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Assignment 5

1. Newton forward - difference formula is used to approximate the birth rate in 1980

Let x = Year

y = Birth rate

k	0	1	2	3	4
x_k	1975	1985	1995	2005	2015
y_k	30.7	31.5	26.1	18.2	16.7

k	x_k	y_k	Δy_k	$\Delta^2 y_k$	$\Delta^3 y_k$	$\Delta^4 y_k$
0	1975	30.7	0.8	-6.2	3.7	5.2
1	1985	31.5	-5.4	-2.5	8.9	
2	1995	26.1	-7.9	6.4		
3	2005	18.2	-1.5			
4	2015	16.7				

$\Delta y_0 = 31.5 - 30.7 = 0.8$ $\qquad \Delta^2 y_0 = -5.4 - 0.8 = -6.2$ $\qquad \Delta^3 y_0 = -2.5 - (-6.2) = 3.7$

$\Delta y_1 = 26.1 - 31.5 = -5.4$ $\qquad \Delta^2 y_1 = -7.9 - (-5.4) = -2.5$ $\qquad \Delta^3 y_1 = 6.4 - (-2.5) = 8.9$

$\Delta y_2 = 18.2 - 26.1 = -7.9$ $\qquad \Delta^2 y_2 = -1.5 - (-7.9) = 6.4$

$\Delta y_3 = 16.7 - 18.2 = -1.5$ $\qquad \Delta^4 y_0 = 8.9 - 3.7 = 5.2$

$x = 1980$ is between 1975 and 1985

Choose $x_0 = 1975$ as reference value

$h = 1985 - 1975 = 10$

$r = \frac{(x - x_0)}{h} = \frac{1980 - 1975}{10} = 0.5$

$$P_4(x) = y_0 + r\Delta y_0 + \frac{r(r-1)}{2!}\Delta^2 y_0 + \frac{r(r-1)(r-2)}{3!}\Delta^3 y_0 + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 y_0$$

$$P_4(1980) = 30.7 + (0.5)(0.8) + \frac{(0.5)(0.5-1)}{2}(-6.2) + \frac{(0.5)(0.5-1)(0.5-2)}{6}(3.7) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{24}(5.2)$$

$= 30.7 + 0.4 + 0.775 + 0.23125 - 0.203125$

$= 31.9031$

≈ 31.9

$\therefore y(1980) \approx P_4(1980) = 31.9$ (1 d.p.)

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Newton backward-difference formula is used to approximate birth rate in 2010

Let x = Year

y = Birth rate

k	x_k	y_k	∇y_k	$\nabla^2 y_k$	$\nabla^3 y_k$	$\nabla^4 y_k$
0	1975	30.7				
1	1985	31.5	0.8			
2	1995	26.1	-5.4	-6.2		
3	2005	18.2	-7.9	-2.5	3.7	
4	2015	16.7	-1.5	6.4	8.9	5.2

$\nabla y_0 = 31.5 - 30.7 = 0.8$

$\nabla y_1 = 26.1 - 31.5 = -5.4$

$\nabla y_2 = 18.2 - 26.1 = -7.9$

$\nabla y_3 = 16.7 - 18.2 = -1.5$

$\nabla^2 y_0 = -5.4 - 0.8 = -6.2$

$\nabla^2 y_1 = -7.9 - (-5.4) = -2.5$

$\nabla^2 y_2 = -1.5 - (-7.9) = 6.4$

$\nabla^3 y_0 = -2.5 - (-6.2) = 3.7$

$\nabla^3 y_1 = 6.4 - (-2.5) = 8.9$

$\nabla^4 y_0 = 8.9 - 3.7 = 5.2$

$x = 2010$ is between 2005 and 2015

Choose $x_4 = 2015$ as reference value

$h = 2015 - 2005 = 10$

$r = \frac{(x - x_4)}{h} = \frac{(2010 - 2015)}{10} = -0.5$

$$P_4(x) = y_4 + r\nabla y_4 + \frac{r(r+1)}{2!}\nabla^2 y_4 + \frac{r(r+1)(r+2)}{3!}\nabla^3 y_4 + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 y_4$$

$$P_4(2010) = 16.7 + (-0.5)(-1.5) + \frac{(-0.5)(0.5)}{2}(6.4) + \frac{(-0.5)(0.5)(1.5)}{6}(8.9) + \frac{(0.5)(0.5)(1.5)(2.5)}{24}(5.2)$$

$$= 16.7 + 0.75 - 0.8 - 0.55625 - 0.203125$$

$$= 15.890625$$

$$\approx 15.9$$

$\therefore y(2010) \approx P_4(2010) = 15.9$ (1 d.p.)

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2.(a) Let x = Year

f = Total Index

k	0	1	2	3	4	5	6	7	8
x_k	1970	1975	1980	1985	1990	1995	2000	2005	2010
f_k	6	8.6	13.6	18.7	31.6	53.2	78.8	100	107.1

$$\begin{bmatrix} S_0 & S_1 \\ S_1 & S_2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$$

where $S_j = \sum_{k=0}^{8} x_k^j$, $j = 0, 1, 2$

$v_l = \sum_{k=0}^{8} x_k^l f_k$, $l = 0, 1$

k	x_k^0	x_k^1	x_k^2	f_k	$x_k^0 f_k$	$x_k^1 f_k$
0	1	1970	3880900	6	6	11820
1	1	1975	3900625	8.6	8.6	16985
2	1	1980	3920400	13.6	13.6	26928
3	1	1985	3940225	18.7	18.7	37119.5
4	1	1990	3960100	31.6	31.6	62884
5	1	1995	3980025	53.2	53.2	106134
6	1	2000	4000000	78.8	78.8	157600
7	1	2005	4020025	100	100	200500
8	1	2010	4040100	107.1	107.1	215271
$\sum_{k=0}^{8}$	9	17910	35642400	-	417.6	835241.5

$x_0^0 = (1970)^0 = 1$ $\quad x_0^1 = (1970)^1 = 1970$ $\quad x_0^2 = (1970)^2 = 3880900$

$x_1^0 = (1975)^0 = 1$ $\quad x_1^1 = (1975)^1 = 1975$ $\quad x_1^2 = (1975)^2 = 3900625$

$x_0^0 f_0 = (1)(6) = 6$ $\quad x_0^1 f_0 = (1970)(6) = 11820$

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Matrix form :

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

$9a_0 + 17910a_1 = 417.6$ $\qquad$ $17910a_0 + 35642400a_1 = 835241.5$ — ②

$$a_0 = \frac{417.6 - 17910a_1}{9} \quad \text{— ①}$$

Sub ① into ②

$$17910\left(\frac{417.6 - 17910a_1}{9}\right) + 35642400a_1 = 835241.5$$

$$831024 - 35640900a_1 + 35642400a_1 = 835241.5$$

$$1500a_1 = 4217.5$$

$$a_1 = 2.8117$$

Sub $a_1 = 2.8117$ into ①

$$a_0 = \frac{417.6 - 17910(2.8117)}{9} = -5548.8830$$

$\therefore a_0 = -5548.8830$, $a_1 = 2.8117$

$\therefore$ Polynomial expression : $p(x) = 2.8117x - 5548.8830$

(b) For $x = 1979$

$$p(1979) = 2.8117(1979) - 5548.8830$$

$$= 15.4713$$

$$\therefore f(1979) \approx p(1979) = 15.4713$$

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3.

k	0	1	2	3	4
x_k	1.5	2.5	4	5	6.5
y_k	0.455	1.615	3.750	5.333	7.857

$$P_4(x) = \sum_{i=0}^{4} L_i(x)\, y_i$$

$$P_4(x) = L_0(x)y_0 + L_1(x)y_1 + L_2(x)y_2 + L_3(x)y_3 + L_4(x)y_4$$

where $L_i(x) = \prod_{\substack{j=0\\ j\neq i}}^{4} \dfrac{(x-x_j)}{(x_i-x_j)}$

$$L_0(3.0) = \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)} = -0.080$$

$$L_1(3.0) = \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)} = 0.700$$

$$L_2(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)} = 0.560$$

$$L_3(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)} = -0.200$$

$$L_4(3.0) = \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)} = 0.020$$

$$\sum_{i=0}^{4} L_i(3.0) = L_0(3.0) + L_1(3.0) + L_2(3.0) + L_3(3.0) + L_4(3.0)$$

$$= -0.080 + 0.700 + 0.560 + (-0.200) + 0.020$$

$$= 1$$

Thus, $P_4(3.0) = \sum_{i=0}^{4} L_i(3.0)\, y_i$

$$= (-0.080)(0.455) + (0.700)(1.615) + (0.560)(3.750) + (-0.200)(5.333)$$

$$+ (0.020)(7.857)$$

$$= 2.2846$$

$\therefore\ y(3.0) \approx P_4(3.0) = 2.2846$ (4 d.p.)

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4. $f(x) = 2x^3 + x^2 - 4$

(a) $x = 2.5$, $h = 0.5$

x	1.0	1.5	2.0	2.5	3.0	3.5
$f(x)$	-1	5	16	33.5	59	94

Central three-point :

$$f'(x_i) \approx \frac{f(x_i+h) - f(x_i-h)}{2h}$$

$$f'(2.5) \approx \frac{f(2.5+0.5) - f(2.5-0.5)}{2(0.5)}$$

$$\approx \frac{f(3) - f(2)}{1}$$

$$\approx 59 - 16$$

$$\approx 43$$

$$f''(x_i) \approx \frac{f(x_i-h) - 2f(x_i) + f(x_i+h)}{h^2}$$

$$f''(2.5) \approx \frac{f(2.5-0.5) - 2f(2.5) + f(2.5+0.5)}{0.5^2}$$

$$\approx \frac{f(2.0) - 2f(2.5) + f(3.0)}{0.25}$$

$$\approx \frac{16 - 2(33.5) + 59}{0.25}$$

$$\approx 32$$

Central five-point :

$$f'(x_i) \approx \frac{1}{12h}\left[f(x_i-2h) - 8f(x_i-h) + 8f(x_i+h) - f(x_i+2h)\right]$$

$$f'(2.5) \approx \frac{1}{12(0.5)}\left[f(2.5-2(0.5)) - 8f(2.5-0.5) + 8f(2.5+0.5) - f(2.5+2(0.5))\right]$$

$$\approx \frac{1}{6}\left[f(1.5) - 8f(2.0) + 8f(3.0) - f(3.5)\right]$$

$$\approx \frac{1}{6}\left[5 - 8(16) + 8(59) - 94\right]$$

$$\approx 42.5$$

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$$f''(x_i) \approx \frac{1}{12h^2}\left[-f(x_i-2h) + 16f(x_i-h) - 30f(x_i) + 16f(x_i+h) - f(x_i+2h)\right]$$

$$f''(2.5) \approx \frac{1}{12(0.5)^2}\left[-f(2.5-2(0.5)) + 16f(2.5-0.5) - 30f(2.5) + 16f(2.5+0.5) - f(2.5+2(0.5))\right]$$

$$\approx \frac{1}{3}\left[-f(1.5) + 16f(2.0) - 30f(2.5) + 16f(3.0) - f(3.5)\right]$$

$$\approx \frac{1}{3}\left[-5 + 16(16) - 30(33.5) + 16(59) - 94\right]$$

$$\approx 32$$

(b) $f(x) = 2x^3 + x^2 - 4$

$f'(x) = 6x^2 + 2x$ $\quad$ $f'(2.5) = 6(2.5)^2 + 2(2.5) = 42.5$

$f''(x) = 12x + 2$ $\quad$ $f''(2.5) = 12(2.5) + 2 = 32$

(c) Central three-point :

$\Rightarrow f'(2.5)$

True value , $\alpha = 42.5$

Estimated value, $\bar{\alpha} = 43$

Error , $\varepsilon = |\alpha - \bar{\alpha}|$

$= |42.5 - 43|$

$= 0.5$

$\Rightarrow f''(2.5)$

True value , $\alpha = 32$

Estimated value, $\bar{\alpha} = 32$

Error , $\varepsilon = |\alpha - \bar{\alpha}|$

$= |32 - 32|$

$= 0$

Central five-point :

$\Rightarrow f'(2.5)$

True value , $\alpha = 42.5$

Estimated value, $\bar{\alpha} = 42.5$

Error , $\varepsilon = |\alpha - \bar{\alpha}|$

$= |42.5 - 42.5|$

$= 0$

$\Rightarrow f''(2.5)$

True value , $\alpha = 32$

Estimated value, $\bar{\alpha} = 32$

Error , $\varepsilon = |\alpha - \bar{\alpha}|$

$= |32 - 32|$

$= 0$

5. (a) $\int_0^3 \frac{2}{1+5x^2}\, dx$

$a = 0$, $b = 3$, $h = \frac{b-a}{N} = \frac{3-0}{6} = 0.5$

i	x_i	$f(x_i) = \frac{2}{1+5x_i^2}$	
0	0	2	
1	0.5		0.8889
2	1.0		0.3333
3	1.5		0.1633
4	2.0		0.0952
5	2.5		0.0620
6	3.0	0.0435	
Total		2.0435	1.5427

$$\int_0^3 \frac{2}{1+5x^2}\, dx = \int_0^3 f(x)\, dx$$

$$= \frac{h}{2}\left[f_0 + f_6 + 2(f_1 + f_2 + f_3 + f_4 + f_5)\right]$$

$$= \frac{0.5}{2}\left[2.0435 + 2(1.5427)\right]$$

$$= 1.2822$$

(b) $\int_0^3 \frac{2}{1+5x^2}\, dx$

$a = 0$, $b = 3$, $h = \frac{b-a}{N} = \frac{3-0}{6} = 0.5$

i	x_i	$f_i = f(x_i) = \frac{2}{1+5x_i^2}$		
0	0	2		
1	0.5		0.8889	
2	1.0			0.3333
3	1.5		0.1633	
4	2.0			0.0952
5	2.5		0.0620	
6	3.0	0.0435		
Total		2.0435	1.1142	0.4285

$$\int_0^3 \frac{2}{1+5x^2}\, dx = \frac{h}{3}\left[(f_0+f_6) + 4\sum_{i=1}^{3} f_{2i-1} + 2\sum_{i=1}^{2} f_{2i}\right]$$

$$= \frac{h}{3}\left[(f_0+f_6) + 4(f_1+f_3+f_5) + 2(f_2+f_4)\right]$$

$$= \frac{0.5}{3}\left[2.0435 + 4(1.1142) + 2(0.4285)\right]$$

$$= 1.2262$$

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(c) $\int_0^3 \frac{2}{1+5x^2}\,dx$

$a=0$, $b=3$, $N=\frac{b-a}{h}=\frac{3-0}{0.25}=12$, $h=0.25$

i	x_i	$f_i = f(x_i) = \frac{2}{1+5x_i^2}$		
0	0	2		
1	0.25		1.5238	
2	0.50		0.8889	
3	0.75			0.5246
4	1.00		0.3333	
5	1.25		0.2270	
6	1.50			0.1633
7	1.75		0.1226	
8	2.00		0.0952	
9	2.25			0.0760
10	2.50		0.0620	
11	2.75		0.0515	
12	3.00	0.0435		
Total		2.0435	3.3043	0.7639

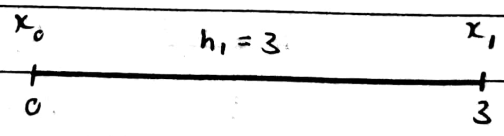
$$\int_0^3 \frac{2}{1+5x^2}\,dx = \frac{3h}{8}\left[(f_0+f_{12}) + 3\sum_{i=1}^{4}(f_{3i-2}+f_{3i-1}) + 2\sum_{i=1}^{3} f_{3i}\right]$$

$$= \frac{3(0.25)}{8}\left[2.0435 + 3(3.3043) + 2(0.7639)\right]$$

$$= 1.2641$$

(d) $\int_0^3 \frac{2}{1+5x^2}\,dx$, $a=0$, $b=3$

$h_1 = 3-0 = 3$



$R_{1,1} = \frac{h_1}{2}(f_0+f_1)$

$= \frac{3}{2}(2+0.0435)$

$= 3.0653$

$f(x) = \frac{2}{1+5x^2}$

$f_0 = f(0) = \frac{2}{1+5(0)^2} = 2$

$f_1 = f(3) = \frac{2}{1+5(3)^2} = 0.0435$

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$h_2 = \frac{1}{2} h_1 = \frac{1}{2}(3) = 1.5$

$x_0 \quad h_2 = 1.5 \quad x_1 \quad x_2$

$0 \quad 1.5 \quad 3$

$$R_{2,1} = \frac{1}{2}\left[R_{1,1} + h_1 \sum_{k=1}^{1} f_{2k-1}\right]$$
$$= \frac{1}{2}\left[R_{1,1} + h_1 (f_1)\right]$$
$$= \frac{1}{2}\left[R_{1,1} + h_1 (f(1.5))\right]$$
$$= \frac{1}{2}\left[3.0653 + 3(0.1633)\right]$$
$$= 1.7776$$

$$f_1 = f(1.5) = \frac{2}{1+5(1.5)^2} = 0.1633$$

$$R_{2,2} = \frac{4R_{2,1} - R_{1,1}}{4^{2-1}-1} = \frac{4(1.7776) - 3.0653}{3} = 1.3484$$

$h_3 = \frac{1}{2} h_2 = \frac{1}{2}(1.5) = 0.75$

$x_0 \quad h_3 = 0.75 \quad x_1 \quad x_2 \quad x_3 \quad x_4$

$0 \quad 0.75 \quad 1.5 \quad 2.25 \quad 3$

$$R_{3,1} = \frac{1}{2}\left[R_{2,1} + h_2 \sum_{k=1}^{2} f_{2k-1}\right]$$
$$= \frac{1}{2}\left[R_{2,1} + h_2 (f_1 + f_3)\right]$$
$$= \frac{1}{2}\left[R_{2,1} + h_2 \left(f(0.75) + f(2.25)\right)\right]$$
$$= \frac{1}{2}\left[1.7776 + 1.5(0.5246 + 0.0760)\right]$$
$$= 1.3393$$

$$f_1 = f(0.75) = \frac{2}{1+5(0.75)^2} = 0.5246$$
$$f_3 = f(2.25) = \frac{2}{1+5(2.25)^2} = 0.0760$$

$$R_{3,2} = \frac{4R_{3,1} - R_{2,1}}{4^{2-1}-1} = \frac{4(1.3393) - 1.7776}{3} = 1.1932$$

$$R_{3,3} = \frac{4^{3-1} R_{3,2} - R_{2,2}}{4^{3-1}-1} = \frac{16(1.1932) - 1.3484}{15} = 1.1829$$

i	$h_i = \frac{b-a}{2^{i-1}}$	$R_{i,1}$	$R_{i,2}$	$R_{i,3}$
1	3	3.0653		
2	1.5	1.7776	1.3484	
3	0.75	1.3393	1.1932	1.1829

$\therefore$ The solution is $\int_0^3 \frac{2}{1+5x^2}\,dx = R_{3,3} = 1.1829$