SEM 2 2020/2021 : SECTION 01

COMPUTATIONAL MATHEMATICS (SECI1113-01)

ASSIGNMENT 2

GROUP: T3

NAME:

ERICA DESIRAE MAURITUS (A20EC0032)

HUDA NAJIHAH BINTI AHMAD ASRI (A20EC0045)

SAYANG ELYIANA AMIERA BINTI HELMEY (A20EC0143)

LECTURER:

DR RAZANA ALWEE

Question 1

Axiom 3 : $(u+v)+w = u+(v+w)$

$$\left(\begin{bmatrix}1+t_1\\2-t_1\\3+2t_1\end{bmatrix}+\begin{bmatrix}1+t_2\\2-t_2\\3+2t_2\end{bmatrix}\right)+\begin{bmatrix}1+t_3\\2-t_3\\3+2t_3\end{bmatrix}$$

$$=\begin{bmatrix}1+(t_1+t_2)\\2-(t_1+t_2)\\3+(2t_1+2t_2)\end{bmatrix}+\begin{bmatrix}1+t_3\\2-t_3\\3+2t_3\end{bmatrix}$$

$$=\begin{bmatrix}1+(t_1+t_2)+1+t_3\\2-(t_1+t_2)+2+t_3\\3+(2t_1+2t_2)+3+2t_3\end{bmatrix}$$

$$=\begin{bmatrix}1+t_1+1+(t_2+t_3)\\2-t_1+2-(t_2+t_3)\\3+2t_1+3+(2t_2+2t_3)\end{bmatrix}$$

$$=\begin{bmatrix}1+t_1\\2-t_1\\3+2t_1\end{bmatrix}+\begin{bmatrix}1+(t_2+t_3)\\2-(t_2+t_3)\\3+(2t_2+2t_3)\end{bmatrix}$$

$$=\begin{bmatrix}1+t_1\\2-t_1\\3+2t_1\end{bmatrix}+\left(\begin{bmatrix}1+t_2\\2-t_2\\3+2t_2\end{bmatrix}+\begin{bmatrix}1+t_3\\2-t_3\\3+2t_3\end{bmatrix}\right)$$

$\therefore$ The vector is satisfied axiom 3.

Axiom 7: $k(u+v) = ku + kv$

$$c\left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}\right)$$

$$= c\begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+c(t_1+t_2) \\ 2-c(t_1+t_2) \\ 3+c(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct_1+ct_2) \\ 2-(ct_1+ct_2) \\ 3+(2ct_1+2ct_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{bmatrix} + \begin{bmatrix} 1+ct_2 \\ 2-ct_2 \\ 3+2ct_2 \end{bmatrix}$$

$$= c\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + c\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$$

$\therefore$ This vector is satisfied axiom 7

Axiom 8 : $(k+m)u = ku + mu$

$$(c+m)\begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} = \begin{bmatrix} 1+(c+m)t \\ 2-(c+m)t \\ 3+(c+m)2t \end{bmatrix}$$

$$c\begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} + m\begin{bmatrix} 1+t \\ 2-t \\ 3+2t \end{bmatrix} = \begin{bmatrix} 1+ct \\ 2-ct \\ 3+2ct \end{bmatrix} + \begin{bmatrix} 1+mt \\ 2-mt \\ 3+2mt \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct+mt) \\ 2-(ct+mt) \\ 3+(2ct+2mt) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(c+m)t \\ 2-(c+m)t \\ 3+(c+m)2t \end{bmatrix}$$

∴ This vector is satisfied axiom 8

∴ V is a vector space

Question 2

a. let $w_1 = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix}$ and $w_2 = \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 + c_2 & a_2 - c_2 \end{bmatrix}$

$$w_1 + w_2 = \begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix} + \begin{bmatrix} a_2 - b_2 & a_2 \\ b_2 + c_2 & a_2 - c_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1 - b_1) + (a_2 - b_2) & a_1 + a_2 \\ (b_1 + c_1) + (b_2 + c_2) & (a_1 - c_1) + (a_2 - c_2) \end{bmatrix}$$

$$= \begin{bmatrix} (a_1 + a_2) - (b_1 + b_2) & a_1 + a_2 \\ (b_1 + b_2) + (c_1 + c_2) & (a_1 + a_2) - (c_1 + c_2) \end{bmatrix}$$

let $a = a_1 + a_2$, $b = b_1 + b_2$, $c = c_1 + c_2$

$$= \begin{bmatrix} a - b & a \\ b + c & a - c \end{bmatrix} \in S$$

$\therefore$ S is closed under addition

$$cw_1 = c\begin{bmatrix} a_1 - b_1 & a_1 \\ b_1 + c_1 & a_1 - c_1 \end{bmatrix}$$

$$= \begin{bmatrix} ca_1 - cb_1 & ca_1 \\ cb_1 - cc_1 & ca_1 - cc_1 \end{bmatrix}$$

let $a = ca_1$, $b = cb_1$, $c = cc_1$

$$= \begin{bmatrix} a - b & a \\ b + c & a - c \end{bmatrix} \in S$$

$\therefore$ S is closed under multiplication

$\therefore$ Thus, S is a subspace of $M_{2\times 2}$

b. $\begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ $\begin{bmatrix} a - b & a \\ b + c & a - c \end{bmatrix}$

$a - b = 5$ —(1)

$a = 3$ —(2)

$b + c = -2$ —(3)

$a - c = 3$ —(4)

let (2) in (1)

$3 - b = 5$

$-b = 2$

$b = -2$

let (2) in (4)

$3 - c = 3$

$-c = 0$

$c = 0$

(1) $a - b = 5$

$3 - (-2) = 5$

$5 = 5$

(2) $a = 3$

$3 = 3$

(3) $b + c = -2$

$-2 + 0 = -2$

$-2 = -2$

(4) $a - c = 3$

$3 - 0 = 3$

$3 = 3$

$\therefore \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S

Question 3

a)

$$C_1\begin{bmatrix}1\\2\\3\end{bmatrix}+C_2\begin{bmatrix}-1\\3\\-1\end{bmatrix}+C_3\begin{bmatrix}1\\2\\-1\end{bmatrix}+C_4\begin{bmatrix}0\\6\\1\end{bmatrix}+C_5\begin{bmatrix}-3\\4\\5\end{bmatrix}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&x_1\\2&3&2&6&4&x_2\\3&-1&-1&1&5&x_3\end{array}\right]\begin{array}{c}R_2\to R_2+m_{21}R_1\\m_{21}=-\frac{2}{1}=-2\end{array}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&x_1\\0&5&0&6&10&-2x_1+x_2\\3&-1&-1&1&5&x_3\end{array}\right]$$

$$\begin{array}{c}R_3\to R_3+m_{31}R_1\\m_{31}=\frac{-3}{1}=-3\end{array}\left[\begin{array}{ccccc|c}1&-1&1&0&-3&x_1\\0&5&0&6&10&-2x_1+x_2\\0&2&-4&1&14&-3x_1+x_3\end{array}\right]R_2\to\frac{1}{5}R_2\left[\begin{array}{ccccc|c}1&-1&1&0&-3&x_1\\0&1&0&\frac{6}{5}&2&\frac{-2x_1+x_2}{5}\\0&2&-4&1.&14&-3x_1+x_3\end{array}\right]$$

$$R_1\to R_1+R_2\left[\begin{array}{ccccc|c}1&0&1&\frac{6}{5}&-1&x_1+\frac{-2x_1+x_2}{2}\\0&1&0&\frac{6}{5}&2&\frac{-2x_1+x_2}{5}\\0&2&-4&1&14&-3x_1+x_3\end{array}\right]\begin{array}{c}R_3\to R_3+m_{32}R_2\\m_{32}=\frac{-2}{1}=-2\end{array}\left[\begin{array}{ccccc|c}1&0&1&\frac{6}{5}&-1&x_1+\frac{-2x_1+x_2}{5}\\0&1&0&\frac{6}{5}&2&\frac{-2x_1+x_2}{5}\\0&0&-4&-\frac{7}{5}&10&-\frac{11}{5}x_1-\frac{2}{5}x_2+x_3\end{array}\right]$$

$$\left(-4C_3-\frac{7}{5}C_4+10C_5=-\frac{11}{5}x_1-\frac{2}{5}x_2+x_3\right)\times 5$$

$$-20C_3-7C_4+50C_5=-11x_1-2x_2+5x_3 \quad — ①$$

$$\left(C_2+\frac{6}{5}C_4+2C_5=\frac{-2x_1+x_2}{5}\right)\times 5$$

$$5C_2+6C_4+10C_5=-2x_1+x_2 \quad — ②$$

$$\left(C_1+C_3+\frac{6}{5}C_4-C_5=x_1+\frac{-2x_1+x_2}{5}\right)\times 5$$

$$5C_1+5C_3+6C_4-5C_5=5x_1-2x_1+x_2$$

$$5C_1+5C_3+6C_4-5C_5=3x_1+x_2 \quad — ③$$

③ − ②, $\left(5C_1-5C_2+5C_3-15C_5=5x_1\right)\div 5$

$$C_1-C_2+C_3-3C_5=x_1 \quad — ④$$

Sub ④ into ①,

$$-20c_3 - 7c_4 + 50c_5 = -11(c_1 - c_2 + c_3 - 3c_5) - 2x_2 + 5x_3$$

$$-20c_3 - 7c_4 + 50c_5 = -11c_1 + 11c_2 - 11c_3 + 33c_5 - 2x_2 + 5x_3$$

$$11c_1 - 11c_2 - 20c_3 + 11c_3 - 7c_4 + 50c_5 - 33c_5 = -2x_2 + 5x_3$$

$$11c_1 - 11c_2 - 9c_3 - 7c_4 + 17c_5 = -2x_2 + 5x_3 \quad \text{— ⑤}$$

Sub ④ into ②,

$$5c_2 + 6c_4 + 10c_5 = -2(c_1 - c_2 + c_3 - 3c_5) + x_2$$

$$5c_2 + 6c_4 + 10c_5 = -2c_1 + 2c_2 - 2c_3 + 6c_5 + x_2$$

$$2c_1 + 5c_2 - 2c_2 + 2c_3 + 6c_4 + 10c_5 - 6c_5 = x_2$$

$$2c_1 + 3c_2 + 2c_3 + 6c_4 + 4c_5 = x_2 \quad \text{— ⑥}$$

Sub ⑥ into ⑤

$$11c_1 - 11c_2 - 9c_3 - 7c_4 + 17c_5 = -2(2c_1 + 3c_2 + 2c_3 + 6c_4 + 4c_5) + 5x_3$$

$$11c_1 - 11c_2 - 9c_3 - 7c_4 + 17c_5 = -4c_1 - 6c_2 - 4c_3 - 12c_4 - 8c_5 + 5x_3$$

$$11c_1 + 4c_1 - 11c_2 + 6c_2 - 9c_3 + 4c_3 - 7c_4 + 12c_4 + 17c_5 + 8c_5 = 5x_3$$

$$15c_1 - 5c_2 - 5c_3 + 5c_4 + 25c_5 = 5x_3$$

$$3c_1 - c_2 - c_3 + c_4 + 5c_5 = x_3$$

∴ This shows that a solution exist for all choice of x_1, x_2, x_3.

Hence, span (S) $= R^3$

b) $$c_1\begin{bmatrix}1\\2\\3\end{bmatrix} + c_2\begin{bmatrix}-1\\3\\-1\end{bmatrix} + c_3\begin{bmatrix}1\\2\\-1\end{bmatrix} + c_4\begin{bmatrix}0\\6\\1\end{bmatrix} + c_5\begin{bmatrix}-3\\4\\5\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1 & -1 & 1 & 0 & -3 & 0\\2 & 3 & 2 & 6 & 4 & 0\\3 & -1 & -1 & 1 & 5 & 0\end{array}\right] \begin{array}{l}R_2 \to R_2 + m_{21}R_1\\ m_{21} = \frac{-2}{1} = -2\end{array} \left[\begin{array}{ccccc|c}1 & -1 & 1 & 0 & -3 & 0\\0 & 5 & 0 & 6 & 10 & 0\\3 & -1 & -1 & 1 & 5 & 0\end{array}\right]$$

$$\begin{array}{l}R_3 \to R_3 + m_{31}R_1\\ m_{21} = -\frac{3}{1} = -3\end{array} \left[\begin{array}{ccccc|c}1 & -1 & 1 & 0 & -3 & 0\\0 & 5 & 0 & 6 & 10 & 0\\0 & 2 & -4 & 1 & 14 & 0\end{array}\right] R_2 \to \frac{1}{5}R_2 \left[\begin{array}{ccccc|c}1 & -1 & 1 & 0 & -3 & 0\\0 & 1 & 0 & \frac{6}{5} & 2 & 0\\0 & 2 & -4 & 1 & 14 & 0\end{array}\right]$$

$$R_1 \to R_1 + R_2 \left[\begin{array}{ccccc|c}1 & 0 & 1 & \frac{6}{5} & -1 & 0\\0 & 1 & 0 & \frac{6}{5} & 2 & 0\\0 & 2 & -4 & 1 & 14 & 0\end{array}\right] \begin{array}{l}R_3 \to R_3 + m_{32}R_2\\ m_{32} = \frac{-2}{1} = -2\end{array} \left[\begin{array}{ccccc|c}1 & 0 & 1 & \frac{6}{5} & -1 & 0\\0 & 1 & 0 & \frac{6}{5} & 2 & 0\\0 & 0 & -4 & -\frac{7}{5} & 10 & 0\end{array}\right]$$

$$R_3 \to -\frac{1}{4}R_3 \left[\begin{array}{ccccc|c}1 & 0 & 1 & \frac{6}{5} & -1 & 0\\0 & 1 & 0 & \frac{6}{5} & 2 & 0\\0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & 0\end{array}\right] R_1 \to R_1 - R_3 \left[\begin{array}{ccccc|c}1 & 0 & 0 & \frac{17}{20} & \frac{3}{2} & 0\\0 & 1 & 0 & \frac{6}{5} & 2 & 0\\0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & 0\end{array}\right]$$

c_4, c_5 are free leading variable

$$c_3 + \frac{7}{20}c_4 - \frac{5}{2}c_5 = 0$$
$$c_3 = -\frac{7}{20}c_4 + \frac{5}{2}c_5$$

$$c_2 + \frac{6}{5}c_4 + 2c_5 = 0$$
$$c_2 = -\frac{6}{5}c_4 - 2c_5$$

$$c_1 + \frac{17}{20}c_4 + \frac{3}{2}c_5 = -\frac{17}{20}c_4 - \frac{3}{2}c_5$$

$\therefore$ Since there is a presence of free leading variable, hence the system has many solution and is linearly dependent.

Thus, S is not a basis of R^3.

Question 4

a. $c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$

$\left[\begin{array}{cc|c}1 & 3 & 1\\1 & -1 & 0\end{array}\right]$ $m_{21} = \frac{-1}{1} = -1$

$R_2 \to R_2 + m_{21}R_1$

$\left[\begin{array}{cc|c}1 & 3 & 1\\0 & -4 & -1\end{array}\right]$

$c_1 + 3c_2 = 1$ — ①

$-4c_2 = -1$

$c_2 = 1/4$ — ②

let ② in ①

$c_1 + 3(1/4) = 1$

$c_1 = 1/4$

$\therefore$ Coordinate of $\begin{bmatrix}1\\0\end{bmatrix}$ relative to B_2 is $\begin{bmatrix}1/4\\1/4\end{bmatrix}$

b. $c_1\begin{bmatrix}1\\-1\end{bmatrix} + c_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix}$

$\left[\begin{array}{cc|c}1 & 0 & 1\\-1 & 2 & 1\end{array}\right]$ $m_{21} = \frac{-1}{-1} = 1$

$R_2 \to R_2 + m_{21}R_1$

$\left[\begin{array}{cc|c}1 & 0 & 1\\0 & 2 & 2\end{array}\right]$

$c_1 = 1$

$2c_2 = 2$

$c_2 = 1$

$d_1\begin{bmatrix}1\\-1\end{bmatrix} + d_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}3\\-1\end{bmatrix}$

$\left[\begin{array}{cc|c}1 & 0 & 3\\-1 & 2 & -1\end{array}\right]$ $m_{21} = \frac{-1}{-1} = 1$

$R_2 \to R_2 + m_{21}R_1$

$\left[\begin{array}{cc|c}1 & 0 & 3\\0 & 2 & 2\end{array}\right]$

$d_1 = 3$

$2d_2 = 2$

$d_2 = 1$

$\therefore$ Transition matrix $= \begin{bmatrix}c_1 & d_1\\c_2 & d_2\end{bmatrix} = \begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}$

c. $[V]_{B_1} = [I]^{B_1}_{B_2}[V]_{B_2}$

$= \begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$

$= \begin{bmatrix}(1\times 2) + (3\times 3)\\(1\times 2) + (1\times 3)\end{bmatrix}$

$= \begin{bmatrix}11\\5\end{bmatrix}$