SECI1113-01 MATEMATIK KOMPUTAN

(COMPUTATIONAL MATHEMATICS)

ASSIGNMENT 1

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Assignment 1

Question 1

substitute $x=1$, $y=-1$

① : $2x-y=3$
$2(1)-(-1)=3$
$3=3$

② : $x+3y=5$
$(1)+3(-1)=5$
$-2\neq 5$

Point $(1,-1)$ is not the solution to the given system of equation.

substitute $x=2$, $y=1$

① : $2x-y=3$
$2(2)-(1)=3$
$3=3$

② : $x+3y=5$
$(2)+3(1)=5$
$5=5$

Point $(2,1)$ is the solution to the given system of equation.

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Question 2

(a) $x - y = 2 \quad -(1)$

$2x - 2y = 4 \quad -(2)$

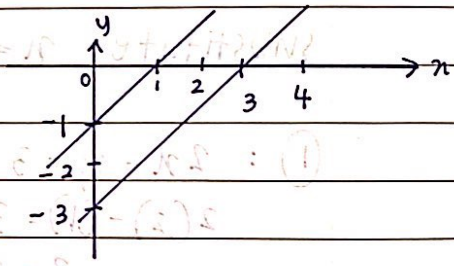
equation (2) $\div 2$: $2x - 2y = 4$

$x - y = 2 \quad -(3)$

(1) = (3) : Both equation is the samo and there is no intersection. Therefore, there is no solution.

(b) $x - y = 2 \quad -(1)$

$2x - 2y = 4 \quad -(2)$



equation (1) and equation (2) are parallel and there is no intersection. Therefore, there is no solution.

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Question 3

(a) $\det A = 1\begin{vmatrix} 1 & 1 \\ 1 & 2 \end{vmatrix} - 0\begin{vmatrix} 5 & 1 \\ 0 & 2 \end{vmatrix} + 3\begin{vmatrix} 5 & 1 \\ 0 & 1 \end{vmatrix}$

$= 1[1(2) - 1(1)] - 0 + 3[5(1) - 0(1)]$

$= 1(1) - 0 + 3(5)$

$= 16$

(b) $\det A = 1\begin{vmatrix} 0 & 1 \\ 1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 0 & -1 \end{vmatrix} + 0\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} =$

$= 1[0(-1) - 1(1)] + 1[-1(-1) - 0(1)] + 0[-1(-1) - 0(0)]$

$= 1(-1) + 1(1) + 0$

$= 0$

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Question 4

(a) $A^{-1} = \dfrac{1}{3(1)-0(0)} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix}$

(b) $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$\text{Det } A = 1 \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} - 0 \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} + 0 \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix}$

$= 1(1) - 0(0) + 0(0)$

$= 1$

$\det A_{11} = \begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1$ $\quad \det A_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$ $\quad \det A_{31} = \begin{vmatrix} 0 & 0 \\ 1 & -2 \end{vmatrix} = 0$

$\det A_{12} = \begin{vmatrix} 0 & -2 \\ 0 & 1 \end{vmatrix} = 0$ $\quad \det A_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$ $\quad \det A_{32} = \begin{vmatrix} 1 & 0 \\ 0 & -2 \end{vmatrix} = -2$

$\det A_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$ $\quad \det A_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$ $\quad \det A_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\text{cofactor matrix } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}$

$\text{Adjoint } A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -2 & 1 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$A^{-1} = \dfrac{1}{1} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

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Question 5

(a) $\left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 2 & -1 & 1 & 0 \\ 4 & -1 & 1 & 4 \end{array}\right] \xrightarrow{R_2 \to R_2 + (-2)R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 4 & -1 & 1 & 4 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-4)R_1} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & -9 & 13 & -32 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (\frac{-9}{5})R_2} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & \frac{2}{5} & \frac{2}{5} \end{array}\right]$

$\xrightarrow{R_3 \to \frac{5}{2}R_3} \left[\begin{array}{ccc|c} 1 & 2 & -3 & 9 \\ 0 & -5 & 7 & -18 \\ 0 & 0 & 1 & 1 \end{array}\right]$

$x_3 = 1$

$-5x_2 + 7x_3 = -18$
$-5x_2 + 7(1) = -18$
$x_2 = \frac{-25}{-5}$
$x_2 = 5$

$x_1 + 2x_2 - 3x_3 = 9$
$x_1 + 2(5) - 3(1) = 9$
$x_1 = 2$

(b) $\left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 3 & -7 & 4 & 10 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-3R_1)} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-10)R_2} \left[\begin{array}{ccc|c} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & 0 & -52 & -104 \end{array}\right]$

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$-52x_3 = -104$ $\quad -x_2 + 5x_3 = 9$ $\quad x_1 + x_2 + 2x_3 = 8$

$x_3 = 2$ $\quad -x_2 + 5(2) = 9$ $\quad x_1 + (1) + 2(2) = 8$

$x_2 = 1$ $\quad x_1 = 3$

Question 6

(a) $\left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ -1 & 2 & 1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right] \xrightarrow{R_2 \to R_2 + R_1} \left[\begin{array}{ccc|c} 1 & -3 & 2 & 0 \\ 0 & -1 & -1 & 0 \\ 2 & 4 & 6 & 0 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-2)R_1} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 10 & 10 & 0 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + 10R_2} \left[\begin{array}{ccc|c} 1 & -3 & -2 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\xrightarrow{R_1 \to R_1 + (-3)R_2} \left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$

$\therefore$ no solution.

(b) $\left[\begin{array}{cc|c} 2 & 1 & 3 \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right] \xrightarrow{R_1 \to \frac{1}{2}R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 4 & 1 & 7 \\ 2 & 5 & -1 \end{array}\right]$

$\xrightarrow{R_2 \to R_2 + (-4)R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & -1 \\ 2 & 5 & -1 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + (-2)R_1} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 1 \\ 0 & 4 & 4 \end{array}\right]$

$\xrightarrow{R_3 \to R_3 + 4R_2} \left[\begin{array}{cc|c} 1 & \frac{1}{2} & \frac{3}{2} \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right]$

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$$\xrightarrow{R_1 \to R_1 + \frac{1}{2}R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & -1 & 1 \\ 0 & 0 & 0 \end{array}\right]$$

$$\xrightarrow{R_2 \to -R_2} \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -1 \\ 0 & 0 & 0 \end{array}\right]$$

$\therefore r = 2, \; s = -1$

Question 7

(a) Doolittle

$$\underset{A}{\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}} \underset{X}{\begin{bmatrix} x \\ y \\ z \end{bmatrix}} = \underset{B}{\begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}}$$

$LU = A$

$$\begin{bmatrix} 1 & 0 & 0 \\ l_{21} & 1 & 0 \\ l_{31} & l_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$u_{11} = 2$

$l_{21} \cdot u_{11} = 8$

$2l_{21} = 8$

$l_{21} = 4$

$u_{12} = 1$

$u_{13} = 4$

$l_{13} \cdot 2 = 4$

$l_{13} = 2$

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$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & l_{32} & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$4 + u_{22} = -3$
$u_{22} = -7$

$2 + l_{32}(-7) = 11$
$-7 l_{32} = 9$
$l_{32} = 9/7$

$4 \cdot 4 + u_{23} = 2$
$u_{23} = -14$

$2(14) + 18 + u_{33} = -1$
$u_{33} = -27$

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -9/7 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$LY = B$ (forward subs)

$$\begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 2 & -\frac{9}{7} & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$y_1 = 12$

$4y_1 + y_2 = 20$
$4(12) + y_2 = 20$
$y_2 = -28$

$2y_1 + (-\frac{9}{7}) y_2 + y_3 = 33$
$2(12) - \frac{9}{7}(-28) + y_3 = 33$
$y_3 = -27$

$$y = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

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$UX = y$ (back subs)

$$\begin{bmatrix} 2 & 1 & 4 \\ 0 & -7 & -14 \\ 0 & 0 & -27 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ -28 \\ -27 \end{bmatrix}$$

$-27x_3 = -27$ $\quad$ $-7x_2 - 14x_3 = -28$ $\quad$ $2x_1 + x_2 + 4x_3 = 12$

$x_3 = -1$ $\quad$ $-7x_2 - 14(-1) = -28$ $\quad$ $2x_1 + 2 + 4(-1) = 12$

$x_2 = 2$ $\quad$ $x_1 = 3$

$x = [3 \quad 2 \quad 1]^T$

b) Crout

$AX = B$

$$\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$LU = A$

$$\begin{bmatrix} l_{11} & 0 & 0 \\ l_{21} & l_{22} & 0 \\ l_{31} & l_{32} & l_{33} \end{bmatrix} \begin{bmatrix} 1 & U_{12} & U_{13} \\ 0 & 1 & U_{23} \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}$$

$l_{11} = 2$ $\quad$ $l_{21} = 8$ $\quad$ $l_{31} = 4$

$l_{11} \cdot U_{12} = 1$ $\quad$ $l_{21} \cdot U_{12} + l_{22} = -3$

$2U_{12} = 1$ $\quad$ $8\left(\frac{1}{2}\right) + l_{22} = -3$

$U_{12} = \frac{1}{2}$ $\quad$ $l_{22} = -3 - 4$

$= -7$

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$4\left(\frac{1}{2}\right) + l_{32} = 11$

$l_{32} = 9$

$2 \cdot u_{13} = 4$

$u_{13} = 2$

$8(2) - 7u_{23} = 2$

$-7u_{23} = 2 - 16$

$u_{23} = 2$

$4(2) + 9(2) + l_{33} = -1$

$l_{33} = -27$

$$\underset{L}{\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -27 \end{bmatrix}} \underset{U}{\begin{bmatrix} 1 & 1/2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}} = \underset{A}{\begin{bmatrix} 2 & 1 & 4 \\ 8 & -3 & 2 \\ 4 & 11 & -1 \end{bmatrix}}$$

$LY = B$

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -7 & 0 \\ 4 & 9 & -2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 12 \\ 20 \\ 33 \end{bmatrix}$$

$2y_1 = 12$

$y_1 = 6$

$8y_1 - 7y_2 = 20$

$8(6) - 7(y_2) = 20$

$-7y_2 = -28$

$y_2 = 4$

$4y_1 + 9y_2 - 27y_3 = 33$

$4(6) + 9(4) - 27y_3 = 33$

$-27y_3 = -27$

$y_3 = 1$

$y = [6 \quad 4 \quad 1]^T$

$UX = Y$

$$\begin{bmatrix} 1 & 1/2 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ 4 \\ 1 \end{bmatrix}$$

$x_3 = 1$

$x_2 + 2x_3 = 4$

$x_2 + 2(1) = 4$

$x_2 = 2$

$x_1 + \frac{1}{2}x_2 + 2x_3 = 6$

$x_1 + \frac{1}{2}(2) + 2(1) = 6$

$x_1 = 3$

$X = [3 \quad 2 \quad 1]^T$

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Question 8

$d(u,v) = \sqrt{(\sqrt{2}-0)^2 + (1-2)^2 + (-1-2)^2}$

$= \sqrt{2+1+9}$

$= \sqrt{12}$

Question 9

(a) $u \cdot v = 2(3) + 1(3) + 2(1)$

$= 11$

(b) $u \times v = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 3 & 3 & 1 \end{vmatrix}$

$= \begin{vmatrix} 1 & 2 \\ 3 & 1 \end{vmatrix} i - \begin{vmatrix} 2 & 2 \\ 3 & 1 \end{vmatrix} j + \begin{vmatrix} 2 & 1 \\ 3 & 3 \end{vmatrix} k$

$= -5i + 4j + 3k$

$= (-5 \quad 4 \quad 3)$

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Question 10

(a) $V = C_1 U_1 + C_2 U_2 + C_3 U_3$

$$\begin{bmatrix} 1 \\ -2 \\ 5 \end{bmatrix} = C_1 \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + C_2 \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + C_3 \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 1 & 2 & -1 & -2 \\ 1 & 3 & 1 & 5 \end{array}\right] \quad R_2 \to R_2 + (-1)R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 1 & 3 & 1 & 5 \end{array}\right]$$

$$R_3 \to R_3 + (-1)R_1 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$$R_3 \to R_3 + (-2)R_2 \quad \left[\begin{array}{ccc|c} 1 & 1 & 2 & 1 \\ 0 & 1 & -3 & -3 \\ 0 & 0 & 5 & 10 \end{array}\right]$$

$5C_3 = 10$ $\quad$ $C_2 - 3C_3 = -3$ $\quad$ $C_1 + C_2 + 2C_3 = 1$

$C_3 = 2$ $\quad$ $C_2 - 3(2) = -3$ $\quad$ $C_1 + 3 + 2(2) = 1$

$C_2 = 3$ $\quad$ $C_1 = -6$

$\therefore V(1, -2, 5)$ is a linear combination

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(b) $C_1 u + C_2 v + C_3 w = 0$

$$C_1\begin{bmatrix}1\\2\\5\end{bmatrix} + C_2\begin{bmatrix}2\\5\\1\end{bmatrix} + C_3\begin{bmatrix}1\\5\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1&2&1&0\\2&5&5&0\\5&1&2&0\end{array}\right] \quad R_2 \to R_2 + (-2)R_1 \quad \left[\begin{array}{ccc|c}1&2&1&0\\0&1&3&0\\5&1&2&0\end{array}\right]$$

$$R_3 \to R_3 + (-5)R_1 \quad \left[\begin{array}{ccc|c}1&2&1&0\\0&1&3&0\\0&-9&-3&0\end{array}\right]$$

$$R_3 \to R_3 + 9R_2 \quad \left[\begin{array}{ccc|c}1&2&1&0\\0&1&3&0\\0&0&24&0\end{array}\right]$$

$$R_1 \to R_1 + (-2)R_2 \quad \left[\begin{array}{ccc|c}1&0&-5&0\\0&1&3&0\\0&0&24&0\end{array}\right]$$

$$R_3 \to \frac{1}{24}R_3 \quad \left[\begin{array}{ccc|c}1&0&-5&0\\0&1&3&0\\0&0&1&0\end{array}\right]$$

$$R_2 \to R_2 + (-3)R_3 \quad \left[\begin{array}{ccc|c}1&0&-5&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$

$C_1 = 0$, $C_2 = 0$, $C_3 = 0$

$\therefore$ The vectors are linearly independent because the system has only trivial solution.

$$R_1 \to R_1 + 5R_3 \quad \left[\begin{array}{ccc|c}1&0&0&0\\0&1&0&0\\0&0&1&0\end{array}\right]$$