SECI 1113
COMPUTATIONAL MATHEMATICS

ASSIGNMENT 4

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1. a.) Relative error $= \dfrac{2\,mph}{60\,mph}$

$= 0.0333 \times 100\%$

$= 3.33\%$

b) $f(x) = x$

$f(2) = 2$

$h = 0.1$

$f'(2) = \dfrac{f(2+0.1) - f(2)}{0.1}$

$= \dfrac{f(2.1) - f(2)}{0.1}$

$= \dfrac{2.1 - 2}{0.1}$

$= 1$

c.) $6.232 + 7.0321 + 5.43 - 3.385 = 15.3091$

$|E_s| \leq 0.0005 + 0.00005 + 0.005 + 0.0005$

$= 0.00605$

$15.3091 + 0.00605 = 15.31515$

$15.3091 - 0.00605 = 15.30305$

$\therefore$ The best approximate value is 15.3 (1 d.p.)

2. $a = 1$, $b = 2$

$f(a) = 1^6 - 1 - 1$

$= -1$

$f(b) = 2^6 - 2 - 1$

$= 61$

$f(a) \times f(b) = -1 \times 61$

$= -61$

$\therefore$ The intermediate theorem is satisfied and there is a root lies between $[1, 2]$ since 1

$f(a) \times f(b) < 0$.

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i	a	f(a)	b	f(b)	p	f(p)
1	1	-1	2	61	1.5	8.8906
2	1	-1	1.5	8.8906	1.25	1.5647
3	1	-1	1.25	1.5647	1.125	-0.0977
4	1.125	-0.0977	1.25	1.5647	1.1875	0.6167
5	1.125	-0.0977	1.1875	0.6166	1.1562	0.2327
6	1.125	-0.0977	1.1562	0.2327	1.1406	0.0613
7	1.125	-0.0977	1.1406	0.0613	1.1328	-0.0197
8	1.1328	-0.0197	1.1406	0.0613	1.1367	0.0204
9	1.1328	-0.0197	1.1367	0.0204	1.1348	0.0008
10	1.1328	-0.0197	1.1348	0.0204	1.1338	-0.0095
11	1.1338	-0.0095	1.1348	0.0204	1.1343	-0.0044
12	1.1343	-0.0044	1.1348	0.0204	1.1345	-0.0023

$\therefore x_{12} - x_{11} < 0.001$, threfore $p = P_{12} = 1.1345 \approx 1.13$ (2.d.p.) ✗

3. a.) $x_0 = 0$, $x_1 = -0.1$, $\varepsilon = 0.005$

$f(0) = \cos(0) + 2\sin(0) + 0^2$ $\quad$ $f(-0.1) = \cos(-0.1) + 2\sin(-0.1) + (-0.1)^2$

$= 1$ $\quad$ $= 0.8053$

i	P_{i-1}	P_i	P_{i+1}	$f(P_{i+1})$
1	0	-0.1	-0.5136	0.1521
2	-0.1	-0.5136	-0.6099	0.0461
3	-0.5136	-0.6099	-0.6518	0.0066
4	-0.6099	-0.6518	-0.6588	0.0004

$|f(P_5)| = 0.0004 < 0.005$

$P_5 = -0.6588$

$= -0.66$ (2.d.p.)

b.) $f(x) = \cos(x) + 2\sin x + x^2$, $f'(x) = -\sin(x) + 2\cos(x) + 2x$, $\varepsilon = 0.005$

i	P_i	$f(P_i)$	$f'(P_i)$
0	-0.1	0.8053	1.8898
1	-0.5261	0.1372	1.1795
2	-0.6424	0.0151	0.9156
3	-0.6589	0.0003	

$f(-0.6589) = 0.0003 (< 0.005)$

$P_3 = -0.6589$

$= -0.66$ (2.d.p.)

$\therefore$ Both Secant method and Newton method shows the same results.

4. i.) $f(x) = 1.37 - \frac{1}{x}$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$
$$= x_n - \frac{1.37 - \frac{1}{x_n}}{\frac{1}{x^2_n}}$$

$$= x_n - x^2_n\left(1.37 - \frac{1}{x_n}\right)$$
$$= x_n(2 - 1.37x_n) \text{ proven}$$

ii) $x_0 = 0.75$

$$x_1 = 0.75(2 - 1.37(0.75))$$
$$= 0.729375$$
$$x_2 = 0.75(2 - 1.37(0.729375))$$
$$= 0.729926589$$
$$x_3 = 0.75(2 - 1.37(0.729926589))$$
$$= 0.729927007$$
$$x_4 = 0.75(2 - 1.37(0.729927007))$$
$$= 0.729927007$$
$$|x_4 - x_3| < 0.00000005$$
$$x_4 = 0.729927007 = 0.72992701 \ (8 \text{ d.p.})$$

5. $S - \lambda I$

$$= \begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1-\lambda & -1 & 0 \\ -1 & 2-\lambda & -1 \\ 0 & -1 & 1-\lambda \end{bmatrix}$$

$$P(\lambda) = \det|S - \lambda I|$$
$$= 1-\lambda \begin{vmatrix} 2-\lambda & -1 \\ -1 & 1-\lambda \end{vmatrix} - (-1)\begin{vmatrix} -1 & -1 \\ 0 & 1-\lambda \end{vmatrix} + 0\begin{vmatrix} -1 & 2-\lambda \\ 0 & -1 \end{vmatrix}$$
$$= 1-\lambda(\lambda^2 + 3\lambda + 1) - (-1)(-1+\lambda) + 0$$
$$= \lambda^2 - 3\lambda + 1 - \lambda^3 - 3\lambda^2 - \lambda - 1 + \lambda$$
$$= -\lambda^3 + 4\lambda^2 - 3\lambda$$

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$-\lambda^3 + 4\lambda^2 - 3\lambda = 0$

$\lambda^3 - 4\lambda^2 + 3\lambda = 0$

$\lambda(\lambda-1)(\lambda-3) = 0$

$\lambda - 1 = 0 \quad \lambda - 3 = 0 \quad \lambda = 0$

$\lambda = 1 \quad \lambda = 3$

Eigen values of B: $\lambda = 0, \lambda = 1, \lambda = 3$.

$(S - \lambda I)V = 0$

when $\lambda = 0$

$$\left(\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix} - \begin{bmatrix}0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\right)\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}1 & -1 & 0 & 0\\ -1 & 2 & -1 & 0\\ 0 & -1 & 1 & 0\end{array}\right] R_2 \to R_2 + R_1$$

$$\left[\begin{array}{ccc|c}1 & -1 & 0 & 0\\ 0 & 1 & -1 & 0\\ 0 & -1 & 1 & 0\end{array}\right] \begin{array}{l}R_1 \to R_1 + R_2\\ \\ R_3 \to R_3 + R_2\end{array}$$

$$\left[\begin{array}{ccc|c}1 & 0 & -1 & 0\\ 0 & 1 & -1 & 0\\ 0 & 0 & 0 & 0\end{array}\right]$$

let $r = s$

$q - s = 0$

$q = s$

$p - s = 0$

$p = s$

$$\begin{bmatrix}p\\ q\\ r\end{bmatrix} = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}s$$

$$V_1 = \begin{bmatrix}1\\ 1\\ 1\end{bmatrix}$$ #

when $\lambda = 1$

$$\left(\begin{bmatrix}1 & -1 & 0\\ -1 & 2 & -1\\ 0 & -1 & 1\end{bmatrix} - \begin{bmatrix}1 & 0 & 0\\ 0 & 1 & 0\\ 0 & 0 & 1\end{bmatrix}\right)\begin{bmatrix}x\\ y\\ z\end{bmatrix} = \begin{bmatrix}0\\ 0\\ 0\end{bmatrix}$$

$$\left[\begin{array}{ccc|c}0 & -1 & 0 & 0\\ -1 & 1 & -1 & 0\\ 0 & -1 & 0 & 0\end{array}\right] R_1 \leftrightarrow R_2$$

$$\left[\begin{array}{ccc|c}-1 & 1 & -1 & 0\\ 0 & -1 & 0 & 0\\ 0 & -1 & 0 & 0\end{array}\right] R_1 \to R_1 \div -1$$

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$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] R_2 \to R_2 \div -1$$

$$\left[\begin{array}{ccc|c} 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{array}\right] \begin{array}{l} R_1 \to R_1 + R_2 \\ \\ R_3 \to R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $z = t$

$x + t = 0$

$x = -t$

$y = 0$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} t$$

$$V_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$ #

when $\lambda = 3$

$$\left(\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} \right) \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} -2 & -1 & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_1 \to R_1 \div -2$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ -1 & -1 & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_2 \to R_2 + R_1$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & -1 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] R_2 \to R_2 \div -\tfrac{1}{2}$$

$$\left[\begin{array}{ccc|c} 1 & \frac{1}{2} & 0 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & -1 & -2 & 0 \end{array}\right] \begin{array}{l} R_1 \to R_1 - \frac{1}{2} R_2 \\ \\ R_3 \to R_3 + R_2 \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 0 & -1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

let $c = u$

$a - u = 0$

$a = u$

$b + 2u = 0$

$b = -2u$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$

$$V_3 = \begin{bmatrix} 1 \\ -2 \\ 1 \end{bmatrix}$$ #

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6) a.) $C_1 = |\lambda - (-18)| < |20| + |-6|$

$= |\lambda + 18| < 26$

$= -44 < \lambda < 8$

$C_2 = |\lambda - 14| < |-12| + |-3|$

$= |\lambda - 14| < 15$

$= -1 < \lambda < 29$

$C_3 = |\lambda - 13| < |30| + |-30|$

$= |\lambda - 13| < 60$

$= -47 < \lambda < 73$

$\therefore$ Region containing all the eigenvalues of B is $[-47, 73]$.

b.)

k	$(v^{(k)})^T$			$(Bv^{(k)})^T$			M_{k+1}
0	0	0	1	−6	−3	13	13
1	−0.4615	−0.2308	1	−2.309	−0.6932	6.079	6.079
2	−0.3798	−0.1140	1	−1.4436	−0.0384	5.026	5.026
3	−0.2872	−0.0076	1	−0.9824	0.34	4.612	4.612
4	−0.2130	0.0737	1	−0.6920	0.5878	4.399	4.399
5	−0.1573	0.1336	1	−0.4966	0.7580	4.273	4.273
6	−0.1162	0.1774	1				

$|V_6 - V_5| < 0.05$

$M_6 = 4.2730 \qquad \lambda_1 = 4$

≈ 4

$V_1 \approx v^6 = (-0.1162, 0.1774, 1)^T$

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c) $A = B - \lambda I$

$$\begin{bmatrix} -18 & 20 & -6 \\ -12 & 14 & -3 \\ 30 & -30 & 13 \end{bmatrix} - 4\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -22 & 20 & -6 \\ -12 & 10 & -3 \\ 30 & -30 & 9 \end{bmatrix}$$

k	$(v^{(k)})^T$			$(Av^{(k)})^T$			M_{k+1}
0	0	0	1	-6	-3	9	9
1	-0.6667	-0.3333	1	2.0014	1.6674	-1.002	2.0014
2	1	0.8331	-0.5006	-2.3344	-2.1670	0.5016	-2.3344
3	1	0.9284	-0.2149	-2.1426	-2.0713	0.2139	-2.1426
4	1	-0.9667	-0.0998	-2.0672	-2.0336	0.1008	-2.0672
5	1	0.9837	-0.0488	-2.0332	-2.0166	-0.0498	-2.0332
6	1	0.9918	-0.0245				

$M_8 = -2.0332$ $\quad\quad \lambda_{shifted} = -2$

$\simeq -2$

$\lambda_3 = \lambda_{shifted} + \lambda_1$

$= -2 + 4$

$= 2$

$v_3 = [1, 0.9918, -0.0245]^T$

d) $-18 + 14 + 13 = 4 + \lambda_2 + 2$

$\lambda_2 = 3_{\#}$

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