #

$f''(2.5) \approx 32$

$f''(2.5) = 32$

$\varepsilon = 32 - 32$

$= 0$ #

central five-point:

$f'(2.5) \approx 42.5$

$f'(2.5) = 42.5$

$\varepsilon = 42.5 - 42.5$

$= 0$ #

$f''(2.5) \approx 32$

$f''(2.5) = 32$

$\varepsilon = 32 - 32$

$= 0$ #

Question 5

Question 5

a. $N = 6,\ a = 0,\ b = 3$

$$N = \frac{b-a}{h} = \frac{3-0}{h}$$

$$h = 0.5$$

i	x_i	$f(x_i) = \frac{2}{1+5x^2}$	
0	0.0000	2	
1	0.5000		0.8889
2	1.0000		0.3333
3	1.5000		0.1633
4	2.0000		0.0952
5	2.5000		0.0620
6	3.0000	0.0435	
		2.0435	1.5427

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{0.5}{2}\left[2.0435 + 2(1.5427)\right]$$

$$= 1.2822$$

b. $N = 6,\ a = 0,\ b = 3$

$$N = \frac{b-a}{h} = \frac{3-0}{h}$$

$$h = 0.5$$

i	x_i	$f_i = f(x_i) = \frac{2}{1+5x^2}$		
0	0.0000	2		
1	0.5000		0.8889	
2	1.0000			0.3333
3	1.5000		0.1633	
4	2.0000			0.0952
5	2.5000		0.0620	
6	3.0000	0.0435		
		2.0435	1.1142	0.4285

$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{0.5}{3}\left[(2.0435) + 4(1.1142) + 2(0.4285)\right]$$

$$= 1.2262$$

c. $a = 0$, $b = 3$, $h = 0.25$

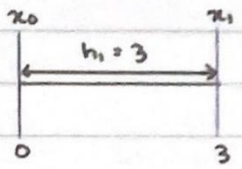
$$N = \frac{b-a}{h} = \frac{3-0}{0.25} = 12$$

i	x_i	$f_i = f(x_i) = \frac{2}{1+5x^2}$		
0	0.0000	2		
1	0.2500		1.5238	
2	0.5000		0.8889	
3	0.7500			0.5246
4	1.0000		0.3333	
5	1.2500		0.2270	
6	1.5000			0.1633
7	1.7500		0.1226	
8	2.0000		0.0952	
9	2.2500			0.0760
10	2.5000		0.0620	
11	2.7500		0.0513	
12	3.0000	0.0435		
		2.0435	3.3043	0.7639

$$\int_0^3 \frac{2}{1+5x^2} = \frac{3(0.25)}{8}\left[2.0435 + 3(3.043) + 2(0.7639)\right]$$

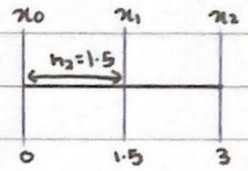
$$= 1.2641$$

d. $h_1 = 3 - 0 = 3$



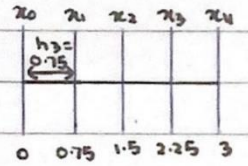
$$R_{1,1} = \frac{h_1}{2}(f_0 + f_1)$$
$$= \frac{3}{2}(2 + 0.0435)$$
$$= 3.0653$$

$$R_{2,1} = \frac{1}{2}[R_{1,1} + h_1(f_1)]$$
$$= \frac{1}{2}[3.0653 + 3(0.1633)]$$
$$= 1.7776$$



$$R_{2,2} = \frac{4R_{2,1} - R_{1,1}}{3}$$
$$= \frac{4(1.7776) - 3.0653}{3}$$
$$= 1.3484$$

$$R_{3,1} = \frac{1}{2}[R_{2,1} + h_2(f(0.75) + f(2.25))]$$
$$= \frac{1}{2}[1.7776 + 1.5(0.5246 + 0.0760)]$$
$$= 1.3393$$



$$R_{3,2} = \frac{4R_{3,1} - R_{2,1}}{3}$$
$$= \frac{4(1.3393) - 1.7776}{3}$$
$$= 1.1932$$

$$R_{3,3} = \frac{16R_{3,2} - R_{2,2}}{15}$$
$$= \frac{16(1.1932) - 1.3484}{15}$$
$$= 1.1829$$

$$\therefore \int_0^3 \frac{2}{1 + 5x^2}\,dx = R_{3,3} = 1.1829$$