SECI1113-01 MATEMATIK KOMPUTAN

(COMPUTATIONAL MATHEMATICS)

ASSIGNMENT 3

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Assignment 3

Question 1

$T(0) = 0$

$$T(0) = T\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right) = T\left(\begin{bmatrix} 2(0)-0 \\ 0+3(0) \end{bmatrix}\right)$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$\therefore$ Since $T(0)=0$, then T is a linear transformation

Question 2

Let $u = (x_1, y_1, z_1)$ and $v = (x_2, y_2, z_2)$

$T(u) + T(v) = T(u+v)$

LS.

$$T(u) + T(v) = T\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + T\begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$

$$= T\begin{bmatrix} x_1+y_1-z_1 \\ 2x_1y_1 \\ x_1+z_1+1 \end{bmatrix} + T\begin{bmatrix} x_2+y_2-z_2 \\ 2x_2y_2 \\ x_2+z_2+1 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+x_2+y_1+y_2-z_1-z_2 \\ 2x_1y_1+2x_2y_2 \\ x_1+x_2+z_1+z_2+2 \end{bmatrix}$$

RS:

$$T(u+v) = T\left(\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} + \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}\right)$$

$$= T\begin{bmatrix} x_1+x_2 \\ y_1+y_2 \\ z_1+z_2 \end{bmatrix}$$

$$= \begin{bmatrix} x_1+x_2+y_1+y_2-z_1-z_2 \\ 2x_1y_1+2x_1y_2+2x_2y_1+2x_2y_2 \\ x_1+x_2+z_1+z_2+1 \end{bmatrix}$$

$\therefore T(u)+T(v) \neq T(u+v)$

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$T(cu) = cT(u)$

<u>LS.</u>

$$T(cu) = T\begin{bmatrix} cx_1 \\ cy_1 \\ cz_1 \end{bmatrix}$$

$$= \begin{bmatrix} cx_1 + cy_1 - cz_1 \\ 2c^2x_1y_1 \\ cx_1 + cz_1 + 1 \end{bmatrix}$$

<u>RS.</u>

$$cT(u) = cT\begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$

$$= c\begin{bmatrix} x_1 + y_1 - z_1 \\ 2x_1y_1 \\ x_1 + z_1 + 1 \end{bmatrix}$$

$$= \begin{bmatrix} cx_1 + cy_1 - cz_1 \\ 2cx_1y_1 \\ cx_1 + cz_1 + c \end{bmatrix}$$

$$T(cu) \neq cT(u)$$

Therefore T is not a linear transformation between the vector space since $T(u+v) \neq T(u)+T(v)$ and $T(cu) \neq cT(u)$.

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Question 3

a) let

$e_1 = \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$ $e_2 : \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$ $e_3 = \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$

$$T(e_1) = Ae_1$$
$$: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix}$$
$$= \begin{bmatrix} x_1 + 2y_1 + 3z_1 \\ 2x_1 + y_1 + 3z_1 \\ x_1 + 3y_1 + 2z_1 \end{bmatrix}$$

$$T(e_2) = Ae_2$$
$$: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix}$$
$$\begin{bmatrix} x_2 + 2y_2 + 3z_2 \\ 2x_2 + y_2 + 3z_2 \\ x_2 + 3y_2 + 2z_2 \end{bmatrix}$$

$$T(e_3) = Ae_3$$
$$: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix}$$
$$\begin{bmatrix} x_3 + 2y_3 + 3z_3 \\ 2x_3 + y_3 + 3z_3 \\ x_3 + 3y_3 + 2z_3 \end{bmatrix}$$

b) $T(3e_1 - 4e_2 + 6e_3)$

$$: T\left(3 \begin{bmatrix} x_1 \\ y_1 \\ z_1 \end{bmatrix} - 4 \begin{bmatrix} x_2 \\ y_2 \\ z_2 \end{bmatrix} + 6 \begin{bmatrix} x_3 \\ y_3 \\ z_3 \end{bmatrix} \right)$$

$$: \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} 3x_1 - 4x_2 + 6x_3 \\ 3y_1 - 4y_2 + 6y_3 \\ 3z_1 - 4z_2 + 6z_3 \end{bmatrix}$$

$$\begin{bmatrix} 3x_1 + 6y_1 + 9z_1 - 4x_2 - 8y_2 - 12z_2 + 6x_3 + 12y_3 + 18z_3 \\ 6x_1 + 3y_1 + 9z_1 - 8x_2 - 4y_2 - 12z_2 + 12x_3 + 6y_3 + 18z_3 \\ 3x_1 + 9y_1 + 6z_1 - 4x_2 - 12y_2 - 8z_2 + 6x_3 + 18y_3 + 12z_3 \end{bmatrix}$$

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Question 4

$$T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x-2y \\ -2x+4y \end{bmatrix}$$

$$T\begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 1-2(3) \\ -2(1)+4(3) \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \end{bmatrix}$$

$\therefore$ Since $T\begin{bmatrix} 1 \\ 3 \end{bmatrix} \neq 0$, so vector v is not in $N(T)$.

Question 5

a) $c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3) = W$

$$c_1\begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} = \begin{bmatrix} -2 \\ 1 \\ 2 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -1 & 0 & -1 & -2 \\ 2 & 5 & -1 & 1 \\ 1 & 0 & 2 & 2 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 2 & 5 & -1 & 1 \\ -1 & 0 & -1 & -2 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 5 & -5 & -3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5}R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 2 \\ 0 & 1 & -1 & -3/5 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

back subs.

$c_3 = 0$ $\quad c_2 - c_3 = -3/5$ $\quad c_1 + 2c_3 = 2$

$\quad c_2 = -3/5$ $\quad c_1 = 2$

$S = \{ c_1 = 2,\ c_2 = -3/5,\ c_3 = 0 \}$

$\therefore W \in R(T)$

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b) $c_1 T(v_1) + c_2 T(v_2) + c_3 T(v_3)$

$$= c_1 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} + c_2 \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} + c_3 \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} -1 & 0 & -1 \\ 2 & 5 & -1 \\ 1 & 0 & 2 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_3} \begin{bmatrix} 1 & 0 & 2 \\ 2 & 5 & -1 \\ -1 & 0 & -1 \end{bmatrix}$$

$$\xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 5 & -5 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to \frac{1}{5} R_2} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_2 \to R_2 + R_3} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\xrightarrow{R_1 \to R_1 - 2R_3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\therefore$ Since leading 1s is in column 1, 2 and 3, a basis for $R(T)$ is

$$R(T) = \text{span} \left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 0 \\ 5 \\ 0 \end{bmatrix} \begin{bmatrix} -1 \\ -1 \\ 2 \end{bmatrix} \right\}$$

c) $$\left[\begin{array}{ccc|c} -1 & 0 & -1 & 0 \\ 2 & 5 & -1 & 0 \\ 1 & 0 & 2 & 0 \end{array}\right] \xrightarrow{R_1 \leftrightarrow R_3} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 2 & 5 & -1 & 0 \\ -1 & 0 & -1 & 0 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 + R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 5 & -5 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right] \xrightarrow{R_2 \to \frac{1}{5} R_2} \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$$\xrightarrow[R_1 \to R_1 - 2R_3]{R_2 \to R_2 + R_3} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{array}\right]$$

$x_1 = 0,\ x_2 = 0,\ x_3 = 0$

$\therefore$ It is unique solution for all choices of c_1, c_2 and c_3.

Hence, no vector span in $N(T)$, and no vector as basis for $N(T)$.

$\therefore \dim(N(T)) = 0$.

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