SEM 2 2020/2021 : SECTION 01

COMPUTATIONAL MATHEMATICS (SECI1113-01)

ASSIGNMENT 1

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Question 1

Checking (1,-1)

$2x-y=3$
$2(1)-(-1)=3$
$2+1=3$
$3=3$

$x+3y=5$
$1+3(-1)=5$
$1-3=5$
$-2\neq 5$

checking (2, 1)

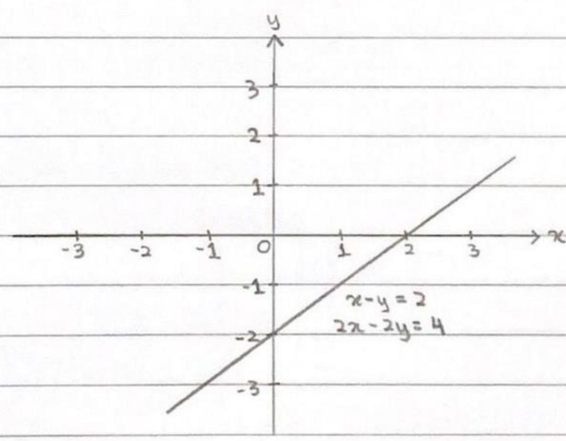
$2x-y=3$
$2(2)-1=3$
$4-1=3$
$3=3$

$x+3y=5$
$2+3(1)=5$
$2+3=5$
$5=5$

$\therefore$ Only (2,1) is a solution to the system

Question 2

a) $x-y=2$ - ①
$2x-2y=4$ - ②
$x=2+y$ - ③
③ into ②
$2(2+y)-2y=4$
$4+2y-2y=4$
$2y-2y=4-4$
$0=0$



$x-y=2$
$\cancel{2}x-\cancel{2}y=\cancel{4}^2$
$x-y=2$

$\therefore$ There are infinite number of solutions and consistent.

b) $x - y = 1$ — ①

$x - y = 3$ — ②

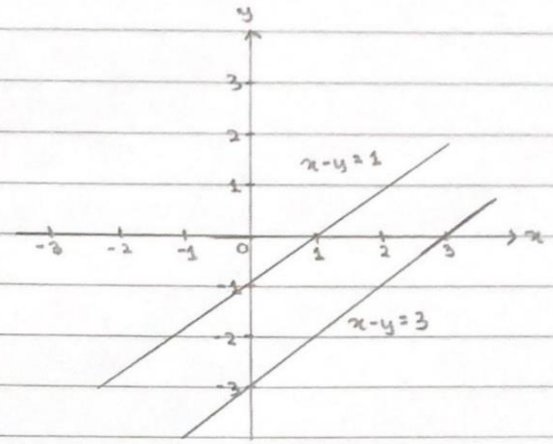
② − ①

$x - y - (x - y) = 3 - 1$

$x - y - x + y = 2$

$x - x - y + y = 2$

$0 = 2$



∴ Two lines are parallel and do not intersect, thus there is no solution and inconsistent.

Question 3

a) $\det A = a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$

$= 1 \det A_{11} - 0 \det A_{12} + 3 \det A_{13}$

$= 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1(1) - 0 + 3(5)$

$= 1 - 0 + 15$

$= 16$

b) $\det B = b_{11}C_{11} + b_{12}C_{12} + b_{13}C_{13}$

$= 1 \det B_{11} - (-1) \det B_{12} + 0 \det B_{13}$

$= 1\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 0\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix}$

$= 1(-1) + 1(1) + 0$

$= -1 + 1 + 0$

$= 0$

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Question 4

a) $A^{-1} = \dfrac{1}{(3)(1)-(0)(0)} \begin{bmatrix} 1 & -0 \\ -0 & 3 \end{bmatrix}$

$= \dfrac{1}{3} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix}$

$= \begin{bmatrix} 1/3 & 0/3 \\ 0/3 & 3/3 \end{bmatrix}$

$= \begin{bmatrix} 1/3 & 0 \\ 0 & 1 \end{bmatrix}$

b) $\det B = 1 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

$= 1(1)$

$= 1$

$$C_B = \begin{bmatrix} \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \\ -\begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} \\ \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} & \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

$$C_B^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$B^{-1} = 1 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

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Question 5

a) $x_1 + 2x_2 - 3x_3 = 9$

$2x_1 - x_2 + x_3 = 0$

$4x_1 - x_2 + x_3 = 4$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & -1 & 1 \\ 4 & -1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right] \begin{array}{l} R_2 \to R_2 + m_{21}R_1 \\ m_{21} = -\frac{2}{1} = -2 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 4 & -1 & 1 & 4 \end{array}\right] \begin{array}{l} R_3 \to R_3 + m_{31}R_1 \\ m_{31} = -\frac{4}{1} = -4 \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & -9 & 13 & -32 \end{array}\right]$$

$$R_2 \to -R_2 \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 5 & -7 & 18 \\ 0 & -9 & 13 & -32 \end{array}\right] \begin{array}{l} R_3 \to R_3 + m_{32}R_2 \\ m_{32} = -\frac{9}{-5} = \frac{9}{5} \end{array} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 5 & -7 & 18 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} \end{array}\right] R_2 \to \frac{5}{2}R_3 \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & 5 & -7 & 18 \\ 0 & 0 & 1 & 1 \end{array}\right]$$

$x_3 = 1$

$5x_2 - 7x_3 = 18$ $\quad\quad\therefore x_1 = 2,\ x_2 = 5,\ x_3 = 1$

$5x_2 - 7(1) = 18$

$5x_2 = 25$

$x_2 = 5$

$x_1 + 2x_2 - 3x_3 = 9$

$x_1 + 2(5) - 3(1) = 9$

$x_1 = 2$

b) $x_1 + x_2 + 2x_3 = 8$

$-x_1 - 2x_2 + 3x_3 = 1$

$3x_1 - 7x_2 + 4x_3 = 10$

$$\begin{bmatrix} 1 & 1 & 1 \\ -1 & -2 & 3 \\ 3 & -7 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 8 \\ 1 \\ 10 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow[m_{21} = \frac{-1}{-1} = 1]{R_2 \to R_2 + m_{21}R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & -1 & 4 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow[m_{31} = -\frac{3}{1} = -3]{R_2 \to R_3 + m_{31}R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & -1 & 4 & 9 \\ 0 & -10 & 1 & -14 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & 1 & -4 & -9 \\ 0 & -10 & 1 & -4 \end{array}\right] \xrightarrow[m_{32} = \frac{-10}{-1} = 10]{R_3 \to R_3 + m_{32}R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 8 \\ 0 & 1 & -4 & -9 \\ 0 & 0 & -39 & -104 \end{array}\right]$$

$-39x_3 = -104$

$x_3 = \frac{-104}{-39}$

$= \frac{8}{3}$

$\therefore\ x_1 = \frac{11}{3},\ x_2 = \frac{5}{3},\ x_3 = \frac{8}{3}$

$x_2 - 4x_3 = -9$

$x_2 - 4\left(\frac{8}{3}\right) = -9$

$x_2 = \frac{5}{3}$

$x_1 + x_2 + x_3 = 8$

$x_1 + \frac{5}{3} + \frac{8}{3} = 8$

$x_1 = \frac{11}{3}$

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Question 6

a) $x_1 - 3x_2 - 2x_3 = 0$

$-x_1 + 2x_2 + x_3 = 0$

$2x_1 + 4x_2 + 6x_3 = 0$

$$\begin{bmatrix} 1 & -3 & -2 \\ -1 & 2 & 1 \\ 2 & 4 & 6 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \begin{array}{l} R_2 \to R_2 + m_{21}R_1 \\ m_{21} = \frac{-1}{-1} = 1 \end{array} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] R_3 \to \tfrac{1}{2}R_3 \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 1 & 2 & 3 & 0 \end{array}\right]$$

$$R_3 \to R_3 - R_1 \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] R_2 \leftrightarrows -R_2 \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right] \begin{array}{l} R_1 \to R_1 + m_{12}R_2 \\ m_{12} = \frac{-3}{-1} = 3 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & -5 & -5 & 0 \end{array}\right]$$

$$\begin{array}{l} R_3 \to R_3 + m_{32}R_2 \\ m_{32} = \frac{-5}{-1} = 5 \end{array} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore x_3$ is a free variable, $x_2 + x_3 = 0$

$x_1 + x_3 = 0$

$\therefore$ Since the final row of row reduced form is equal to 0 Thus, the system has many solutions.

Let $x_3 = b$

$x_2 + b = 0$ $\quad$ $x_1 + b = 0$

$x_2 = -b$ $\quad$ $x_1 = -b$

$\therefore x_1 = -b,\ x_2 = -b,\ x_3 = b$, where $b \in R$

b) $2r + s = 3$

$4r + s = 7$

$2r + 5s = -1$

$$\begin{bmatrix} 2 & 1 \\ 4 & 1 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} r \\ s \end{bmatrix} = \begin{bmatrix} 3 \\ 7 \\ -1 \end{bmatrix}$$

$$\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right] \begin{array}{l} R_2 \to R_2 + m_{21}R_1 \\ m_{21} = \frac{-4}{2} = -2 \end{array} \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 2 & 5 & -1 \end{array}\right] R_3 \to R_3 - R_1 \left[\begin{array}{cc|c} 2 & 1 & 3 \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{array}\right]$$

$$R_1 \to \tfrac{1}{2}R_1 \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 1 \\ 0 & 4 & -4 \end{array}\right] R_2 \to -R_2 \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & 1 & -1 \\ 0 & 4 & -4 \end{array}\right] \begin{array}{l} R_1 \to R_1 + m_{12}R_2 \\ m_{12} = \frac{-\frac{1}{2}}{1} = -\frac{1}{2} \end{array} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & -4 & 4 \end{array}\right]$$

$$\begin{array}{l} R_3 \to R_3 + m_{32}R_2 \\ m_{32} = \frac{-4}{-1} = 4 \end{array} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$r = 2$

$s = -1$

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Question 7

a) $2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ L_{21} & 1 & 0 \\ L_{31} & L_{32} & 1 \end{bmatrix} \begin{bmatrix} U_{11} & U_{12} & U_{13} \\ 0 & U_{22} & U_{23} \\ 0 & 0 & U_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$U_{11} = 2$ | $L_{21}U_{12} + U_{22} = -3$ | $L_{31}U_{11} = 4$ | $L_{31}U_{13} + L_{32}U_{23} + U_{33} = -1$

$U_{12} = 1$ | $(4)(1) + U_{22} = -3$ | $L_{31}(2) = 4$ | $(2)(4) + (-\frac{9}{7})(-14) + U_{33} = -1$

$U_{13} = 4$ | $U_{22} = -7$ | $L_{31} = 2$ | $U_{33} = -27$

$L_{21}U_{11} = 8$ | $L_{21}U_{13} + U_{23} = 2$ | $L_{31}U_{12} + L_{32}U_{22} = 11$

$L_{21}(2) = 8$ | $(4)(4) + U_{23} = 2$ | $(2)(1) + L_{32}(-7) = 11$

$L_{21} = 4$ | $U_{23} = -14$ | $-7L_{32} = 9$

$L_{32} = -\frac{9}{7}$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$\therefore Y = \begin{bmatrix} 12 & -28 & -27 \end{bmatrix}^T$

$Y_1 = 12$ | $4Y_1 + Y_2 = 20$ | $2Y_1 - \frac{9}{7}Y_2 + Y_3 = 33$

$4(12) + Y_2 = 20$ | $2(12) - \frac{9}{7}(-28) + Y_3 = 33$

$Y_2 = -28$ | $Y_3 = -27$

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$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$-27z = -27$
$z = 1$

$-7y - 14z = -28$
$-7y - 14(1) = -28$
$-7y = -14$
$y = 2$

$2x + y + 4z = 12$
$2x + 2 + 4(1) = 12$
$2x = 6$
$x = 3$

$X = \begin{bmatrix} 3 & 2 & 1 \end{bmatrix}^T$

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Question 7

b) $2x + y + 4z = 12$

$8x - 3y + 2z = 20$

$4x + 11y - z = 33$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$$\begin{bmatrix} L_{11} & 0 & 0 \\ L_{21} & L_{22} & 0 \\ L_{31} & L_{32} & L_{33} \end{bmatrix}\begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$L_{11} = 2$

$L_{11}U_{12} = 1$

$(2)U_{12} = 1$

$\therefore U_{12} = \frac{1}{2}$

$L_{11}U_{13} = 4$

$(2)U_{13} = 4$

$U_{13} = 2$

$L_{21} = 8$

$L_{21}U_{12} + L_{22} = -3$

$(8)(\frac{1}{2}) + L_{22} = -3$

$L_{22} = -7$

$L_{21}U_{13} + L_{22}U_{23} = 2$

$(8)(2) + (-7)U_{23} = 2$

$-7U_{23} = -14$

$U_{23} = 2$

$L_{31} = 4$

$L_{31}U_{12} + L_{32} = 11$

$(4)(\frac{1}{2}) + L_{32} = 11$

$L_{32} = 9$

$L_{31}U_{13} + L_{32}U_{23} + L_{33} = -1$

$(4)(2) + (9)(2) + L_{33} = -1$

$L_{33} = -27$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$\therefore Y = [6 \quad 4 \quad 1]^T$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}\begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$2Y_1 = 12$

$Y_1 = 6$

$8Y_1 - 7Y_2 = 20$

$8(6) - 7Y_2 = 20$

$-7Y_2 = -28 \rightarrow Y_2 = 4$

$4Y_1 + 9Y_2 - 27Y_3 = 33$

$4(6) + 9(4) - 27Y_3 = 33$

$-27Y_3 = -27 \rightarrow Y_3 = 1$

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$$\begin{bmatrix} 1 & \frac{1}{2} & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$z = 1$

$y + 2z = 4$

$y + 2(1) = 4$

$y = 2$

$x + \frac{1}{2}y + 2z = 6$

$x + \frac{1}{2}(2) + 2(1) = 6$

$x = 3$

8. $u = (\sqrt{2}, 1, -1)$, $v = (0, 2, 2)$

$$d = \sqrt{(u_1 - v_1)^2 + (u_2 - v_2)^2 + (u_3 - v_3)^2}$$

$$= \sqrt{(\sqrt{2} - 0)^2 + (1-2)^2 + (-1-2)^2}$$

$$= \sqrt{3} _*$$

9. $u = (2, 1, 2)$, $v = (3, 3, 1)$

a) $u \cdot v = u_1 v_1 + u_2 v_2 + u_3 v_3$

$= (2 \times 3) + (1 \times 3) + (2 \times 1)$

$= 6 + 3 + 2$

$= 11 _*$

b) $u \times v = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} k$

$= -5i + 4j + 3k$

$= (-5, 4, 3) _*$

10. a) $v = (1, -2, 5)$

$u_1 = (1, 1, 1)$, $u_2 = (1, 2, 3)$, $u_3 = (2, -1, 1)$

$v = c_1 u_1 + c_2 u_2 + c_3 u_3$

$$\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right]$$

$m_{21} = \frac{-1}{1} = -1$

$R_2 \to R_2 + m_{21} R_1$

$R_2 : 1 + (-1)(1) = 0$

$2 + (-1)(1) = 1$

$-1 + (-1)(2) = -3$

$-2 + (-1)(1) = -3$

$m_{31} = \frac{-1}{1} = -1$

$R_3 \to R_3 + m_{31} R_1$

$R_3 : 1 + (-1)(1) = 0$

$3 + (-1)(1) = 2$

$1 + (-1)(2) = -1$

$5 + (-1)(1) = 4$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right] \quad R_3 \to (-1)R_3 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & -2 & 1 & -4 \end{array}\right]$$

$m_{12} = \frac{-1}{1} = -1$

$R_1 \to R_1 + m_{12} R_2$

$R_1 : 1 + (-1)(0) = 1$

$1 + (-1)(1) = 0$

$2 + (-1)(-3) = 5$

$1 + (-1)(-3) = 4$

$m_{32} = -\frac{(-2)}{1} = 2$

$R_3 \to R_3 + m_{32} R_2$

$R_3 : 0 + (2)(0) = 0$

$-2 + (2)(1) = 0$

$1 + (2)(-3) = -5$

$-4 + (2)(-3) = -10$

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & -5 & -10 \end{array}\right] \quad R_3 \to (-1/5)R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 4 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 1 & 2 \end{array}\right]$$

$m_{13} = -\frac{5}{1} = -5$

$R_1 \to R_1 + m_{13} R_3$

$R_1 : 1 + (-5)(0) = 1$

$0 + (-5)(0) = 0$

$5 + (-5)(1) = 0$

$4 + (-5)(2) = -6$

$m_{23} = -\frac{(-3)}{1} = 3$

$R_2 \to R_2 + m_{23} R_3$

$R_2 : 0 + (3)(0) = 0$

$1 + (3)(0) = 1$

$-3 + (3)(1) = 0$

$-3 + (3)(2) = 3$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & -6 \\ 0 & 1 & 0 & -4 \\ 0 & 0 & 1 & 2 \end{array}\right] \quad \begin{array}{l} c_1 = -6 \\ c_2 = -4 \\ c_3 = 2 \end{array}$$

$$\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = -6\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} - 3\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + 2\begin{bmatrix} 2 \\ 4 \\ 1 \end{bmatrix} \; *$$

∴ Thus v can be express as linear combination of u_1, u_2 and u_3.

10. b) $u = (1, 2, 5)$, $v = (2, 5, 1)$, $w = (1, 5, 2)$

$u(1, 2, 5) + v(2, 5, 1) + w(1, 5, 2) = 0$

$$\begin{array}{l} u + 2v + w = 0 \\ 2u + 5v + 5w = 0 \\ 5u + v + 2w = 0 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 2 & 5 & 5 & 0 \\ 5 & 1 & 2 & 0 \end{array}\right]$$

$m_{21} = \frac{-2}{1} = -2$ $\qquad$ $m_{31} = -\frac{5}{1} = -5$

$R_2 \to R_2 + m_{21}R_1$

R_2 : $2 + (-2)(1) = 0$

$5 + (-2)(2) = 1$

$5 + (-2)(1) = 3$

$0 + (-2)(0) = 0$

$R_3 \to R_3 + m_{31}R_1$

R_3 : $5 + (-5)(1) = 0$

$1 + (-5)(2) = -9$

$2 + (-5)(1) = -3$

$0 + (-5)(0) = 0$

$$\left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & -9 & -3 & 0 \end{array}\right] \quad R_3 \to (-1/3)R_3 \quad \left[\begin{array}{ccc|c} 1 & 2 & 1 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 3 & 1 & 0 \end{array}\right]$$

$m_{12} = -\frac{2}{1} = -2$

$R_1 \to R_1 + m_{12}R_2$

R_1 : $1 + (-2)(0) = 1$

$2 + (-2)(1) = 0$

$1 + (-2)(3) = -5$

$m_{32} = -\frac{3}{1} = -3$

$R_3 \to R_3 + m_{32}R_2$

R_3 : $0 + (-3)(1) = 0$

$3 + (-3)(1) = 0$

$1 + (-3)(3) = -8$

NO:

DATE:

$$\left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 8 & 0 \end{array}\right] \quad R_3 \to (1/8) R_3 \quad \left[\begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$m_{13} = -\frac{5}{1} = -5$

$R_1 \to R_1 + m_{13} R_3$

$R_1 : 1 + (-5)(0) = 1$

$0 + (-5)(0) = 0$

$5 + (-5)(1) = 0$

$m_{23} = -\frac{3}{1} = -3$

$R_2 \to R_2 + m_{23} R_3$

$R_2 : 0 + (-3)(0) = 0$

$1 + (-3)(0) = 1$

$3 + (-3)(1) = 0$

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$c_1 = 0$

$c_2 = 0$

$c_3 = 0$

∴ Thus these three vectors are linearly independent.