SECI1113-01 MATEMATIK KOMPUTAN

(COMPUTATIONAL MATHEMATICS)

ASSIGNMENT 4

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Question 1

(a) $|\varepsilon| = |\alpha - \bar{\alpha}| = 2$

$\alpha = 60$

$$\varepsilon_{rel} = \frac{|\alpha - \bar{\alpha}|}{|\alpha|} = \frac{2}{|60|} = 0.33$$

Percentage $= 0.33 \times 100\% = 3.3\%$

(b) $$f'(x) = \frac{f(x+h) - f(x)}{h}$$

$f(x) = x$

$f'(x) = 1$

$$f'(2) = \frac{f(2+0.1) - 2}{0.1}$$

$$= \frac{f(2.1) - 2}{0.1}$$

$$= \frac{2.1 - 2}{0.1}$$

$$= 1$$

$|\varepsilon| = |\text{true value} - \text{approximate value}|$

$= |1-1|$

$= 0$

$\therefore$ Absolute error $= 0$

(c) $6.232 + 7.0321 + 5.43 - 3.385 = 15.3091$

Modulus error :

$|\varepsilon_3| \leq 0.0005 + 0.00005 + 0.005 + 0.0005 = 0.00605$

$15.3091 + 0.00605 = 15.31515$

$15.3091 - 0.00605 = 15.30305$

$\therefore$ Both of these values are same if rounded to 1 decimal point. Therefore, the most accurate value is 15.3 (1dp)

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Question 2

$f(x) = x^6 - x - 1$, $[1, 2]$, $\varepsilon = 0.001$

$a = 1$ $\quad f(a) = 1^6 - 1 - 1 = -1$ $\quad f(a)f(b) < 0$

$b = 2$ $\quad f(b) = 2^6 - 2 - 1 = 61$ $\quad p \in [1, 2]$

$p_1 = \frac{1+2}{2} = 1.5$

$p_1 = 1.5$ $\quad f(p_1) = 1.5^6 - 1.5 - 1 = 8.8906$ $\quad f(a)f(p_1) < 0$

$p_2 = \frac{1+1.5}{2} = 1.25$ $\quad p \in [1, 1.5]$

$p_2 = 1.25$ $\quad f(p_2) = 1.25^6 - 1.25 - 1 = 1.5647$ $\quad f(a)f(p_2) < 0$

$p_3 = \frac{1+1.25}{2} = 1.125$ $\quad p \in [1, 1.25]$

$p_3 = 1.125$ $\quad f(p_3) = 1.125^6 - 1.125 - 1 = -0.0977$ $\quad f(a)f(p_3) < 0$

$f(1.125) = -0.0977$, $f(1.25) = 1.5647$ $\quad p \in [1.125, 1.25]$

$p_4 = \frac{1.125 + 1.25}{2} = 1.1875$

$p_4 = 1.1875$ $\quad f(p_4) = 1.1875^6 - 1.1875 - 1 = 0.6167$ $\quad f(a)f(b) < 0$

$f(1.25) = -0.0977$, $f(1.1875) = 0.6167$ $\quad p \in [1.125, 1.1875]$

$p_5 = \frac{1.125 + 1.1875}{2} = 1.1563$

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$p_5 = 1.1563$ $f(p_5) = 1.1563^6 - 1.1563 - 1 = 0.2338$

$f(1.125) = -0.0977$, $f(1.1563) = 0.2338$ $f(a)f(b) < 0$

$p_6 = \frac{1.125 + 1.1563}{2} = 1.1407$ $p \in [1.125, 1.1563]$

$p_6 = 1.1407$ $f(p_6) = 1.1407^6 - 1.1407 - 1 = 0.0624$

$f(1.125) = -0.0977$, $f(1.1407) = 0.0624$ $f(a)f(b) < 0$

$p_7 = \frac{1.125 + 1.1407}{2} = 1.1329$ $p \in [1.125, 1.1407]$

$p_7 = 1.1329$ $f(p_7) = 1.1329^6 - 1.1329 - 1 = -0.0187$ $f(a)f(b) < 0$

$f(1.1329) = -0.0187$, $f(1.1407) = 0.0624$ $p \in [1.1329, 1.1407]$

$p_8 = \frac{1.1329 + 1.1407}{2} = 1.1368$

$p_8 = 1.1368$ $f(p_8) = 1.1368^6 - 1.1368 - 1 = 0.0215$ $f(a)f(b) < 0$

$f(1.1329) = -0.0187$, $f(1.1368) = 0.0215$ $p \in [1.1329, 1.1368]$

$p_9 = \frac{1.1329 + 1.1368}{2} = 1.1349$

$p_9 = 1.1349$ $f(p_9) = 1.1349^6 - 1.1349 - 1 = 0.0018$ $f(a)f(b) < 0$

$f(1.1329) = -0.0187$, $f(1.1349) = 0.0018$ $p \in [1.1329, 1.1349]$

$p_{10} = \frac{1.1329 + 1.1349}{2} = 1.1339$

$p_{10} = 1.1339$ $f(p_{10}) = 1.1339^6 - 1.1339 - 1 = -0.0085$ $f(a)f(b) < 0$

$f(1.1339) = -0.0085$, $f(1.1349) = 0.0018$ $p \in [1.1339, 1.1349]$

$p_{11} = \frac{1.1339 + 1.1349}{2} = 1.1344$

$p_{11} = 1.1344$ $f(p_{11}) = 1.1344^6 - 1.1344 - 1 = -0.0033$ $f(a)f(b) < 0$

$f(1.1344) = -0.0033, f(1.1349) = 0.0018$ $p \in [1.1344, 1.1349]$

$$p_{12} = \frac{1.1344 + 1.1349}{2} = 1.1347$$

$p_{12} = 1.1347$ $f(p_{12}) = 1.1347^6 - 1.1347 - 1 = -0.0002$ $f(a)f(b) < 0$

$f(1.1347) = -0.0002, f(1.1349) = -0.00018$ $p \in [1.1347, 1.1349]$

i	a	$f(a)$	b	$f(b)$	p_i	$f(p_i)$
0	1	-1	2	61	1.5	8.891
1	1	-1	1.5	8.8906	1.25	1.565
2	1	-1	1.25	1.5647	1.125	-0.0977
3	1.125	-0.0977	1.125	1.5647	1.1875	0.6167
4	1.125	-0.0977	1.1375	0.6167	1.1563	0.2338
5	1.125	-0.0977	1.1563	0.2338	1.1407	0.0624
6	1.125	-0.0977	1.1407	0.0624	1.1329	-0.0187
7	1.1329	-0.0177	1.1407	0.0624	1.1368	0.0215
8	1.1329	-0.0187	1.1368	0.0215	1.1349	0.0018
9	1.1329	-0.0087	1.1349	0.0018	1.1339	-0.0085
10	1.1329	-0.0085	1.1349	0.0018	1.1344	-0.0033
11	1.1344	-0.0033	1.1349	0.0018	1.1347	-0.0002

$|f(1.1347)| = 0.0002 < 0.001$

$\therefore p = p_{11} = 1.13$ (2 decimal points)

Question 3

$f(x) = \cos(x) + 2\sin(x) + x^2$ $\varepsilon = \frac{1}{2} \times 10^{-2}$

(a) Secant Method, $x_0 = 0$, $x_1 = -0.1$ $= 0.005$

$x_0 = 0$

$f(x_0) = \cos(0) + 2\sin(0) + (0)^2 = 1$

$x_1 = -0.1$

$f(x_1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2 = 0.8053$

$x_2 = \dfrac{0(0.8053) - (-0.1)(1)}{0.8053 - 1} = -0.5136$

$f(x_2) = \cos(-0.5136) + 2\sin(-0.5136) + (-0.5136)^2 = 0.1521$

$|f(x_2)| = 0.1521 > 0.005$

$x_3 = \dfrac{-0.1(0.1521) - (-0.5136)(0.8053)}{0.1521 - 0.8053} = -0.6099$

$f(x_3) = \cos(-0.6099) + 2\sin(-0.6099) + (-0.6099)^2 = 0.0461$

$|f(x_3)| = 0.0461 > 0.005$

$x_4 = \dfrac{-0.5136(0.0461) - (-0.6099)(0.1521)}{0.0461 - 0.1521} = -0.6518$

$f(x_4) = \cos(-0.6518) + 2\sin(-0.6518) + (-0.6518)^2 = 0.0066$

$|f(x_4)| = 0.0066 > 0.005$

$x_5 = \dfrac{-0.6099(0.0066) - (-0.6518)(0.0461)}{0.0066 - 0.0461} = -0.6588$

$f(x_5) = \cos(-0.6588) + 2\sin(-0.6588) + (-0.6588)^2 = 0.0004$

$|f(x_5)| = 0.0004 < 0.005$

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$$x_6 = \frac{-0.6518(0.0004) - (-0.6588)(0.0066)}{0.0004 - 0.0066} = -0.6593$$

$f(x_6) = \cos(-0.6593) + 2\sin(-0.6593) + (-0.6593)^2 = -0.00003$

$|f(x_6)| = 0.00003 < 0.005$, stop.

n	x_{n-1}	x_n	x_{n+1}	$\|f(x_{n+1})\|$	$\|x_{n+1} - x_n\|$
1	0	-0.1	-0.5136	0.1521	0.4136
2	-0.1	-0.5136	-0.6099	0.0461	0.0963
3	-0.5136	-0.6099	-0.6518	0.0066	0.0419
4	-0.6099	-0.6518	-0.6588	0.0004	0.007
5	-0.6518	-0.6588	-0.6593	0.00003	0.0005

Both $|f(x_{n+1})| = 0.00003$ and $|x_{n+1} - x_n| = 0.0005 < 0.005$.
Therefore, after five iterations, the approximation to the root is $x_6 = -0.6593 \approx -0.66$ (2 d.p.)

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(b) Newton method, $x_0 = -0.1$

$f(x) = \cos(x) + 2\sin(x) + x^2$

$f'(x) = -\sin(x) + 2\cos(x) + 2x$

$x_0 = -0.1$

$f(x_0) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2 = 0.8053$

$f'(x_0) = -\sin(-0.1) + 2\sin(-0.1) + 2(-0.1)^2 = 1.8898$

$|f(x_0)| = 0.80533 > 0.005$

$x_1 = -0.1 - \dfrac{0.8053}{1.8893} = -0.5261$

$f(x_1) = \cos(-0.5261) + 2\sin(-0.5261) + (-0.5261)^2 = 0.1372$

$f'(x_1) = -\sin(-0.5261) + 2\cos(-0.5261) + 2(-0.5261) = 1.1795$

$|f(x_1)| = 0.1372 > 0.005$

$x_2 = -0.5261 - \dfrac{0.1372}{1.1795} = -0.6424$

$f(x_2) = \cos(-0.6424) + 2\sin(-0.6424) + (-0.6424)^2 = 0.0151$

$f'(x_2) = -\sin(-0.6424) + 2\cos(-0.6424) + 2(-0.6424) = 0.9156$

$|f(x_2)| = 0.0151 > 0.005$

$x_3 = -0.6424 - \dfrac{0.0151}{0.9156} = -0.6589$

$f(x_3) = \cos(-0.6589) + 2\sin(-0.6589) + (-0.6589)^2 = 0.0003$

$|f(x_3)| = 0.0003 < 0.005$, stop

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i	x_i	$f(x_i)$	$f'(x_i)$
0	-0.1	0.8053	1.8898
1	-0.5261	0.1372	1.1795
2	-0.6424	0.0151	0.9156
3	-0.6589	0.0003	

$x = x_3 = -0.6589 \approx -0.66$ (2 d.p.)

$\therefore$ Both Secant Method and Newton's Method get the same result which is -0.66. Secant Method use more step to get the result.

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Question 4

$\frac{1}{x} = 1.37$ $\quad f(x) = \frac{1}{x} - 1.37, \; f'(x) = -\frac{1}{x^2}$

(a) Newton's formula = $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$$= x_n - \frac{\left(\frac{1}{x_n} - 1.37\right)}{-\frac{1}{x_n^2}}$$

$$= x_n - \left(\frac{1 - 1.37x_n}{x_n}\right)\left(-\frac{x_n^2}{1}\right)$$

$$= x_n - (-x_n + 1.37x_n^2)$$

$$= x_n + x_n - 1.37x_n^2$$

$$= 2x_n - 1.37x_n^2$$

$$x_{n+1} = x_n(2 - 1.37x_n)$$

From question, $\alpha = 1.37$

$x_{n+1} = x_n(2 - \alpha x_n)$

$x_{n+1} = x_n(2 - 1.37x_n$ proven

b) 8 decimal places , $x_0 = 0.75$

$$\varepsilon = \frac{1}{2} \times 10^{-8} = 0.000000005$$

$x_0 = 0.75$

$f(x_0) = \frac{1}{0.75} - 1.37 = -0.03666667$ $\quad |f(x_0)| = -0.03666667 >$ 0.000000005

$f'(x_0) = -\frac{1}{0.75^2} = -1.77777778$

$x_1 = 0.75\,(2 - 1.37(0.75)) = 0.729375$

$f(x_1) = \frac{1}{0.75} - 1.37 = 0.00103685$ $\quad |f(x_1)| = 0.00103685 >$ 0.000000005

$f'(x_1) = \frac{-1}{0.75^2} = -1.87974204$

$x_2 = 0.729375\,(2 - 1.37(0.729375)) = 0.72992659$

$f(x_2) = \frac{1}{0.72992659} - 1.37 = 0.00000078$ $\quad |f(x_2)| = 0.0000078 >$ 0.000000005

$f'(x_2) = -\frac{1}{0.72992659^2} = 1.87690216$

$x_3 = 0.72992659\,(2 - 1.37(0.72992659)) = 0.72992701$

$f(x_3) = \frac{1}{0.72992701} - 1.37 = -0.000000005$ $\quad |f(x_3)| = 0.000000005 <$ 0.000000005, stop

$f'(x_3) = -\frac{1}{0.72992701^2} = 1.87693552$

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i	x_i	$f(x_i)$	$f'(x_i)$
0	0.75	-0.03666667	1.77777778
1	0.729375	0.00103685	1.87974204
2	0.72992659	0.00000078	1.87690215
3	0.72992701	-0.000000005	1.87693552

$\therefore x = x_3 = 0.72992701$ (8d.p.)

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Question 5

$$S = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

$$S - \lambda I = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$$\det a_{11} = (1-\lambda)\begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix}$$

$$= (1-\lambda)[(2-\lambda)(1-\lambda) - (-1)(-1)]$$

$$= (1-\lambda)[2-2\lambda-\lambda+\lambda^2-1]$$

$$= 1-3\lambda+\lambda^2-\lambda+3\lambda^2-\lambda^3$$

$$= -\lambda^3+4\lambda^2-4\lambda+1$$

$$\det a_{12} = -1\begin{bmatrix} -1 & -1 \\ 0 & 1-\lambda \end{bmatrix}$$

$$= -1(-1(1-\lambda)-0(-1))$$

$$= -1(-1+\lambda)$$

$$= 1-\lambda$$

$$\det a_{13} = 0\begin{bmatrix} -1 & 2-\lambda \\ 0 & -1 \end{bmatrix}$$

$$= 0$$

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$$p(\lambda) = -\lambda^3 + 4\lambda^2 - 4\lambda + 1 - (1-\lambda) + 0$$
$$= -\lambda^3 + 4\lambda^2 + 3\lambda$$

Let $-\lambda^3 + 4\lambda^2 - 3\lambda = 0$
$$\lambda^3 - 4\lambda^2 + 3\lambda = 0$$

$$(\lambda-3)(\lambda-0)(\lambda-1) = 0$$
$$\therefore \lambda = 3, \ \lambda = 0, \ \lambda = 1$$

$(S - \lambda I)x = 0$

For $\lambda = 3$

$$\left[\begin{array}{ccc|c} -2 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \xrightarrow[R_2 \to -R_2]{R_1 \to -R_1} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \leftrightarrow R_2]{R_3 \to -R_3} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & -1 & -2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 1 & 2 & 0 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $x_3 = a$
$x_2 + 2x_3 = 0$
$-2a = x_2$

$x_1 + x_2 + x_3 = 0$
$x_1 - 2a + a = 0$
$x_1 = a$

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$$x = \begin{bmatrix} a \\ -2a \\ a \end{bmatrix} = a\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$\therefore$ The eigenvector for $\lambda=3$ is $\begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$

$\lambda = 0$

$$\left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ -1 & 2 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + 1} \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & -1 & 1 & 0 \end{array}\right]$$

$$R_3 \to R_3 + 1 \quad \left[\begin{array}{ccc|c} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

Let $x_3 = a$

$x_2 - x_3 = 0$ $\quad$ $x_1 - x_2 = 0$

$x_2 = a$ $\quad$ $x_1 = a$

$$x = \begin{bmatrix} a \\ a \\ a \end{bmatrix}, \quad a = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$\therefore$ The eigenvector for $\lambda = 0$ is $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

$\lambda = 1$

$$\left[\begin{array}{ccc|c} 0 & -1 & 0 & 0 \\ -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \xrightarrow{R_1 \Leftrightarrow R_2} \left[\begin{array}{ccc|c} -1 & 1 & -1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

$$\begin{array}{l} R_1 \to -R_1 \\ R_2 \to -R_2 \end{array} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right]$$

$$R_3 \to R_3 + R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

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let $x_3 = a$ $\qquad x_1 + x_2 + x_3 = 0$

$x_2 = 0$ $\qquad x_1 = -a$

$$x = \begin{bmatrix} -a \\ 0 \\ a \end{bmatrix} ; = a \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$\therefore$ The eigenvector for $\lambda = 1$ is

$$\begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}.$$

Question 6

$$B = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix}$$

a) Row 1 , $-18 \quad 20 \quad -6$

B_1 , $a_{11} = -18$ $\quad |\lambda - (-18)| < |20| + |-6|$

$|\lambda + 18| < 26$

$-44 < \lambda 8$

$-44 < \lambda < 8$

$\lambda \in (-44, 8)$

Row 2

B_2 , $a_{22} = 14$ $\quad |\lambda - 14| < |-12| + |-3|$

$|\lambda - 14| < 15$

$-1 < \lambda < 29$

$\lambda \in (-1, 29)$

Row3

B_3, $a_{33} = 13$ $\quad |\lambda - 13| < |30| + |-30|$

$|\lambda - 13| < 60$

$-47 < \lambda < 73$

$\lambda \in (-47, 73)$

$\therefore \lambda \in (-47, 73)$ is the region containing all eigenvalues of B.

(b) $V^{(0)} = [0, 0, 1]^T$ $\quad \varepsilon = 0.05$

$$V = Bv^{[0]}$$

$$= \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -3 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ -13 \end{bmatrix}$$

$m_1 = 13 \qquad V^{(1)} = \frac{1}{13} \begin{bmatrix} -6 \\ -3 \\ 13 \end{bmatrix} \qquad |V^{(1)} - V^{(0)}| = 0.5160 > 0.05$

$$= \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1.0000 \end{bmatrix}$$

$$V = Bv^{(1)}$$

$$= \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -3 & 13 \end{bmatrix} \begin{bmatrix} -0.4615 \\ -0.2308 \\ 1.0000 \end{bmatrix} = \begin{bmatrix} -2.309 \\ -0.6932 \\ 6.079 \end{bmatrix} \quad m_2 = 6.079$$

$$V \times \frac{1}{m_2} = \frac{1}{6.079} \begin{bmatrix} -2.309 \\ -0.6932 \\ 6.079 \end{bmatrix} = \begin{bmatrix} -0.3798 \\ -0.114 \\ 1 \end{bmatrix}$$

K	$(V^{(K)})^T$			$(Av^{(K)})^T$			m_{K+1}
0	0	0	1	−6	−3	13	13
1	−0.4615	−0.2308	1	−2.309	−0.6932	6.0790	6.0790
2	−0.3798	−0.1140	1	−1.4436	−0.0384	5.0260	5.0260
3	−0.2872	−0.0076	1	−0.9824	0.3400	4.6210	4.6210
4	−0.2130	0.0737	1	−0.6920	0.5878	4.3990	4.3990
5	−0.1573	0.1336	1	−0.4966	0.7580	4.2730	4.2730
6	−0.1162	0.1774	1	−0.3604	0.8780	4.1920	4.1920
7	−0.0860	0.2094	1				

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check $|v^{(7)} - v^{(6)}| = \sqrt{(-0.0860+0.1162)+(0.2094-0.1774)^2+(1-1)^2}$
$= 0.0440 < 0.05$

$\lambda_1 = m_7 = 4.1920 \approx 4$

$V_1 \approx V^{(7)} = \begin{bmatrix} -0.0860 \\ 0.2094 \\ 1 \end{bmatrix}$

(c) $V^{(0)} = [0,0,1]^T$, $\lambda_1 = 4$

let A = B shifted

$A = B - \lambda_1 I$
$= B - 4I$

$\begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix} = \begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

let $v = Av^{(0)}$
$= \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix}$ $m_1 = 9$

$V^{(1)} = v \times \frac{1}{m_1} = \begin{bmatrix} -6 \\ -3 \\ 9 \end{bmatrix} \times \frac{1}{9} = \begin{bmatrix} -0.6667 \\ -0.3333 \\ 1 \end{bmatrix}$

$|v' - v^0| = 0.7454 > 0.05$

$V = Bv^{(1)} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix} \begin{bmatrix} -0.6667 \\ -0.3333 \\ 1 \end{bmatrix} = \begin{bmatrix} 2.0014 \\ 1.6674 \\ -1.0020 \end{bmatrix}$

$m_2 = 2.0014$

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$$V^{(2)} = \frac{V}{m_2}$$

$$= \frac{1}{2.0014}\begin{bmatrix} 2.0014 \\ 1.6674 \\ -0.0020 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.8331 \\ -0.5006 \end{bmatrix}$$

$$\|V^{(2)} - V^{(1)}\| = \sqrt{(1+0.6667)^2 + (0.8331+0.3333)^2 + (0.5006-1)^2}$$
$$= 2.5278 > \varepsilon$$

$$V = BV^{(2)}$$
$$= \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}\begin{bmatrix} 1 \\ 0.8331 \\ -0.5006 \end{bmatrix} = \begin{bmatrix} -2.3844 \\ -2.1672 \\ 0.5016 \end{bmatrix} \quad m_3 = -2.3844$$

$$V^{(3)} = \frac{V}{m_3}$$

$$= \frac{1}{-2.3844}\begin{bmatrix} -2.3844 \\ -2.1672 \\ 0.5016 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.9284 \\ -0.2149 \end{bmatrix}$$

$$\|V^{(3)} - V^{(2)}\| = \sqrt{(1-1)^2 + (0.9284-0.8331)^2 + (-0.2149+0.5006)^2}$$
$$= 0.3012 > \varepsilon$$

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k	$(v^{(k)})^T$			$(Bv^{(k)})^T$			m_{k+1}
0	0	0	1	-6	-3	9	9
1	-0.6667	-0.3333	1	2.0014	1.6674	-1.0020	2.0014
2	1	0.8331	-0.5006	-2.3344	-2.1672	0.5016	-2.3344
3	1	0.9284	-0.2149	-2.1426	-2.0713	0.2139	-2.1426
4	1	0.9667	-0.0998	-2.0672	-2.0336	0.1008	-2.0672
5	1	0.9837	-0.0488	-2.0332	-2.0166	0.0498	-2.0332
6	1	0.9918	-0.0245				

$\|v^{(6)} - v^{(5)}\| = \sqrt{(1-1)^2 + (-0.9918 - 0.9837)^2 + (-0.0245 + 0.0488)^2}$
$= 0.0256 < 0.05$

λ shifted $= m_6 = -2.0332 \approx -2$

$\lambda = \lambda$ shifted $+ \lambda_1$ $\qquad v = [1 \quad 0.9918 \quad -0.0245]^T$
$= -2 + 4 = 2$

(d) Since the extreme eigenvalue for the opposite is $\lambda = 2$, and has the same sign as largest magnitude of original matrix, $\lambda_1 = 4$, then all the eigenvalues of B are positive and $\lambda = 2$ is the smallest eigenvalue of matrix A.

Matrix A has 3 eigenvalue.

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(d) Let $|\lambda_1| \geq |\lambda_2| \geq |\lambda_3|$

$$\sum \lambda_i = \sum a_{ii}$$

$$\lambda_1 + \lambda_2 + \lambda_3 = a_{11} + a_{12} + a_{13}$$
$$= -18 + 14 + 13$$
$$= 9$$
$$4 + \lambda_2 + 2 = 9$$
$$\lambda_2 = 9 - 4 - 2$$
$= 3$ is the intermediate eigenvalue (λ_2)