SECI 1113
COMPUTATIONAL MATHEMATICS

ASSIGNMENT 5

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Assignment 5 (Answer)

Question 1

1. i) $f(1980)$

$f(x)$

Newton's Forward Difference table

k	x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$
0	1975	30.7	0.8	-6.2	3.7	5.2
1	1985	31.5	-5.4	-2.5	8.9	
2	1995	26.1	-7.9	6.4		
3	2005	18.2	-1.5			
4	2015	16.7				

$h = 1985 - 1975$

$h = 10$

$r = (x - x_0)/h = (1980 - 1975)/10 = 0.5$

$r = 0.5$

$$P_4(x) = f(x_0) + r\Delta f(x_0) + \frac{r(r-1)}{2!}\Delta^2 f(x_0) + \frac{r(r-1)(r-2)}{3!}\Delta^3 f(x_0) + \frac{r(r-1)(r-2)(r-3)}{4!}\Delta^4 f(x_0)$$

$$P_4(1980) = 30.7 + 0.5(0.8) + \frac{0.5(0.5-1)}{2!}(-6.2) + \frac{0.5(0.5-1)(0.5-2)}{3!}(3.7) + \frac{0.5(0.5-1)(0.5-2)(0.5-3)}{4!}(5.2)$$

$= 30.7 + 0.4 + 0.775 + 0.23125 + (-0.203125)$

$= 31.903125$

$= 31.9031$

$\therefore f(1980) \approx P_4(1980) = 31.9031$

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ii) $f(2010) = f(x)$

Newton's Backward Difference Table

k	x	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
0	1975	30.7				
1	1985	31.5	0.8			
2	1995	26.1	-5.4	-6.2		
3	2005	18.2	-7.9	-2.5	3.7	
4	2015	16.7	-1.5	6.4	8.9	5.2

$h = 1985 - 1975 = 10$

$r = (x - x_4)/h = (2010 - 2015)/10 = -0.5$

$$P_4(x) = f(x_4) + r\nabla f(x_4) + \frac{r(r+1)}{2!}\nabla^3 f(x_4) + \frac{r(r+1)(r+2)}{3!}\nabla^3 f(x_4) + \frac{r(r+1)(r+2)(r+3)}{4!}\nabla^4 f(x_4)$$

$$P_4(2010) = 16.7 + (-0.5)(-1.5) + \frac{(-0.5)(-0.5+1)}{2!}(6.4) + \frac{(-0.5)(-0.5+1)(-0.5+2)}{3!}(8.9)$$

$$+ \frac{(-0.5)(-0.5+1)(-0.5+2)(-0.5+3)}{4!}(5.2)$$

$$= 16.7 + 0.75 - 0.8 - 0.55625 - 0.203125$$

$$= 15.8906$$

$\therefore f(2010) \simeq P_4(2010) = 15.8906$

Question 2

$p(x) = a_0 + a_1 x \Rightarrow a_0$ and a_1 (two coefficients to solve) $\Rightarrow$ matrix size is 2×2 matrix

a) ① In matrix form

$$\begin{bmatrix} s_0 & s_1 \\ s_1 & s_2 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} v_0 \\ v_1 \end{bmatrix}$$

with $s_j = \sum_{k=0}^{8} x_k^j$; $j = 0,1,2$ and $v_l = \sum_{k=0}^{8} x_k^l f_k$, $l = 0,1$

②

	x_k^0	x_k^1	x_k^2	$x_k^0 f_k$	$x_k^1 f_k$
	1	1970	3880900	6	11820
	1	1975	3900625	8.6	16985
	1	1980	3920400	13.6	26928
	1	1985	3940225	18.7	37119.5
	1	1990	3960100	31.6	62884
	1	1995	3980025	53.2	106134
	1	2000	4000000	78.8	157600
	1	2005	4020025	100	200500
	1	2010	4040100	107.1	215271
$\sum_{k=0}^{8}$	9	17910	35642400	417.6	835241.5

$$\begin{bmatrix} 9 & 17910 \\ 17910 & 35642400 \end{bmatrix}\begin{bmatrix} a_0 \\ a_1 \end{bmatrix} = \begin{bmatrix} 417.6 \\ 835241.5 \end{bmatrix}$$

$9a_0 + 17910\, a_1 = 417.6 \rightarrow$ ①

$17910\, a_0 + 35642400 a_1 = 835241.5 \rightarrow$ ②

$a_0 = \dfrac{417.6 - 17910 a_1}{9} \rightarrow$ ③

Subs. ③ into ②

$$17910\left(\frac{417.6 - 17910\, a_1}{9}\right) + 35642400\, a_1 = 835241.5$$

$$\left(\left(\frac{7479216 - 320768100 a_1}{9}\right) + 35642400 a_1 = 835241.5\right) \times 9$$

$$7479216 - 320768100 a_1 + 320781600 a_1 = 7517173.5$$

$$13500 a_1 = 7517173.5 - 7479216$$

$$a_1 = \frac{37957.5}{13500}$$

$$a_1 = 2.8117$$

Subs. $a_1 = 2.8117$ into ③

$$a_0 = \frac{417.6 - 17910(2.8117)}{9} = -5548.883$$

Question 2

a) ③ $a_0 = -5548.883$, $a_1 = 2.8117$

④ $P(x) = -5548.883 + 2.8117x$

b) $P(1979) = -5548.883 + 2.8117(1979)$

$= 15.47$

$= 15.5$

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Question 3

① $\cancel{P_4(x) = L_0 x (y_0)}$

$$P_4(x) = L_0(x) y_0 + L_1(x) y_1 + L_2(x) y_2 + L_3(x) y_3 + L_4(x) y_4 = \sum_{i=0}^{4} L_i(x) y_i$$

with $L_i(x) = \prod_{\substack{j=0 \\ j \neq i}}^{4} \frac{(x - x_j)}{(x_i - x_j)}$

$$P_4(3.0) = \sum_{i=0}^{4} L_i(3.0) y_i \quad \text{with} \quad L_i(3.0) = \prod_{\substack{j=0 \\ j \neq i}}^{4} \frac{(3.0 - x_j)}{(x_i - x_j)}$$

② $L_0(3.0) = \frac{(3.0 - x_1)(3.0 - x_2)(3.0 - x_3)(3.0 - x_4)}{(x_0 - x_1)(x_0 - x_2)(x_0 - x_3)(x_0 - x_4)}$

$\cancel{= (3.0-1.5)(3.0-2.5)(3.0-4)}$

$= \frac{(3.0-2.5)(3.0-4)(3.0-5)(3.0-6.5)}{(1.5-2.5)(1.5-4)(1.5-5)(1.5-6.5)}$

$= -0.08$

$L_1(3.0) = \frac{(3.0 - x_0)(3.0 - x_2)(3.0 - x_3)(3.0 - x_4)}{(x_1 - x_0)(x_1 - x_2)(x_1 - x_3)(x_1 - x_4)}$

$= \frac{(3.0-1.5)(3.0-4)(3.0-5)(3.0-6.5)}{(2.5-1.5)(2.5-4)(2.5-5)(2.5-6.5)}$

$= 0.70$

$L_2(3.0) = \frac{(3.0 - x_0)(3.0 - x_1)(3.0 - x_3)(3.0 - x_4)}{(x_2 - x_0)(x_2 - x_1)(x_2 - x_3)(x_2 - x_4)}$

$= \frac{(3.0-1.5)(3.0-2.5)(3.0-5)(3.0-6.5)}{(4-1.5)(4-2.5)(4-5)(4-6.5)}$

$= 0.56$

$L_3(3.0) = \frac{(3.0 - x_0)(3.0 - x_1)(3.0 - x_2)(3.0 - x_4)}{(x_3 - x_0)(x_3 - x_1)(x_3 - x_2)(x_3 - x_4)}$

$= \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-6.5)}{(5-1.5)(5-2.5)(5-4)(5-6.5)}$

$= -0.20$

$L_4(3.0) = \frac{(3.0 - x_0)(3.0 - x_1)(3.0 - x_2)(3.0 - x_3)}{(x_4 - x_0)(x_4 - x_1)(x_4 - x_2)(x_4 - x_3)}$

$= \frac{(3.0-1.5)(3.0-2.5)(3.0-4)(3.0-5)}{(6.5-1.5)(6.5-2.5)(6.5-4)(6.5-5)}$

$= 0.02$

③ $\sum_{i=0}^{4} L_i(3.0) = L_0(3.0) + L_1(3.0) + L_2(3.0) + L_3(3.0) + L_4(3.0)$

$= -0.08 + 0.70 + 0.56 + (-0.20) + 0.02$

$= 1$

④ Thus, $P_4(3.0) = -0.08(0.455) + 0.70(1.615) + 0.56(3.750) - 0.20(5.333) + 0.02(7.857)$

$= \cancel{2.28464}$

$= 2.28464$

$\therefore y(3.0) \approx P_4(3.0) = 2.2846$

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Question 4

$h = 0.5$

Table

x	$f(x) = 2x^3 + x^2 - 4$
1.5	5
2.0	16
2.5	33.5
3.0	59
3.5	94

First derivative for $f'(2.5)$

a) ① Central three-point formula

$$f'(x_i) \simeq \frac{f(x_i+h) - f(x_i-h)}{2h}$$

$$f'(2.5) \approx \frac{f(2.5+0.5) - f(2.5-0.5)}{2(0.5)}$$

$$f'(2.5) = \frac{f(3.0) - f(2.0)}{1}$$

$$= \frac{59 - 16}{1}$$

$$= 43$$

② Central five-point formula

$$f'(x_i) \approx \frac{1}{12h}\left[f(x_i-2h) - 8f(x_i-h) + 8f(x_i+h) - f(x_i+2h)\right]$$

$$f'(2.5) \approx \frac{1}{12(0.50)}\left[f(2.5-2(0.5)) - 8f(2.5-0.5) + 8f(2.5+0.5) - f(2.5+2(0.5))\right]$$

$$= \frac{1}{6}\left[f(1.5) - 8f(2.0) + 8f(3.0) - f(3.5)\right]$$

$$= \frac{1}{6}\left[5 - 8(16) + 8(59) - 94\right]$$

$$= 42.5$$

Second derivative for $f''(2.5)$

① Central three-point formula

$$f''(x_i) \approx \frac{1}{h^2}\left[f(x_i-h) - 2f(x_i) + f(x_i+h)\right]$$

$$f''(2.5) \approx \frac{1}{(0.5)^2}\left[f(2.5-0.5) - 2f(2.5) + f(2.5+0.5)\right]$$

$$= \frac{1}{0.25}\left[f(2.0) - 2f(2.5) + f(3.0)\right] \Rightarrow = \frac{1}{0.25}\left[16 - 2(33.5) + 59\right] = 32$$

(2) Central five-point formula

$$f''(x_i) \approx \frac{1}{12h^2}\left[-f(x_i-2h)+16f(x_i-h)-30f(x_i)+16f(x_i+h)-f(x_i+2h)\right]$$

$$f''(2.5) \approx \frac{1}{12(0.5)^2}\left[-f(2.5-2(0.5))+16f(2.5-0.5)-30f(2.5)+16f(2.5+0.5)-f(2.5+2(0.5))\right]$$

$$= \frac{1}{3}\left[-f(1.5)+16f(2.0)-30f(2.5)+16f(3.0)-f(3.5)\right]$$

$$= \frac{1}{3}\left[-5+16(16)-30(33.5)+16(59)-94\right]$$

$$= 32$$

b) (1) exact value of $f'(2.5)$

$$f(x) = 2x^3+x^2-4 \rightarrow f'(x) = 6x^2+2x \rightarrow f''(x) = 12x+2$$

(1) exact value of $f'(2.5)$: $f'(2.5) = 6(2.5)^2+2(2.5)$
$= 42.5$

(2) exact value of $f''(2.5) = 12(2.5)+2$
$= 32$

c) First derivatives

Difference Formula	$f'(2.5)$ $h=0.5$	|error|
~~Three~~ Three-point central	43	0.5
Five-point central	42.5	0.0

Second derivative

Difference Formula	$h=0.5$ $f''(2.5)$	|error|
central difference three-point formula	32	0.0
Central difference five-point formula	32	0.0

Question 5

$\int_0^3 \frac{2}{1+5x^2}\,dx$

$a=0,\ b=3,\ N=6,\ h=\frac{b-a}{N}=\frac{3-0}{6}=0.5$

$h=0.5$

$f(x)=\frac{2}{1+5x^2}$

a) ~~Trapezoidal~~ Trapezoidal rule with N=6

① $a=0, b=3, N=6, h=0.5$

② Table for h=0.5

i	x_i	$f(x_i)=\frac{2}{1+5x_i^2}$	
0	0	2	
1	0.5		0.8889
2	1.0		0.3333
3	1.5		0.1633
4	2.0		0.0952
5	2.5		0.0620
6	3.0	0.0435	
	Total	2.0435	1.5427

$$\int_a^b f(x)\,dx=\frac{h}{2}\left[f_0+f_N+2\sum_{i=1}^{N-1}f_i\right]$$

$$\int_0^3 \frac{2}{1+5x^2}\,dx=\int_0^3 f(x)\,dx$$

$$\approx \frac{h}{2}\left[f_0+f_6+2(f_1+f_2+f_3+f_4+f_5)\right]$$

$$=\frac{0.5}{2}\left[2.0435+2(1.5427)\right]$$

$$=1.2822$$

b) Simpson's $\frac{1}{3}$ Formula with N=6

① $a=0, b=3, N=6, h=0.5$

② Table for h= 0.5

i	x_i	$f(x_i)=\frac{2}{1+5x_i^2}$		
0	0	2		
1	0.5		0.8889	
2	1.0			0.3333
3	1.5		0.1633	
4	2.0			0.0952
5	2.5		0.0620	
6	3.0	0.0435		
	Total	2.0435	1.1142	0.4285

③ $$\int_0^3 \frac{2}{1+5x^2}\,dx\approx\frac{h}{3}\left[(f_0+f_6)+4\sum_{i=1}^{3}f_{2i-1}+2\sum_{i=1}^{2}f_{2i}\right]$$

$$=\frac{0.5}{3}\left[2.0435+4(1.1142)+2(0.4285)\right]$$

$$=1.2262$$

c) Simpson's $\frac{3}{8}$ formula with $h = 0.25$

$a = 0, b = 3, h = 0.25, N = \frac{3.0}{0.25} = 12$
($N = 12$, must be multiple of 3).

① Table for $h = 0.25$

i	x_i	$f_i = f(x_i) = \frac{2}{1+5x_i^2}$		
0	0	2		
1	0.25		1.5238	
2	0.50		0.8889	
3	0.75			0.5246 → f_3
4	1.00		0.3333	
5	1.25		0.2270 ~~0.226~~	
6	1.50			0.1633 → f_6
7	1.75		0.1226	
8	2.00		0.0952	
9	2.25			0.0760 → f_9
10	2.50		0.0620	
11	2.75		0.0515	
12	3.00	0.0435		
Total		2.0435	3.3043	0.7639

② $\int_0^3 \frac{2}{1+5x^2}\,dx \approx \frac{3h}{8}\left[(f_0 + f_{12}) + 3\sum_{i=1}^{4}(f_{3i-2} + f_{3i-1}) + 2\sum_{i=1}^{3} f_{3i}\right]$

$= \frac{3(0.25)}{8}\left[2.0435 + 3(3.3043) + 2(0.7639)\right]$

$= 1.2641$

d) Romberg until R_{33}

$\int_0^3 \frac{2}{1+5x^2}\,dx$

① $h_1 = b - a = 3 - 0 = 3$
$h_1 = 3$

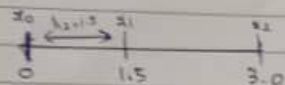
② i) $i = 1 \Rightarrow$ $x_0 = 0$, $h_1 = 3$, $x_1 = 3$ $\Rightarrow f(x) = \frac{2}{1+5x^2}$ → $f(x_0) = f(0) = \frac{2}{1+5(0)^2} = 2$
→ $f(x_1) = f(3) = \frac{2}{1+5(3)^2} = 0.0435$

ii) $R_{1,1} = \frac{h_1}{2}(f_0 + f_1)$
$= \frac{3}{2}(2 + 0.0435)$

$= 3.06525$

③ i) $R_{2,1} \Rightarrow i=2,\ N_2 = 2^{2-1} = 2,\ h_2 = \frac{1}{2}h_1 = \frac{1}{2}(3) = \frac{3}{2}$

$h_2 = 1.5$



$\Rightarrow f(x) = \frac{2}{1+5x^2}$

$f(x_1) = f(1) = f(1.5) = \frac{2}{1+5(1.5)^2}$

$f(1.5) = 0.1633$

ii) $R_{2,1} = \frac{1}{2}\left[R_{1,1} + h_1 f_1\right]$

$= \frac{1}{2}\left[3.06525 + 3(0.1633)\right]$

$= 1.777575 = 1.77758$

$= 1.77758$

iii) $R_{2,2} = \frac{4R_{2,1} - R_{1,1}}{4-1}$

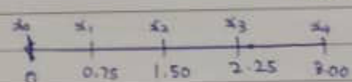
$\cancel{= \frac{4(1.7776) - 3.06525}{3}}$

$= \frac{4(1.77758) - 3.06525}{3}$

$= 1.34836$

④ i) $R_{3,1} \Rightarrow i=3,\ N_3 = 2^{3-1} = 4,\ h_3 = \frac{1}{2}h_2 = \frac{1}{2}(1.5) = 0.75$

$h_3 = 0.75$



$\Rightarrow f(x) = \frac{2}{1+5x^2}$

$f(x_1) = f(1) = f(0.75) = \frac{2}{1+5(0.75)^2} = 0.52459$

$f(x_3) = f(3) = f(2.25) = \frac{2}{1+5(2.25)^2} = 0.0760$

ii) $R_{3,1} = \frac{1}{2}\left[R_{2,1} + h_2(f_1 + f_2)\right]$

$= \frac{1}{2}\left[1.77758 + 1.5(0.52459 + 0.0760)\right]$

$= 1.33923$

iii) $R_{3,2} = \frac{4R_{3,1} - R_{2,1}}{3}$

$= \frac{4(1.33923) - 1.77758}{3}$

$= 1.19311$

iv) $R_{3,3} = \frac{4^2 R_{3,2} - R_{2,2}}{4^{3-1} - 1}$

$= \frac{16(1.19311) - 1.34836}{15}$

$= 1.16276$