SEM 2 2020/2021 : SECTION 01

COMPUTATIONAL MATHEMATICS (SECI1113-01)

ASSIGNMENT 3

GROUP: T3

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Question 1

Question 1

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 2x - y \\ x + 3y \end{bmatrix}$$

$$T(u+v) = T(u) + T(v)$$

$$T\left(\begin{bmatrix} x_1 + x_2 \\ y_1 + y_2 \end{bmatrix}\right) = \begin{bmatrix} 2(x_1 + x_2) - (y_1 + y_2) \\ (x_1 + x_2) - 3(y_1 + y_2) \end{bmatrix}$$

$$= \begin{pmatrix} 2x_1 - y_1 \\ x_1 + 3y_1 \end{pmatrix} + \begin{pmatrix} 2x_2 - y_2 \\ x_2 + 3y_2 \end{pmatrix}$$

$$= T\begin{pmatrix} x_1 \\ y_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$

$$T(cu) = cT(u)$$

$$T\left(c\begin{bmatrix} x \\ y \end{bmatrix}\right) = T\left(\begin{bmatrix} cx \\ cy \end{bmatrix}\right)$$

$$= \begin{bmatrix} 2cx - cy \\ cx + 3cy \end{bmatrix}$$

$$= cT\begin{bmatrix} x \\ y \end{bmatrix}$$

$\therefore$ T is a linear transformation

Question 2

Question 2

$$T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+y-z \\ 2xy \\ x+z+1 \end{bmatrix}$$

$T(u+v) = T(u) + T(v)$

$$T\left(\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}\right) = \begin{bmatrix} (x_1+x_2)+(y_1+y_2)-(z_1+z_2) \\ 2(x_1+x_2)(y_1+y_2) \\ (x_1+x_2)+(z_1+z_2)+1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+y_1-z_1 \\ 2x_1y_1 \\ x_1+z_1+1 \end{bmatrix} + \begin{bmatrix} x_2+y_2-z_1 \\ 2x_1+y_1 \\ x_2+z_2 \end{bmatrix}$$

$$\neq T\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} + T\begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}$$

$T(cu) = cT(u)$

$$T\left(c\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = T\left(\begin{bmatrix} cx \\ cy \\ cz \end{bmatrix}\right)$$

$$= \begin{bmatrix} cx+cy-cz \\ 2cxy \\ cx+cz+c \end{bmatrix}$$

$$= cT\begin{bmatrix} x+y+z \\ 2xy \\ x+z+1 \end{bmatrix}$$

$\therefore$ T is not a linear transformation between vector space as it is not satisfied addition property.

Question 3

Question 3

a. Let $e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $e_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$$T(e_1) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

$$T(e_2) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

$$T(e_3) = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 2 \end{bmatrix}$$

b. $T(3e_1 - 4e_2 + 6e_3)$

$= 3T(e_1) - 4T(e_2) + 6T(e_3)$

$$= 3\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} - 4\begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 3 \\ 2 \\ 2 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -8 & +18 \\ 6 & -4 & +18 \\ 3 & -12 & +12 \end{bmatrix}$$

$$= \begin{bmatrix} 13 \\ 20 \\ 3 \end{bmatrix}$$

Question 4

Question 4

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} x - 2y \\ -2x + 4y \end{bmatrix}$$

$$\begin{bmatrix} 1 & -2 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ Vector V is not in N(T)

$$\begin{bmatrix} 1 - 6 \\ -2 + 12 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} -5 \\ 10 \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

Question 5

Question 5

a)

$$C_1 T(V_1) + C_2 T(V_2) + C_3 T(V_3) = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$C_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + C_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] R_1 <-> R_3 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 2 & 5 & -1 & 1 \\ -1 & 0 & -1 & -2 \end{array}\right] \begin{array}{l} R_2 \to R_2 + m_{21} R_1 \\ m_{21} = \frac{-2}{1} = -2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ -1 & 0 & -1 & -2 \end{array}\right]$$

$$R_3 \to R_3 + R_1 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ 0 & 0 & 1 & 0 \end{array}\right] R_2 \to \tfrac{1}{5} R_2 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & \frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \to R_1 + m_{13} R_3 \\ m_{13} = \frac{-2}{1} = -2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & \frac{3}{5} \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore C_1 = 2$, $C_2 = -\frac{3}{5}$, $C_3 = 0$

$\therefore$ Thus, W is in R(T)

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b)

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} R_1 <-> R_3 \begin{bmatrix} 1 & 0 & 2 \\ 2 & 5 & -1 \\ -1 & 0 & -1 \end{bmatrix} \begin{array}{l} R_2 \to R_2 + m_{21} R_1 \\ m_{21} = \frac{-2}{1} = -2 \end{array} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -5 \\ -1 & 0 & -1 \end{bmatrix}$$

$$R_3 \to R_3 + R_1 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & 1 \end{bmatrix} R_2 \to \tfrac{1}{5} R_2 \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{l} R_1 \to R_1 + m_{13} R_3 \\ m_{13} = \frac{-2}{1} = -2 \end{array} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2 \to R_2 + R_3 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Since the leading 1s are in column 1, 2 and 3, thus, the basis for R(T) is

$$R(T) = \text{span} \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

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c)

$$C_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + C_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] R_1 \leftrightarrow R_3 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 5 & -1 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 + m_{21}R_1 \\ m_{21} = \frac{-2}{1} = -2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 5 & -5 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right]$$

$$R_3 \to R_3 + R_1 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] R_2 \to \frac{1}{5}R_2 \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \begin{array}{l} R_1 \to R_1 + m_{13}R_3 \\ m_{13} = \frac{-2}{1} = -2 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$R_2 \to R_2 + R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$\therefore$ $C_1 = 0$, $C_2 = 0$, $C_3 = 0$

$\therefore$ Since all choices of C_1, C_2, C_3 have unique solution

$\therefore$

Hence, $N(T) = \text{span}\left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

$\therefore \dim(N(T)) = 0$

Question 6

Question 6

a. $T\left(\begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1+1 \\ 2(1)-(-1) \\ 1+1 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$

$T\left(\begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} -1+1 \\ 2(-1)-(-1) \\ -1+1 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$

$T\left(\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}\right) = \begin{bmatrix} 0+1 \\ 2(1)-0 \\ 1+1 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

$$a_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + a_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \\ 2 \end{bmatrix}$$

$-a_3 = 3$ $\quad$ $a_2 - a_3 = 0$ $\quad$ $a_1 - a_2 = 2$

$a_3 = -3$ $\quad$ $a_2 - (-3) = 0$ $\quad$ $a_1 - (-3) = 2$

$a_2 = -3$ $\quad$ $a_1 = -1$

$$b_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + b_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + b_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$$

$-b_3 = -1$ $\quad$ $b_2 - b_3 = 0$ $\quad$ $b_1 - b_2 = 0$

$b_3 = 1$ $\quad$ $b_2 - 1 = 0$ $\quad$ $b_1 - 1 = 0$

$b_2 = 1$ $\quad$ $b_1 = 1$

$$c_1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$$

$-c_3 = 2$ $\quad$ $c_2 - c_3 = 1$ $\quad$ $c_1 - c_2 = 2$

$c_3 = -2$ $\quad$ $c_2 - (-2) = 1$ $\quad$ $c_1 - (-1) = 2$

$c_2 = -1$ $\quad$ $c_1 = 1$

$$[T]_{B'}^{B} = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 1 & -1 \\ -3 & 1 & -2 \end{bmatrix}$$

b. $$T\left(\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}\right) = \begin{bmatrix} -2+3 \\ 2(1)-(-2) \\ 1+3 \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$

$$a_1 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} + a_2 \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix} + a_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}$$

$-a_1 - a_2 = -2$ — ①	③ − ②	a_2 into ①	a_2, a_1 into ②
$a_1 - a_2 + a_3 = 1$ — ②	$2a_2 = 2$	$-a_1 - 1 = -2$	$1 - 1 + a_3 = 1$
$a_1 + a_2 + a_3 = 3$ — ③	$a_2 = 1$	$a_1 = 1$	$a_3 = 1$

$$\begin{bmatrix} -2 \\ 1 \\ 3 \end{bmatrix}_B = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$[T(v)]_{B'} = \begin{bmatrix} -1 & 1 & 1 \\ -3 & 1 & -1 \\ -3 & 1 & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} (-1+1-1) \\ (-3+1-1) \\ (-3+1-2) \end{bmatrix} = \begin{bmatrix} 1 \\ -3 \\ -4 \end{bmatrix}$$

$$T(v) = 1 \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix} - 4 \begin{bmatrix} -1 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} (0-3+4) \\ (0-0+4) \\ (1+3-0) \end{bmatrix} = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix}$$