SECI 1113 (Computational Mathematics)

SEMESTER 2, 2020/2021

GROUP ASSIGNMENT 4

SECTION: 01

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Assignment 4 SECI 1113-01

Question 1

a) $|E| = 2$

$\alpha = 60$

$E_r = \frac{|E|}{\alpha} = \frac{2}{60} = 0.033$

percentage $= 0.033 \times 100$

$= 3.33\%$ *

b) $f'(x) \approx \frac{f(x+h) - f(x)}{h}$, $h = 0.1$

$f'(x) = x$, $f'(2) = ?$

$f'(2) = \frac{f(2+0.1) - f(2)}{0.1}$

$= \frac{2.1 - 2}{0.1}$

$= 1$

$|E| = 1 - 1$

$= 0$ *

c) exact calculation :

$6.232 + 7.0321 + 5.43 - 3.385 = 15.3091$

modulus error :

$|E_3| \leq 0.0005 + 0.00005 + 0.005 + 0.0005$

$= 0.00605$

$15.3091 + 0.00605 = 15.31515$

$15.3091 - 0.00605 = 15.30305$

∴ The best approximate value is 15.3 (1 d.p.)

Question 2

$f(x) = x^6 - x - 1 \qquad \varepsilon = 0.001$

$a = 1, \quad b = 2$

$f(a) = f(1) = 1^6 - 1 - 1 = -1$

$f(b) = f(2) = 2^6 - 2 - 1 = 61$

$\rightarrow f(1)f(2) < 0, \quad p \in [1, 2]$

$$p_1 = \frac{a+b}{2} = \frac{1+2}{2} = 1.5$$

$f(p_1) = f(1.5) = (1.5)^6 - 1.5 - 1 = 8.891$

$\rightarrow f(1)f(1.5) < 0, \quad p \in [1, 1.5]$

i	a	b	f (a)	f (b)	c	f (c)
1	1.000	2.000	-1.000	61.000	1.500	8.891
2	1.000	1.500	-1.000	8.891	1.250	1.565
3	1.000	1.250	-1.000	1.565	1.125	-0.098
4	1.125	1.250	-0.098	1.565	1.188	0.623
5	1.125	1.188	-0.098	0.623	1.157	0.242
6	1.125	1.157	-0.098	0.242	1.141	0.066
7	1.125	1.141	-0.098	0.066	1.133	-0.018
8	1.133	1.141	-0.018	0.066	1.137	0.024
9	1.133	1.137	-0.018	0.024	1.135	0.003
10	1.133	1.135	-0.018	0.003	1.134	-0.007
11	1.134	1.135	-0.007	0.003	1.135	0.003
12	1.134	1.135	-0.007	0.003	1.135	0.003

$\therefore$ Since $|1.135 - 1.135| = 0.000$

$|p_i - p_{i-1}| < 0.001$

therefore $p = p_{12} = 1.14$

Question 3

a. $x_0 = 0$, $x_1 = -0.1$ $\quad \varepsilon = 0.005$

$f(x_0) = \cos(0) + 2\sin(0) + 0^2$

$= 1.000$

$f(x_1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$

$= 0.8053$

$$x_2 = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

$$= \frac{0(0.8053) - (-0.1)(1)}{0.8053 - 1}$$

$= -0.5136$

$f(-0.5136) = \cos(-0.5136) + 2\sin(-0.5136) + (-0.5136)^2$

$= 0.1521 > \varepsilon$

i	x_i	x_{i+1}	x_{i+2}	$f(x_{i+2})$
0	0.0000	-0.1000	-0.5136	0.1521
1	-0.1000	-0.5136	-0.6100	0.0457
2	-0.5136	-0.6100	-0.6514	0.0065
3	-0.6100	-0.6514	-0.6582	0.0013

$\therefore |f(x_5)| = 0.0013 < \varepsilon = 0.005$,

thus $x = x_5 = -0.6582 = -0.66$ (2 decimal places)

b. $x_0 = -0.1 \qquad \varepsilon = 0.005$

$f(x) = \cos(x) + 2\sin(x) + x^2$

$f'(x) = -\sin(x) + 2\cos(x) + 2x$

$f(x_0) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$

$= 0.8053$

$f'(x_0) = -\sin(-0.1) + 2\cos(-0.1) + 2(-0.1)$

$= 1.8898$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = -0.1 - \frac{0.8053}{1.8898} = -0.5261$$

$f(x_1) = f(-0.5261)$

$= \cos(-0.5261) + 2\sin(-0.5261) + (-0.5261)^2$

$= 0.1372 > \varepsilon$

i	x_i	$f(x_i)$	$f'(x_i)$
0	-0.1000	0.8053	1.8898
1	-0.5261	0.1372	1.1795
2	-0.6424	0.0151	0.9156
3	-0.6589	0.0003	0.8758

$\therefore |f(x_3)| = 0.0003 < \varepsilon = 0.005$,

thus $x = x_3 = -0.6589 = -0.66$

→ both methods produced root, $x = -0.66$.

the final $f(x_i)$ is smaller at Newton method (0.0003) than Secant method (0.0013).

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Question 4

i)

$\frac{1}{x} = 1.37$

$f(x) = 1.37 - \frac{1}{x}$

$f'(x) = -x^{-1}$

$= (-1) - x^{-2}$

$= x^{-2}$

$= \frac{1}{x^2}$

$X_{n+1} = X_n - \frac{f(X_n)}{f'(X_n)}$

$= X_n - \left(\left(1.37 - \frac{1}{X_n}\right) \div \frac{1}{X_n^2}\right)$

$= X_n - \left(\left(\frac{1.37X_n - 1}{\cancel{X_n}} \times X_n^{2^1}\right)\right)$

$= X_n - \left(1.37X_n^2 - X_n\right)$

$= X_n - 1.37X_n^2 + X_n$

$= 2X_n - 1.37X_n^2$

$= X_n(2 - 1.37X_n)$

$= X_n(2 - \alpha X_n)$, where $\alpha = 1.37$

$\therefore$ Proven

ii) $X_0 = 0.75$

$X_1 = 0.75(2 - 1.37(0.75))$

$= 0.729375000$

$X_2 = 0.729375000(2 - 1.37(0.729375000))$

$= 0.729926590$

$X_3 = 0.729926590(2 - 1.37(0.729926590))$

$= 0.729927007$

$X_4 = 0.729927007(2 - 1.37(0.729927007))$

$= 0.729927007$

$|X_4 - X_3| = 0.000000000 < 0.000000005$

$\therefore X = X_4 = 0.729927007 \approx 0.72992701$

Question 5

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$S - \lambda I = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$$C_1 = C_1 - (\lambda - 1) C_2$$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ \lambda^2 - 3\lambda + 1 & 2-\lambda & -1 \\ \lambda - 1 & -1 & 1-\lambda \end{bmatrix}$$

$$\det(S - \lambda I) = (0)(-1)^{1+1} \begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} + (-1)(-1)^{1+2} \begin{vmatrix} \lambda^2-3\lambda+1 & -1 \\ \lambda-1 & 1-\lambda \end{vmatrix}$$

$$+ (0)(-1)^{1+3} \begin{vmatrix} \lambda^2-3\lambda+1 & 2-\lambda \\ \lambda-1 & -1 \end{vmatrix}$$

$$= \begin{vmatrix} \lambda^2-3\lambda+1 & -1 \\ \lambda-1 & 1-\lambda \end{vmatrix}$$

$$= (\lambda^2 - 3\lambda + 1)(1-\lambda) - (-1)(\lambda - 1)$$

$$= -\lambda^3 + 4\lambda^2 - 3\lambda$$

$$= \lambda(-\lambda^2 + 4\lambda - 3)$$

roots : $\lambda_1 = 3$, $\lambda_2 = 1$, $\lambda_3 = 0$

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when $\lambda = 3$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 1-3 & -1 & 0 \\ -1 & 2-3 & -1 \\ 0 & -1 & 1-3 \end{bmatrix} = \begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} -2 & -1 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_1 \to \frac{1}{2}R_2} \begin{bmatrix} 1 & 1/2 & 0 \\ -1 & -1 & -1 \\ 0 & -1 & -2 \end{bmatrix} \xrightarrow{R_2 \to R_2 + R_1} \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & -1/2 & -1 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\downarrow R_2 \to -2R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix} \xleftarrow{R_1 \to R_1 - \frac{1}{2}R_2} \begin{bmatrix} 1 & 1/2 & 0 \\ 0 & 1 & 2 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = t$

$x_2 = -2t$

$x_3 = t$

$\therefore$ null space = eigenvector

$$= \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

when $\lambda = 1$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 1-1 & -1 & 0 \\ -1 & 2-1 & -1 \\ 0 & -1 & 1-1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0 & -1 & 0 \\ -1 & 1 & -1 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} -1 & 1 & -1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \xrightarrow{R_1 \to -R_1} \begin{bmatrix} 1 & -1 & 1 \\ 0 & -1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\downarrow R_2 \to -R_2$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \xleftarrow{R_3 \to R_3 + R_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix} \xleftarrow{R_1 \to R_1 + R_2} \begin{bmatrix} 1 & -1 & 1 \\ 0 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = -t$

$x_2 = 0$

$x_3 = t$

$\therefore$ null space = eigen vector

$$= \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

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when $\lambda = 0$

$$\begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix} = \begin{bmatrix} 1-0 & -1 & 0 \\ -1 & 2-0 & -1 \\ 0 & -1 & 1-0 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 + R_1} \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix} \xrightarrow{R_1 = R_1 + R_2} \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$\downarrow R_3 = R_3 + R_2$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$x_1 = t$, $x_2 = t$, $x_3 = t$

$\therefore$ null space = eigenvector = $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

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Question 6

a) $B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$

ROW 1

$|\lambda - (-18)| < |20| + |-6|$

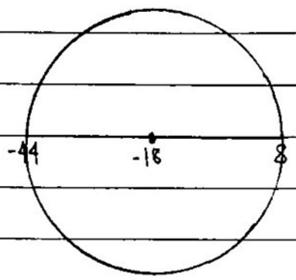
$|\lambda + 18| < 26$

$|\lambda + 18| > -26 \quad , \quad |\lambda + 18| < 26$

$\lambda > -44 \quad , \quad \lambda < 8$

$-44 < \lambda < 8$

$\therefore \lambda \in (-44, 8)$



ROW 2

$|\lambda - 14| < |-12| + |-3|$

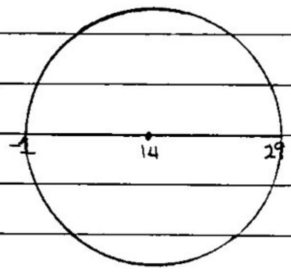
$|\lambda - 14| < 15$

$|\lambda - 14| > -15 \quad , \quad |\lambda - 14| < 15$

$\lambda > -1 \quad , \quad \lambda < 29$

$-1 < \lambda < 29$

$\therefore \lambda \in (-1, 29)$



ROW 3

$|\lambda - 13| < |30| + |-30|$

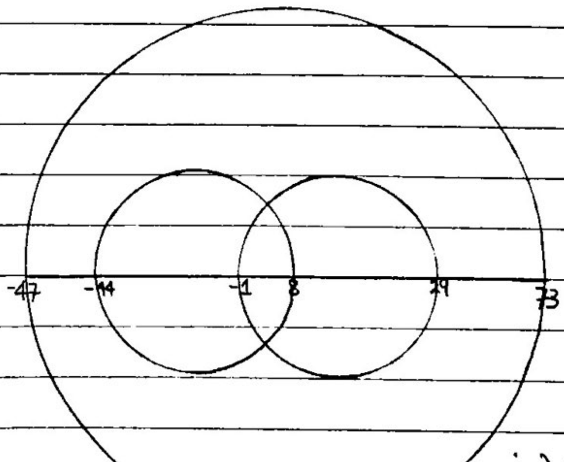
$|\lambda - 13| < 60$

$|\lambda - 13| > -60 \quad , \quad |\lambda - 13| < 60$

$\lambda > -47 \quad , \quad \lambda < 73$

$-47 < \lambda < 73$

$\therefore \lambda \in (-47, 73)$



$\therefore \lambda \in (-47, 73)$

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(b) $v = [0, 0, 1]^T$, $\varepsilon = 0.05$

$$v = Bv^{(0)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} \quad m_1 = 13$$

$$v^{(1)} = v/m_1 = \frac{1}{13} \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} = \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1.0000 \end{bmatrix}$$

$$v = Bv^{(1)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1.0000 \end{bmatrix} = \begin{bmatrix} -2.3090 \\ -0.6932 \\ 6.0790 \end{bmatrix} \quad m_2 = 6.0790$$

$$v^{(2)} = \frac{1}{6.0790} \begin{bmatrix} -2.3090 \\ -0.6932 \\ 6.0790 \end{bmatrix} = \begin{bmatrix} -0.3798 \\ -0.1140 \\ 1.0000 \end{bmatrix}$$

$$Bv^{(2)} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.3798 \\ -0.1140 \\ 1.0000 \end{bmatrix} = \begin{bmatrix} -1.4436 \\ -0.0384 \\ 5.0260 \end{bmatrix} \quad m_3 = 5.0260$$

$$v^{(3)} = \frac{1}{5.0260} \begin{bmatrix} -1.4436 \\ -0.0384 \\ 5.0260 \end{bmatrix} = \begin{bmatrix} -0.2872 \\ -0.0076 \\ 1.0000 \end{bmatrix}$$

k	$(v^{(k)})^T$			$(Bv^{(k)})^T$			m_{k+1}
0	0.0000	0.0000	1.0000	-6.0000	-3.0000	13.0000	13.0000
1	-0.4615	-0.2308	1.0000	-2.3090	-0.6932	6.0790	6.0790
2	-0.3798	-0.1140	1.0000	-1.4436	-0.0384	5.0260	5.0260
3	-0.2872	-0.0076	1.0000	-0.9824	0.3400	4.6120	4.6120
4	-0.2130	0.0737	1.0000	-0.6920	0.5878	4.3990	4.3990
5	-0.1573	0.1336	1.0000	-0.4966	0.7580	4.2730	4.2730
6	-0.1162	0.1774	1.0000	-0.3604	0.8780	4.1920	4.1920
7	-0.0860	0.2094	1.0000				

$|V^{(7)} - V^{(6)}|$

$$\left\| \begin{bmatrix} -0.0860 \\ 0.2904 \\ 1.0000 \end{bmatrix} - \begin{bmatrix} -0.1167 \\ 0.1774 \\ 1.0000 \end{bmatrix} \right\| = \begin{bmatrix} 0.0302 \\ 0.0320 \\ 0.0000 \end{bmatrix} < \varepsilon \ (0.05)$$

$\lambda_1 \approx m_7 = 4.1920 \approx 4$

$$V_1 \approx V^{(7)} = \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1.0000 \end{bmatrix}$$

$Bv_1 = \lambda_1 v_1$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1.0000 \end{bmatrix} = 4 \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1.0000 \end{bmatrix}$$

$$\begin{bmatrix} -0.2640 \\ 0.9636 \\ 4.1380 \end{bmatrix} = \begin{bmatrix} -0.3440 \\ 0.8376 \\ 4.0000 \end{bmatrix} \approx \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$$

$\therefore \lambda_1 \approx m_7 = 4.1920 \approx 4$

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c) Let $A = B_{shifted}$ $V = [0 \ 0 \ 1]^T$, $\varepsilon = 0.05$

$A = B - \lambda_1 I$

$= B - 4I$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}$$

$$V = Av^{(0)} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \quad m_1 = 9$$

$$V^{(1)} = \frac{1}{9}\begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} = \begin{bmatrix} -0.6667 \\ -0.3333 \\ 1.0000 \end{bmatrix}$$

$$Av^{(1)} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} -0.6667 \\ -0.3333 \\ 1.0000 \end{bmatrix} = \begin{bmatrix} 2.0014 \\ 1.6674 \\ -1.0020 \end{bmatrix} \quad m_2 = 2.0014$$

$$V^{(2)} = \frac{1}{2.0014}\begin{bmatrix} 2.0014 \\ 1.6674 \\ -1.0020 \end{bmatrix} = \begin{bmatrix} 1.0000 \\ 0.8331 \\ 0.5006 \end{bmatrix} \quad Av^{(2)} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} 1.0000 \\ 0.8331 \\ 0.5006 \end{bmatrix} = \begin{bmatrix} -2.3344 \\ -2.1672 \\ 0.5016 \end{bmatrix}$$

$m_3 = -2.3344$

k	$(v^{(k)})^T$			$(Av^{(k)})^T$			m_{k+1}
0	0.0000	0.0000	1.0000	-6.0000	-3.0000	9.0000	9.0000
1	-0.6667	-0.3333	1.0000	2.0014	1.6674	-1.0020	2.0014
2	1.0000	0.8331	0.5006	-2.3344	-2.1672	0.5016	-2.3344
3	1.0000	0.9284	-0.2149	-2.1426	-2.0713	0.2139	-2.1426
4	1.0000	0.9667	-0.0998	-2.0672	-2.0336	0.1008	-2.0672
5	1.0000	0.9837	-0.0488	-2.0332	-2.0166	0.0498	-2.0332
6	1.0000	0.9918	-0.0245				

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$$\left| V^{(6)} - V^{(5)} \right| =$$

$$\left\| \begin{bmatrix} 1.0000 \\ 0.9918 \\ -0.0245 \end{bmatrix} - \begin{bmatrix} 1.0000 \\ 0.9837 \\ -0.0488 \end{bmatrix} \right\| = \begin{bmatrix} 0.0000 \\ 0.0081 \\ 0.0243 \end{bmatrix} < \varepsilon \; (0.05)$$

$$\lambda_{shifted} = m_6 = -2.0332 \approx -2$$

$$\lambda = \lambda_{shifted} + \lambda_1$$

$$= -2 + 4$$

$$= 2$$

$$V = V^{(6)} = \begin{bmatrix} 1.0000 \\ 0.9918 \\ -0.0245 \end{bmatrix}$$

$$Av = \lambda v$$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} \begin{bmatrix} 1.0000 \\ 0.9918 \\ -0.0245 \end{bmatrix} = 2 \begin{bmatrix} 1.0000 \\ 0.9918 \\ -0.0245 \end{bmatrix}$$

$$\begin{bmatrix} 1.9830 \\ 1.9587 \\ -0.0725 \end{bmatrix} = \begin{bmatrix} 2.0000 \\ 1.9836 \\ -0.049 \end{bmatrix} \approx \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$$

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d) Since the opposite extreme eigenvalue $\lambda = 2$ has the same sign as the largest magnitude eigenvalue of the original matrix $\lambda_1 = 4$, all eigenvalues of B are positive and $\lambda_3 = 2$ is the smallest eigenvalue of matrix B.

Let $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$

$\lambda_1 = 4$

$\lambda_3 = 2$

$$\sum_{i=1}^{3} \lambda_i = \sum_{i=1}^{3} a_{ii}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{22} + a_{33} = -18 + 14 + 13 = 9$$

$$4 + \lambda_2 + 2 = 9$$

$$6 + \lambda_2 = 9$$

$$\lambda_2 = 3$$

$\therefore \lambda_1 = 4, \lambda_2 = 3, \lambda_3 = 2$