SECI1113-01 MATEMATIK KOMPUTAN

(COMPUTATIONAL MATHEMATICS)

ASSIGNMENT 2

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Assignment 2

1. Axiom 3 : $(u+v)+w = u+(v+w)$

Let

$$u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} \qquad v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} \qquad w = \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$(u+v)+w = \left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} \right) + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(t_1+t_2+t_3) \\ 2-(t_1+t_2+t_3) \\ 3+(2t_1+2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+(t_2+t_3) \\ 2-(t_2+t_3) \\ 3+(2t_2+2t_3) \end{bmatrix}$$

$$= \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \left(\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix} + \begin{bmatrix} 1+t_3 \\ 2-t_3 \\ 3+2t_3 \end{bmatrix} \right)$$

$$= u+(v+w)$$

Axiom 3 is satisfied.

Axiom 7 : $k(u+v) = ku + kv$

Let

$$u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} \qquad v = \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$$

Let c be any real numbers

$$c(u+v) = c\left(\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + \begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}\right)$$

$$= c\begin{bmatrix} 1+(t_1+t_2) \\ 2-(t_1+t_2) \\ 3+(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+c(t_1+t_2) \\ 2-c(t_1+t_2) \\ 3+c(2t_1+2t_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct_1+ct_2) \\ 2-(ct_1+ct_2) \\ 3+(2ct_1+2ct_2) \end{bmatrix}$$

$$= \begin{bmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{bmatrix} + \begin{bmatrix} 1+ct_2 \\ 2-ct_2 \\ 3+2ct_2 \end{bmatrix}$$

$$= c\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + c\begin{bmatrix} 1+t_2 \\ 2-t_2 \\ 3+2t_2 \end{bmatrix}$$

$$= cu + cv$$

Axiom 7 is satisfied.

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Axiom 8 : $(k+m)u = ku + mu$

Let

$$u = \begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

Let c and m be any real number.

$$(c+m)u = (c+m)\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(c+m)t_1 \\ 2-(c+m)t_1 \\ 3+(c+m)2t_1 \end{bmatrix}$$

$$cu + mu = c\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix} + m\begin{bmatrix} 1+t_1 \\ 2-t_1 \\ 3+2t_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+ct_1 \\ 2-ct_1 \\ 3+2ct_1 \end{bmatrix} + \begin{bmatrix} 1+mt_1 \\ 2-mt_1 \\ 3+2mt_1 \end{bmatrix}$$

$$= \begin{bmatrix} 1+(ct_1+mt_1) \\ 2-(ct_1+mt_1) \\ 3+(2ct_1+2mt_1) \end{bmatrix}$$

$$= \begin{bmatrix} 1+(c+m)t_1 \\ 2-(c+m)t_1 \\ 3+(c+m)2t_1 \end{bmatrix}$$

$$= (c+m)u$$

Axiom 8 is satisfied

$\therefore$ V is a vecter space since Axiom 3, Axiom 7 and Axiom 8 are satisfied.

2.(a) Let

$$u = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix} \qquad v = \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$$

$$u+v = \begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix} + \begin{bmatrix} a_2-b_2 & a_2 \\ b_2+c_2 & a_2-c_2 \end{bmatrix}$$

$$= \begin{bmatrix} (a_1-b_1)+(a_2-b_2) & a_1+a_2 \\ (b_1+c_1)+(b_2+c_2) & (a_1-c_1)+(a_2-c_2) \end{bmatrix}$$

$\therefore$ S is closed under addition.

Let k be any real number

$$ku = k\begin{bmatrix} a_1-b_1 & a_1 \\ b_1+c_1 & a_1-c_1 \end{bmatrix}$$

$$= \begin{bmatrix} k(a_1-b_1) & ka_1 \\ k(b_1+c_1) & k(a_1-c_1) \end{bmatrix}$$

$$= \begin{bmatrix} ka_1-kb_1 & ka_1 \\ kb_1+kc_1 & ka_1-kc_1 \end{bmatrix}$$

$\therefore$ S is closed under multiplication.

Hence, S is a subspace of $M_{2\times2}$.

(b) Let

$$S = \begin{bmatrix} a-b & a \\ b+c & a-c \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$$

$a-b=5$ $\quad a=3$ $\quad b+c=-2$ $\quad a-c=3$

When $a=3$: $3-b=5$, $b=-2$

When $b=-2$: $-2+c=-2$, $c=0$

When $a=3$ and $c=0$: $3-0=3$ ($=3$)

$\therefore \begin{bmatrix} 5 & 3 \\ -2 & 3 \end{bmatrix}$ is in S.

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3. (a) Let

$$V_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \quad V_2 = \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} \quad V_3 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \quad V_4 = \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} \quad V_5 = \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix}$$

$$c_1V_1 + c_2V_2 + c_3V_3 + c_4V_4 + c_5V_5 = X$$

$$c_1 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 3 \\ -1 \end{bmatrix} + c_3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} + c_4 \begin{bmatrix} 0 \\ 6 \\ 1 \end{bmatrix} + c_5 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$$\left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 2 & 3 & 2 & 6 & 4 & x_2 \\ 3 & -1 & -1 & 1 & 5 & x_3 \end{array}\right] \xrightarrow{R_2 \to R_2 + (-2)R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 5 & 0 & 6 & 10 & x_2 - 2x_1 \\ 3 & -1 & -1 & 1 & 5 & x_3 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + (-3)R_1} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 5 & 0 & 6 & 10 & x_2 - 2x_1 \\ 0 & 2 & -4 & 1 & 14 & x_3 - 3x_1 \end{array}\right]$$

$$\xrightarrow{R_2 \to \frac{1}{5} R_2} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & \frac{1}{5}x_2 - \frac{2}{5}x_1 \\ 0 & 2 & -4 & 1 & 14 & x_3 - 3x_1 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + (-2)R_2} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & \frac{1}{5}x_2 - \frac{2}{5}x_1 \\ 0 & 0 & -4 & -\frac{7}{5} & 10 & x_3 - \frac{2}{5}x_2 - \frac{11}{5}x_1 \end{array}\right]$$

$$\xrightarrow{R_3 \to (-\frac{1}{4})R_3} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & x_1 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & \frac{1}{5}x_2 - \frac{2}{5}x_1 \\ 0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & -\frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1 \end{array}\right]$$

c_4 and c_5 are non-leading variable. Hence, let

$c_4 = s \qquad c_5 = t$

$$c_3 + \frac{7}{20}c_4 - \frac{5}{2}c_5 = -\frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1$$

$$c_3 + \frac{7}{20}s - \frac{5}{2}t = -\frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1$$

$$c_3 = \frac{5}{2}t - \frac{7}{20}s - \frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1$$

$$c_2 + \frac{6}{5}c_4 + 2c_5 = \frac{1}{5}x_2 - \frac{2}{5}x_1$$
$$c_2 + \frac{6}{5}s + 2t = \frac{1}{5}x_2 - \frac{2}{5}x_1$$
$$c_2 = \frac{1}{5}x_2 - \frac{2}{5}x_1 - \frac{6}{5}s - 2t$$

$$c_1 - c_2 + c_3 - 3c_5 = x_1$$
$$c_1 - \left(\frac{1}{5}x_2 - \frac{2}{5}x_1 - \frac{6}{5}s - 2t\right) + \left(\frac{5}{2}t - \frac{7}{20}s - \frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1\right) - 3t = x_1$$
$$c_1 - \frac{1}{5}x_2 + \frac{2}{5}x_1 + \frac{6}{5}s + 2t + \frac{5}{2}t - \frac{7}{20}s - \frac{1}{4}x_3 + \frac{1}{10}x_2 + \frac{11}{20}x_1 - 3t = x_1$$
$$c_1 - \frac{1}{10}x_2 + \frac{19}{20}x_1 - \frac{1}{4}x_3 + \frac{17}{20}s + \frac{3}{2}t = x_1$$
$$c_1 = \frac{1}{20}x_1 + \frac{1}{10}x_2 + \frac{1}{4}x_3 - \frac{17}{20}s - \frac{3}{2}t$$

$\because$ There are solution exists for all choice of x_1, x_2, x_3

Hence, span $(S) = \mathbb{R}^3$.

(b) $c_1v_1 + c_2v_2 + c_3v_3 + c_4v_4 + c_5v_5 = 0$

$$c_1\begin{bmatrix}1\\2\\3\end{bmatrix} + c_2\begin{bmatrix}-1\\3\\-1\end{bmatrix} + c_3\begin{bmatrix}1\\2\\-1\end{bmatrix} + c_4\begin{bmatrix}0\\6\\1\end{bmatrix} + c_5\begin{bmatrix}-3\\4\\5\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\2&3&2&6&4&0\\3&-1&-1&1&5&0\end{array}\right] \xrightarrow{R_2\to R_2+(-2)R_1} \left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&5&0&6&10&0\\3&-1&-1&1&5&0\end{array}\right]$$

$$\xrightarrow{R_3\to R_3+(-3)R_1} \left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&5&0&6&10&0\\0&2&-4&1&14&0\end{array}\right]$$

$$\xrightarrow{R_2\to \frac{1}{5}R_2} \left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&1&0&\frac{6}{5}&2&0\\0&2&-4&1&14&0\end{array}\right]$$

$$\xrightarrow{R_3\to R_3+(-2)R_2} \left[\begin{array}{ccccc|c}1&-1&1&0&-3&0\\0&1&0&\frac{6}{5}&2&0\\0&0&-4&-\frac{7}{5}&10&0\end{array}\right]$$

$$\xrightarrow{R_3 \to (-\frac{1}{4}) R_3} \left[\begin{array}{ccccc|c} 1 & -1 & 1 & 0 & -3 & 0 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & 0 \\ 0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + R_2} \left[\begin{array}{ccccc|c} 1 & 0 & 1 & \frac{6}{5} & -1 & 0 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & 0 \\ 0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & 0 \end{array}\right]$$

$$\xrightarrow{R_1 \to R_1 + (-1) R_3} \left[\begin{array}{ccccc|c} 1 & 0 & 0 & \frac{17}{20} & \frac{3}{2} & 0 \\ 0 & 1 & 0 & \frac{6}{5} & 2 & 0 \\ 0 & 0 & 1 & \frac{7}{20} & -\frac{5}{2} & 0 \end{array}\right]$$

Let $c_4 = s$, $c_5 = t$

$$c_3 + \frac{7}{20} c_4 - \frac{5}{2} c_5 = 0$$
$$c_3 + \frac{7}{20} s - \frac{5}{2} t = 0$$
$$c_3 = \frac{5}{2} t - \frac{7}{20} s$$

$$c_2 + \frac{6}{5} c_4 + 2 c_5 = 0$$
$$c_2 + \frac{6}{5} s + 2t = 0$$
$$c_2 = -\frac{6}{5} s - 2t$$

$$c_1 + \frac{17}{20} c_4 + \frac{3}{2} c_5 = 0$$
$$c_1 + \frac{17}{20} s + \frac{3}{2} t = 0$$
$$c_1 = -\frac{17}{20} s - \frac{3}{2} t$$

$\therefore$ Span $(S) = R^3$

$\therefore$ The set S is not lineary independent since there is no trivial solution.

Hence, S is not the basis of R^3.

4. (a)

$$c_1\begin{bmatrix}1\\1\end{bmatrix} + c_2\begin{bmatrix}3\\-1\end{bmatrix} = \begin{bmatrix}1\\0\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1 & 3 & 1\\1 & -1 & 0\end{array}\right] \xrightarrow{R_2 \to R_2 + (-1)R_1} \left[\begin{array}{cc|c}1 & 3 & 1\\0 & -4 & -1\end{array}\right]$$

$-4c_2 = -1$ $\qquad\qquad$ $c_1 + 3c_2 = 1$

$c_2 = \frac{1}{4}$ $\qquad\qquad$ $c_1 = 1 - 3(\frac{1}{4}) = \frac{1}{4}$

$\therefore$ coordinates of $\begin{bmatrix}1\\0\end{bmatrix}$ relative to B_2 is

$$[v]_{B_2} = \begin{bmatrix}c_1\\c_2\end{bmatrix} = \begin{bmatrix}\frac{1}{4}\\\frac{1}{4}\end{bmatrix}$$

(b)

$$a_1\begin{bmatrix}1\\-1\end{bmatrix} + a_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}1\\1\end{bmatrix} \qquad b_1\begin{bmatrix}1\\-1\end{bmatrix} + b_2\begin{bmatrix}0\\2\end{bmatrix} = \begin{bmatrix}3\\-1\end{bmatrix}$$

$$\left[\begin{array}{cc|c}1 & 0 & 1\\-1 & 2 & 1\end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{cc|c}1 & 0 & 1\\0 & 2 & 2\end{array}\right] \qquad \left[\begin{array}{cc|c}1 & 0 & 3\\-1 & 2 & -1\end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{cc|c}1 & 0 & 3\\0 & 2 & 2\end{array}\right]$$

$a_1 = 1$ $\qquad$ $a_2 = 1$ $\qquad\qquad$ $b_i = 3$ $\qquad$ $b_2 = 1$

$$[v_1]_{B_1} = \begin{bmatrix}1\\1\end{bmatrix} \qquad\qquad [v_2]_{B_1} = \begin{bmatrix}3\\1\end{bmatrix}$$

Therefore, the transition matrix is

$$[I]_{B_2}^{B_1} = \begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}$$

(c) $[v]_{B_1} = [I]_{B_2}^{B_1}[v]_{B_2}$

$$= \begin{bmatrix}1 & 3\\1 & 1\end{bmatrix}\begin{bmatrix}2\\3\end{bmatrix}$$

$$= \begin{bmatrix}11\\5\end{bmatrix}$$